Nova Scotia Assessment: Mathematics in Grade 3

Lessons Learned

> "For learners to succeed, teachers must assess students' individual abilities and characteristics and choose appropriate and effective instructional strategies accordingly."
>
> – Helene J. Sherman

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NOVA SCOTIA

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Purpose of this document

This *Lessons Learned* document was developed based on an analysis of the Item Description Reports for the 2018-2019 Nova Scotia Assessment: Mathematics in Grade 3 (M3). This document is intended to support all classroom teachers, (in particular at grades Primary–3), and administrators at the school, region, and provincial levels, in using the information gained from this assessment to inform next steps for numeracy instruction.

After the results for each mathematics assessment become available, an Item Description Report is developed to describe each item of the mathematics assessment in relation to the curriculum outcomes and cognitive processes involved with each mathematical strand. The percentage of students across the province who answered each item correctly is also connected to each item. Item description reports for mathematics are made available to school regions for distribution to schools, and they include provincial, regional, and school data. Schools and regions should examine their own data in relation to the provincial data for continued discussions, explorations, and support for mathematics focus at the classroom, school, region, and provincial levels.

This document specifically addresses areas that students across the province found challenging based on provincial assessment evidence. It is essential that teachers consider assessment evidence from a variety of sources to inform the next steps most appropriate for their students. Effective classroom instruction and assessment strategies are responsive to the individual learners within a classroom.

The M3 assessment generates information that is useful in guiding classroom-based instruction and assessment in mathematics. This document provides an overview of the mathematics tasks included in the assessment, information about this year's mathematics assessment results, and a series of Lessons Learned for mathematics. Suggested next steps for classroom instruction and assessment are included for each Lesson Learned.

Overview of the Nova Scotia Assessment: Mathematics in Grade 3

Nova Scotia Assessments are large-scale assessments that provide reliable data about how well all students in the province are learning the mathematics curricula. It is different from many standardized tests in that all questions are written by Nova Scotia teachers to align with curriculum outcomes and the results reflect a snapshot of how well students are learning these outcomes. These results can be counted on to provide a good picture of how well students are learning curriculum outcomes within schools, regions and in the province. Since the assessments are based on the Nova Scotia curriculum, and are developed by Nova Scotia teachers, results can be used to determine whether the curriculum, approaches to teaching and allocation of resources are effective. Furthermore, because individual student results are available, these, in conjunction with other classroom assessment evidence, help classroom teachers understand what each student has under control and identify next steps to inform instruction.

The assessment provides information about mathematics for each student and complements assessment data collected in the classroom. This assessment is administered at the end of Grade 3. It is designed to provide detailed information for every student in the province regarding their progress in achieving a selection of mathematics curriculum outcomes at the end of Grade 3. Information from this assessment can be used by teachers to inform their instruction and next steps in providing support and intervention for their students.

The design of the assessment includes the following:

- mathematical tasks that reflect a selection of outcomes aligned with the curriculum from the end of grade 1 to the end of grade 3 from across all strands of the mathematics curriculum
- due to the timing of the administration in late spring, questions specific to Unit 11 and Unit 12 (multiplication and division) in the Yearly Plan for grade 3 would not be reflected in the Mathematics/Mathématiques Assessment or *Lessons Learned* document.
- all items are in selected response format
- all items are designed to provide a broad range of challenge, thereby providing information about a range of individual student performance

Table 1: Specific Curriculum Outcomes Assessed in 2018–2019

Strand	**Specific Curriculum Outcomes**
Number (N)	2N03, 2N09, 3N02, 3N03, 3N04, 3N05, 3N08, 3N09, 3N13
Patterns and Relations (PR)	2PR01, 3PR01, 3PR02, 3PR03
Measurement (M)	3M01, 3M02, 3M03, 3M04, 3M05
Geometry (G)	2G02, 3G01, 3G02
Statistics and Probability (SP)	3SP01, 3SP02

