

Nova Scotia Examinations Advanced Mathematics 12 Web Sample 1

# Student Booklet



Education Evaluation Services

#### General Instructions - WEB SAMPLE\*

This examination is composed of two sections with the following suggested time allotment:

Selected-Response (Multiple-Choice) Questions	Value 25 pts	(approx. 40 min)*
Constructed-Response Questions	Value 75 pts	(approx. 120 min)

\*note: there are 35 constructed response questions on the Math NSEs as of January 2008

Total time: 3 hours (revision time included)

Use these suggested times to guide you in the completion of the examination; however, you might not find it necessary to spend the suggested time on each section. Plan your time to enable you to complete the examination.

You are not permitted to use your own graphing calculator unless your teacher has cleared the memory immediately prior to this examination. The only graphing calculators permitted are TI-82, TI-83, TI-83 Plus, TI-84, or TI-84 Plus.

If the question indicates that you are not to use a graphing calculator, you are still permitted to use a calculator to perform arithmetic operations.

Calculators are not to be shared.

Graph paper, scrap paper, and formula sheets are provided at the end of this booklet. These pages can be removed from the booklet for your use during the examination.

Note: Diagrams are not necessarily drawn to scale.

#### Selected-Response Questions - WEB SAMPLE (Total Value: 25 points)

In this part of the examination, there are 25 selected-response questions\*, each with a value of 1 point. Read each question carefully, and decide which of the responses best answers the question being asked.

You are provided a separate student answer sheet. In the selected-response section of the student answer sheet, fill in the bubble that corresponds to your choice as shown in the example below. Use an HB pencil only.

Example

1. What are the roots of  $x^2 + 3x - 4 = 0$ ?

А.	4 and 1	В.	-4 and 3
C.	-4 and 1	D.	4 and 3

(On student answer sheet)



If you wish to change an answer, please ensure that you erase your first answer completely on the student answer sheet. Calculations or rough work on the selected-response pages of the examination booklet will not be scored.

\*Note: As of January 2008 there are 35 selected response questions on the NSE Math exams.

1. If in a sequence the second d	lifference is constant, then the sequence is
A arithmetic	B linear
C. quadratic	D. cubic
2. The equation of the axis of s	ymmetry of the parabola represented by the function $y = x^2 - 2$ is
A. $x = 2$	B. $x = -2$
C. $y = 2$	D. $x = 0$
3. Given the equation $x^2 + 48 =$	= 0, then $x$ is equal to
A. $\pm 4\sqrt{3}$	B. $\pm 4\sqrt{3}i$
C 6.9	D no possible value
0. 00	D. no possione tune
4. Which mapping rule describe	es how to map function $y_1$ onto the function $y_2$ ?
A. $(x, y) \to (x-3, 2y-5)$	B. $(x, y) \to (2x-3, y+5)$
C. $(x, y) \to (2x-3, y-5)$	D. $(x, y) \to (x-3, \frac{1}{2}y+5)$
5. For the function $y = ax^2 + bx$	x + c, the y-intercept is always
A. $-\frac{b}{2a}$	B. <i>c</i>

6. If the equation $2kx^2 - 4x + 1 = 0$ has two	equal roots (double root), then the value of $k$ is
A $k = 0$	в k>0
C. $k=2$	D. $k > 2$
7. An olympic diver dives from the high di of the water varies with the time, <i>t</i> , in se according to the equation $d = -2t^2 + 3t +$ during the dive? (rounded to the nearest	ving board. The distance, <i>d</i> , in metres, from the surface conds, that have passed since she left the board, -10. What is her maximum elevation above the water unit)
A. 3 metres	B. 10 metres
C. 11 metres	D. 12 metres
8. Which one of the following is the transfe	formational form of the function $y = 2(x-3)^2 + 5$ ?
A. $\frac{1}{2}y - 5 = (x - 3)^2$	B. $2(y-5) = (x-3)^2$
C. $\frac{1}{2}(y-5) = (x-3)^2$	D. $2y-5=(x-3)^2$
9. What value of b makes the expression $x$ A. $\frac{18}{25}$ C. $\frac{6}{5}$	$^{2} + bx + \frac{36}{25}$ a perfect square? B. $\frac{3}{5}$ D. $\frac{12}{5}$
<ul> <li>10. The students at East High School are platells two other students who in turn each people and so on. What type of sequence A. cubic</li> <li>C. quadratic</li> </ul>	<ul> <li>anning a surprise party for the principal. One student</li> <li>a tell two other people. These people each tell two other</li> <li>b geometric</li> <li>c arithmetic</li> </ul>
11 Which of the following is NOT a second	
11. Which of the following is NOT a geome $\left\{ 1  3  9 \right\}$	tric sequence?
A. $(10, 20, 40, \dots)$	<b>B.</b> $(2-7, 10, 10.5,)$
C. $\{0.8, 0.08, 0.008,\}$	D. $\{\frac{1}{2}, \frac{1}{3}, \frac{1}{4},\}$
12. What is the range of the function $y = 3(2)$	$(2)^{x+1} + 4?$
A. $\{y \in R \mid y < 7\}$	$\mathbf{B}.  \left\{ y \in \mathbf{R} \mid y \ge 4 \right\}$
C. $\{y \in R \mid y > 4\}$	$\mathbf{D.}  \left\{ y \in R  \middle   y > 10 \right\}$

13. Which of the following e	xpressions i	s equival	lent to 2	$x^{-\frac{3}{2}}$ ?		
A. $\frac{1}{2x^{\frac{3}{2}}}$			В. (	$\left(\frac{1}{2x}\right)^{\frac{2}{3}}$		
C. $\frac{2}{x^{\frac{3}{2}}}$			D. $\frac{1}{\sqrt{3}}$	$\frac{2}{\sqrt{x^2}}$		
14. Which expression is equi	valent to 21	$og\left(\frac{4}{x}\right),$	$x \neq 0$ , fo	or all poss	ible valu	ues of <i>x</i> ?
A. $\log 16 - 2\log x$			B. le	$\log 8 - 2 \log$	g x	
C. $\log 16 - \log 2x$			D. (	log 4 – log	$(gx)^2$	
15. The table given shows he interest is compounded a	ow two dolla nnually. Wl	rs invest nat is the	ed over approxi	a period o mate ann	of years gual intere	grows in value when est rate?
	x (years)	1	2	3	4	
	y (\$)	2.06	2.12	2.18	2.25	
A. 1.03% C. 6%			B. 3 D. 9	% .7%		
16. If $\log_x \left(\frac{1}{64}\right) = -\frac{3}{2}$ then x	is equal to					
A. 16			B. 8	1		
C. $\frac{1}{8}$			D. $\overline{1}$	<u>1</u> 6		
17. Which of the following r	elationships	cannot b	e model	led using	an expo	nential function?
A. growth of a population C. half-life of radioactiv	on e elements		B. c D. ti	ompound dal cycles	interest	
18. The Alaska quake of 196 measured 9.0. How many hundredth)	4 measured / times more	9.2 on the intense	e Richte was the	er scale, a Alaska qı	nd the Suake? (ro	ri Lanka quake of 2004 ounded to the nearest
A. 0.20 C. 1.02			B. 1 D. 1	.01 .58		



20. When the circle represented by  $x^2 + y^2 + 10x - 8y = 11$  is translated 7 units to the right, the new circle has its centre at

A.	(12, -4)	B.	(-12, 4)
C.	(-2, 4)	D.	(2,4)



A. 3	B. $\sqrt{14}$
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23. If an event can succeed in s wa	ys and fail in f ways, then the probability of success is
A. $\frac{s}{f}$	B. $\frac{s}{f+s}$
C. $s \times f$	D. $1-f$
24. Two dice are thrown. Given the what is the probability that the	at the sum of the two numbers on the pair of dice is greater than 7, se two numbers are the same?
A. $\frac{1}{5}$	B. $\frac{5}{12}$
C. $\frac{1}{12}$	D. $\frac{1}{2}$
25. In a school of 200 students, 80 the probability of selecting five	have blood type O. If 5 students are chosen at random, what is students with type O blood?
A. $\frac{1}{80} \times \frac{1}{79} \times \frac{1}{78} \times \frac{1}{77} \times \frac{1}{76}$	B. $\frac{5}{_{80}C_5}$
C. $\frac{5}{_{80}P_5}$	D. $\frac{{}_{80}C_5}{{}_{200}C_5}$