Table 2: Specific Curriculum Outcomes Assessed in 2018–2019 by Grade Level

<table>
<tr><th colspan="3">SCO Assessed</th></tr>
<tr><td>2N03, 2N09, 2PR01, 2G02</td><td>Grade 2</td><td>17.4%</td></tr>
<tr><td>3N02, 3N03, 3N04, 3N05, 3N08, 3N09, 3N13
3PR01, 3PR02, 3PR03
3M01, 3M02, 3M03, 3M04, 3M05
3G01, 3G02
3SP01, 3SP02</td><td>Grade 3</td><td>82.6%</td></tr>
<tr><td colspan="2">Total = 23</td><td>100%</td></tr>
</table>

Cognitive levels of questions in mathematics are defined as:

- *Knowledge questions* require students to recall or recognize information, names, definitions, or steps in a procedure.
- *Application questions* require students to make connections, represent a situation in more than one way (translating between representations), or solve contextual problems.
- *Analysis questions* require students to go beyond comprehension and application to higher order thinking skills, such as generalizations and non-routine problem-solving.

Table 3: Distribution of Cognitive Level Questions

Cognitive Level Table of Specifications	
Cognitive Levels	**Percentage**
Knowledge	20–30%
Application	50–60%
Analysis	10–20%

These percentages are also recommended for well-balanced summative classroom-based assessments.

Please refer to Appendix A for further information about cognitive levels of questioning.

The Nova Scotia Assessment: Mathematics 3 includes 70 items distributed over two days for a duration of 60 minutes each day; 35 items on day one and 35 items on day two. The chart below shows the distribution, by mathematical strand and cognitive level, of items each day.

Table 4: Number of Items by Strand and Cognitive Level in 2018–2019

Number of Items Day 1				
	Knowledge	Application	Analysis	Total
Number	4	11	2	17
Patterns and Relations	1	2	1	4
Measurement	1	3	1	5
Geometry	1	3	1	5
Statistics and Probability	1	2	1	4
Number of Items Day 2				
	Knowledge	Application	Analysis	
Number	4	12	2	18
Patterns and Relations	1	3	1	5
Measurement	1	2	1	4
Geometry	1	2	1	4
Statistics and Probability	1	2	1	4
Total	16	42	12	70
Cognitive Level 2018-2019%	22.9%	60%	17.1%	100%
Table of Specifications	(20–30%)	(50–60%)	(10–20%)	

Performance Levels

Below are the Nova Scotia Assessment: Mathematics in Grade 3 Performance Levels

Level 1: Students at Level 1 can generally solve problems when they are simple and clearly stated or where the method to solve the problem is suggested to them. They rely on a limited number of strategies to solve problems. They can do addition and subtraction of whole numbers but may not understand when each operation should be used. They can recognize some mathematical terms and symbols, mainly from earlier grades. They may be able to represent a concept pictorially and concretely, such as place value.

Level 2: Students at Level 2 can generally solve problems similar to problems they have seen before. They depend on a few familiar methods to solve problems. They rely on strategies such as trial and error or guess and check rather than having a variety of strategies to choose from. They can do addition and subtraction of whole numbers and usually understand when each operation should be used. They can understand and use some mathematical terms and symbols, especially those from earlier grades. They can pictorially, concretely, and contextually represent a concept, such as place value.

Level 3: Students at Level 3 can generally solve problems that involve several steps and may solve problems they have not seen before. They can choose appropriate strategies to solve problems. They can apply number operations (+, –,) correctly and can judge whether an answer makes sense. They can understand and use many mathematical terms and symbols, including those at grade level. They pictorially, concretely, and contextually represent a concept, such as place value.

Level 4: Students at Level 4 can solve new and complex problems. They are consistent when choosing efficient strategies to solve problems. They can apply number operations (+, –) with confidence and ease. They can think carefully about whether an answer makes sense. They interpret and represents mathematical concepts using symbolic form with ease. They consistently use all representations with ease to represent a concept, such as place value.

Assessment Results

The Nova Scotia Assessment: Mathematics in Grade 3 was first administered in the 2018–2019 school year. The following is a breakdown of the 2018–2019 M3 results for each performance level (8279 grade 3 students participated in the M3 assessment):

- 72.4% of grade 3 students in the province have a performance level of 3 or above
- Performance Level 1: 11.9% of students in the province are below the expectations of this assessment
- Performance Level 2: 15.7% of students in the province are approaching the expectations of this assessment
- Performance Level 3: 51% of students in the province are at the expectations of this assessment
- Performance Level 4: 21.4% of students in the province are above the expectations of this assessment

The 2019-2020 and 2020-2021 Nova Scotia Assessments: Mathematics in Grade 3 (were not administered) due to Covid restrictions.