#### Constructed-Response Questions (Total Value: 75 points)

Read each question carefully, and be sure to write your response in the box and space provided. If the answer box indicates that you are to show your work, then points will be awarded for your correct work and your correct final answer. The method used to solve a problem must clearly be shown even when using a graphing calculator. If the answer box requires that just a final answer be provided, then points will be awarded for the correct answer only.

When working with decimal values, you may round off to the hundredths place in your final answer only. If any decimal values are rounded prior to the final step of the solution, at least 4 decimal places must be kept.

With the exception of the probability unit, all answers must be given in simplified form.

26. Given the graph below, do the following tasks without using the regression feature on your graphing calculator.



(a) Determine the general form of the function represented by the above graph.

(2 points)

Show your work above and write your conclusion or final answer in the box below.

(b) On the same grid as above, trace a parabola that has the same *x*-intercepts as the given parabola and a maximum value of 8. Indicate the coordinates of the vertex and 2 other points on the curve. (1.5 points)

27. Solve algebraically to find the exact roots of the following equations. Simplify where possible.

(a)  $3x^2 = -6x$ 

(1.5 points)

Show your work above and write your conclusion or final answer in the box below.

(b) 
$$\frac{3}{3-x} + \frac{4}{x+2} = 4$$

(3 points)

20		. 1 6 4 6 1	1 ' C'	<u> </u>		
28.	The number of dot	s in each of the fol	lowing fig	ures forms	a sequence.	
				•	••	
			•	••	• • • •	
			••	• • •	• • • •	
		• 5:	••	•••		
		rigure 1	rigure 2	rigure 3	rigure 4	
	(a) Algebraically d	etermine the funct	ion that ge	nerates the	sequence.	(3 points)
	Show your work a	above and write yo	ur conclus	ion or final a	answer in the box l	pelow.
	(b) If the figures co	ontinue to follow th	ne same pa	ttern, whic	h figure would cor	tain exactly 590
	dots?		I I I	,	8	(1 point
						(1 point)
		Einel Answer				
		rinal Answer				

- 29. A snowball is thrown into the air. The function  $h = -4.9t^2 + 20t + 1.8$  expresses the relationship between height, h, in metres and time, t, in seconds.
  - (a) Algebraically determine the maximum height the snowball reaches. (3 points)

Show your work above and write your conclusion or final answer in the box below.

(b) How long is the snowball in the air?

(2 points)

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30. A piece of land in the shape below has to be fenced. If 600 m of fencing are to be used, find the values of *x* and *y* that will produce a maximum area. (5 points)



31. Bill kicks a football in Tom's direction. The football follows a parabolic path. Tom is unaware that he may be standing in the football's path. After having travelled a horizontal distance of 10 metres, the football reaches a maximum height of 18 metres. Will Tom, who is 1.8 metres tall, get hit by the football if he's standing 1 metre from the spot where the football should hit the ground? Solve this problem algebraically.





(2 points)

Show your work above and write your conclusion or final answer in the box below.

(b)  $8(2)^{x+3} = 120$ 

(2 points)

Show your work above and write your conclusion or final answer in the box below.

(c)  $4^{2x} - 17(4^x) + 16 = 0$ 

(2 points)

33. (a) Describe in words how the graphs of  $y=b^x$  and  $y=b^{-x}$  for b>0, and  $b\neq 1$  are related. You must state a total of 3 similarities and/or differences. (3 points)

(b) Given the function  $y = ab^x$ , for what values of 'a' and 'b' will the graph of the function be an exponential growth curve? (2 points)

34. Susan tried to solve the equation  $x = \log_2(-3)$ . She got the error message 'NONREAL ANS' on her TI-83 calculator when trying to evaluate  $\log_2(-3)$ . Explain why. (2 points) 35. When Drug 1 enters the bloodstream, it gradually dilutes, decreasing by 20% every 5 days. A second drug, after entering the bloodstream, also decreases but by 10% every 7 days. If Billy takes 200 mg of Drug 1 and Maria takes 150 mg of Drug 2 at the same time, when will the amount of the drug remaining in their bloodstreams be the same? (5 points)

36. An accident at a nuclear reactor released 20 g/km<sup>2</sup> of a radioactive material in the area around the reactor. After 5 months the contamination had dropped to 10 g/km<sup>2</sup> and scientists agree that the level must drop below 2 g/km<sup>2</sup> before it is safe for human habitation. How long before the workers can safely return to work? (3.5 points)



38. A circle has a diameter with endpoints A (-3, 5) and B (15, -3). Determine whether the point (7, 8) is located inside, outside, or on the circumference of the circle.

(3 points)





(a) Determine the equation of the perpendicular bisector of chord  $\overline{AB}$ . (3 points)

Show your work above and write your conclusion or final answer in the box below.

(b) Determine algebraically if the perpendicular bisector of chord  $\overline{AB}$  passes through the point (-4, -2). (1 point)

40. Rewrite each equation in transformational form. State whether the equation represents a circle o
an ellipse. If it is a circle, state the centre and radius. If it is an ellipse, state the centre and the
lengths of the two axes.

(a) 
$$x^2 + y^2 - 8y = 1$$

(2 points)

Show your work above and write your conclusion or final answer in the box below.

(b) 
$$4x^2 + y^2 - 8x + 4y - 8 = 0$$

(3 points)

41. Refer to the follow	ving chart.			(3 points)
		Event A	Event A	
	Event B	10	20	
	Event B	15	25	
Calculate (a) P(A or B)				
	Final Answer			
(b) <b>P(A B)</b>				
	Final Answer			
42. From a group of 5	men and 6 women, w	hat is the probab	ility that a comm	nittee formed at random
42. From a group of 5 will consist of 3 m	men and 6 women, w	hat is the probab	ility that a comm	nittee formed at random (3.5 points)

43. Joe, Mary, and George are among the seven finalists for a random draw to win three different prizes. What is the probability that Joe will win 1<sup>st</sup> prize, Mary will win 2<sup>nd</sup> prize, and George will win 3<sup>rd</sup> prize? Express your answer in fraction form. (2 points)

44.	John,	Amy,	and	Fred	tried	to	solve	the	follo	owing	problem	:
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In a certain city, during a person's lifetime the probability of having diabetes is 0.10 and the probability of having cancer is 0.05. What is the probability of a person having either diabetes or cancer in his/her lifetime ?

Suppose that event C is 'person having cancer' and event D is 'person having diabetes'.

Their proposed solutions are as follows:

John's solution:  $P(C \text{ and } D) = 0.10 \times 0.05 = 0.005$ 

Amy's solution: P(C or D) = 0.10 + 0.05 = 0.15

Fred's solution: P(C or D) = 0.10 + 0.05 - 0.005 = 0.145

(a) Which student has the correct answer?

Final Answer

(b) Explain why the other two solutions are NOT correct.

You have reached the end of the ADVANCED WEB SAMPLE 1 Examination. Please check your work to ensure you have completed all questions.

(2 points)

(1 point)

#### **Formula Sheet – Advanced Mathematics 12**

#### **Quadratic Unit**

General form:  $y = ax^2 + bx + c$ Standard form:  $y = a(x-h)^2 + k$ Transformational form:  $\frac{1}{a}(y-k) = (x-h)^2$ If  $ax^2 + bx + c = 0$ , then  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ 

### **Exponential Growth Unit**

$$y = ab^{x}$$

$$A(y-C) = b^{B(x-D)}$$

$$\log_{a}(xy) = \log_{a} x + \log_{a} y$$

$$\log_{a}(x \div y) = \log_{a} x - \log_{a} y \text{ or } \log_{a}\left(\frac{x}{y}\right) = \log_{a} x - \log_{a} y$$

$$\log_{a} x^{b} = b(\log_{a} x)$$

#### **Circle Geometry Unit**

d =  $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ The coordinates of M are:  $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ General form:  $Ax^2 + Ay^2 + Dx + Ey + F = 0$ 

$$Ax^2 + By^2 + Dx + Ey + F = 0$$

Standard form:  $(x - h)^{2} + (y - k)^{2} = r^{2}$ 

Transformational form: 
$$\left[\frac{1}{r}(x-h)\right]^2 + \left[\frac{1}{r}(y-k)\right]^2 = 1$$
$$\left[\frac{1}{a}(x-h)\right]^2 + \left[\frac{1}{b}(y-k)\right]^2 = 1$$

$$m = \frac{\Delta y}{\Delta x}$$

#### **Probability Unit**

$$P(A \text{ and } B) = P(A) \times P(B)$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

$${}_{n}P_{r} = \frac{n!}{(n-r)!} \qquad {}_{n}C_{r} = \frac{n!}{r!(n-r)!}$$



Nova Scotia Examinations Advanced Mathematics 12

# Web Sample 1

# Marking Guide



Education Evaluation Services

Teacher Name: \_

# Selected Response Answers

1.	С	14. A
2.	D	15. B
3.	В	16. A
4.	А	17. D
5.	В	18. D
6.	С	19. B
7.	С	20. D
8.	С	21. A
9.	D	22. C
10.	В	23. B
11.	D	24. A
12.	С	25. D
13.	С	



# Question 26 (b)

(1.5 points)



## (1.5 points)



(3 points)



# Question 28 (a)

(3 points)



Question 28 (b)

(1 point)



# (3 points)

(2 points)



# Question 29 (b)

 $x = \frac{-20 \pm \sqrt{20^2 - 4(-4.9)(1.8)}}{2(-4.9)}$  $-\frac{1}{4.9}(0-22.2082) = (t-2.0408)^2$ 1 pt  $4.5323 = (t - 2.0408)^2$ 1 pt  $=\frac{-20\pm\sqrt{435.28}}{-9.8}$  $\pm 2.1289 = t - 2.0408$  $t = 2.0408 \pm 2.1289$ x=0.09 x = 4.171 pt t = 4.17t =>0.09 1 pt The snowball is in the air for 4.17 seconds. The snowball is in the air for 4.17 seconds. OR  $y = -4.9x^2 + 20x + 1.8$ 1 pt (4.17, 0)1 pt The snowball is in the air for 4.17 seconds.

### Question 30



# Question 31



# Question 32 (a)

(2 points)



# Question 32 (b)

#### $2^{x+3} = 15$ 0.5 pt $\log 2^{x+3} = \log 15$ \_\_\_\_\_\_ 0.5 pt 0.5 pt $2^{x+3} = 15$ $(x+3)\log 2 = \log 15 -$ 0.5 pt 0.5 pt $\log_2 15 = x + 3^{-1}$ $x+3 = \frac{\log 15}{\log 2}$ OR $3.9069 \doteq x + 3 =$ 0.5 pt $3.91 \doteq x$ -0.5 pt $x + 3 \doteq 3.9069$ $x \doteq 0.91$ 0.5 pt

Question 32 (c)

(2 points)

(2 points)



#### Note: The similarities and differences must refer to the graphs and <u>not</u> the equations.

Examples of acceptable answers (1 pt each):

- $\checkmark$  They share the same *y*-intercept.
- ✓ When  $y = b^x$  is increasing,  $y = b^{-x}$  is decreasing.
- ✓ Each is a reflection of the other in x = 0 (y axis).
- ✓ Same horizontal asymptote.
- ✓ Neither have *x*-intercepts.

# Question 33 (b)

# (2 points)



## Question 34

# (2 points)



# Question 35

(5 points)



# Question 36



(4 points)

Statement(s)	Reason(s)	
$\overline{MQ} \cong \overline{NP}$	Given	
$\angle KPQ \cong \angle KQP$	Given	
$\Delta KPQ$ is isosceles	Definition of an isosceles triangle	
$\overline{PK} \cong \overline{KQ}$	$\triangle KPQ$ is an isosceles triangle	
$\triangle MQK \cong \triangle NPK$	SAS	
$\overline{MK} \cong \overline{NK}$	CPCTC	
	OR	
Statement(s)	OR Reason(s)	
$Statement(s)$ $\overline{MQ} \cong \overline{NP}$	OR Reason(s) Given	
$Statement(s)$ $\overline{MQ} \cong \overline{NP}$ $\angle KPQ \cong \angle KQP$	OR       Reason(s)       Given       Given	
Statement(s) $\overline{MQ} \cong \overline{NP}$ $\angle KPQ \cong \angle KQP$ $\triangle KPQ$ is isosceles	OR       Reason(s)         Given       Given         Given       Definition of an isosceles triangle	
Statement(s) $\overline{MQ} \cong \overline{NP}$ $\angle KPQ \cong \angle KQP$ $\Delta KPQ$ is isosceles $\overline{PK} \cong \overline{KQ}$	OR       Reason(s)         Given       Given         Given       Definition of an isosceles triangle $\Delta KPQ$ is an isosceles triangle	
Statement(s) $\overline{MQ} \cong \overline{NP}$ $\angle KPQ \cong \angle KQP$ $\triangle KPQ$ is isosceles $\overline{PK} \cong \overline{KQ}$ $\angle KPM \cong \angle KQN$	OR       Reason(s)         Given       Given         Given       Definition of an isosceles triangle $\Delta KPQ$ is an isosceles triangle       Supplementary angles	
Statement(s) $\overline{MQ} \cong \overline{NP}$ $\angle KPQ \cong \angle KQP$ $\triangle KPQ$ is isosceles $\overline{PK} \cong \overline{KQ}$ $\angle KPM \cong \angle KQN$ $\overline{MP} \cong \overline{NQ}$	ORReason(s)GivenGivenDefinition of an isosceles triangle $\Delta KPQ$ is an isosceles triangleSupplementary anglesSegment subtraction	
Statement(s) $\overline{MQ} \cong \overline{NP}$ $\angle KPQ \cong \angle KQP$ $\triangle KPQ$ is isosceles $\overline{PK} \cong \overline{KQ}$ $\angle KPM \cong \angle KQN$ $\overline{MP} \cong \overline{NQ}$ $\triangle MPK \cong \triangle NQK$	ORReason(s)GivenGivenDefinition of an isosceles triangle $\Delta KPQ$ is an isosceles triangleSupplementary anglesSegment subtractionSAS	

# Question 38



## Question 39 (a)

# (3 points)



### Question 39 (b)

#### (1 point)



# Question 40 (a)



# Question 40 (b)

# (3 points)

$$4x^{2} - 8x + y^{2} + 4y = 8$$

$$4(x^{2} - 2x) + y^{2} + 4y = 8$$

$$4(x^{2} - 2x + 1) + y^{2} + 4y + 4 = 8 + 4 + 4$$

$$4(x - 1)^{2} + (y + 2)^{2} = 16$$

$$\left[\frac{(x - 1)}{2}\right]^{2} + \left[\frac{(y + 2)}{4}\right]^{2} = 1$$
Ellipse   
0.5 pt
Minor axis measures 4 units.
Centre (1, -2)   
0.5 pt
$$Centre (1, -2)$$

# Question 41 (a)

# (1.5 points)



## Question 41 (b)

(1.5 points)



# Question 42

(3.5 points)



# Question 43

(2 points)



# Question 44 (a)

# (1 point)

Question 44 (b)

# (2 points)

Note: Res	ponses should show understanding of the following:
$\checkmark$	John's solution is not correct because he calculated the probability of C and D not C or D.
$\checkmark$	Amy's solution considers C or D but includes cases where a person gets both.