Mathematics in Grade 3 Lessons Learned

The assessment information gathered from the Nova Scotia Assessment: Mathematics in Grade 3 data has been organized into 8 ***Lessons Learned***:

- Translating Between and Among Representations
- Representing and Partitioning Whole Numbers
- Whole Number Operations
- Patterns and Relations
- Measurement
- Geometry
- Statistics and Probability.
- Problem Solving

Each Lesson Learned is divided into four sections that address the following questions

A. What conclusions can be drawn from the Nova Scotia Assessment: Mathematics in Grade 3?
B. Do students have any misconceptions or errors in their thinking?
C. What are the next steps in instruction for the class and for individual students?
D. What are the most appropriate methods and activities for assessing student learning?

Lessons Learned

1. **Translating Between and Among Representations**: Students did well when translating and moving flexibly between and among all representations of a concept. But some students still need to be encouraged to translate between words, pictures, or symbols. If asked to represent a question using representations, (words, symbols, or pictures), most students provide words, pictures, or symbols. Many students realize that they can use varied representations (words, pictures, or symbols) when solving a question.

2. **Representing and Partitioning Whole Numbers:** Students have improved when asked to apply their knowledge of basic facts and skills and to represent a situation or the steps in a procedure when given application questions. Students were successful when translating between representations of a concept (contextually, concretely, pictorially, symbolically, and verbally). When students are working to partition whole numbers and to perform operations, it is very important for students to understand that numbers can be broken down into two or more parts in many ways.

3. **Whole Number Operations**: Students were challenged when asked to apply basic skills, knowledge, and computational procedures to application and analysis questions. Students need to be able to apply the higher order thinking skills of problem solving, creativity, and reasoning to do application and analysis items. Students should have experiences with all the story structures for addition and subtraction. Students also need to be encouraged to estimate before calculating an answer to a question.

4. **Patterns and Relations:** Students were challenged when asked to transfer their visual representation of patterns to numerical patterns. Students improved when asked to describe either an increasing pattern or a decreasing pattern, and need to recognize that each term has a numeric value. They still have difficulty when asked to identify in an increasing pattern a specific element. Students seemed to forget that a pattern rule must have a starting point. Without the starting point, the pattern rule is incomplete. They need to be encouraged to provide all the information for a pattern rule. Students need to continue to work with representations of patterns, concretely, contextually, pictorially, symbolically, and verbally.

5. **Measurement**: Students are expected to build conceptual understanding of what it means to measure with a ruler. Emphasis should be placed on counting the intervals between the numbers, rather than looking at the number on the ruler that is aligned with the end of the object. Students also need to use a ruler to measure the length of a pencil or other objects with and without using zero as the starting point. Students need to recognize which mass unit (gram or kilogram) is appropriate for measuring and comparing the mass of a specific item. Students need to be able to explain that perimeter is the distance around a shape or the length of the boundary of an enclosed region. Students need to find the perimeter of many different regular, and composite shapes, before being introduced to questions in pictorial form. Students need to work with perimeter in application and analysis questions.

6. **Geometry**: Students need to continue developing their knowledge of 2-D shapes and 3-D objects by describing and sorting them according to their geometric attributes. Students need experiences to identify the faces, edges, and vertices of 3-D objects including spheres, cones, cylinders, pyramids, cubes, and other prisms. Students need to be provided with opportunities to explore these attributes through sorting and constructing activities. Students need to extend their knowledge of regular polygons. They need to focus on comparing the number of sides as the key attribute for classifying polygons. Students need more experiences with regular polygons, so that they begin to realize that a polygon, regardless of its dimensions, remains the same shape.

7. **Statistics and Probability:** Students were challenged using tally marks, lists, charts, line plots, and bar graphs to organize data relevant to their everyday life. Students need opportunities and experiences to interpret information collected, organized, and displayed in tally charts, charts, line plots and bar graphs. Students need to develop the skill of interpreting graphs and answering questions and drawing conclusions from those tally charts, line plots and bar graphs. They need to be encouraged to ask or write questions that go beyond simplistic reading of a graph. Both literal and inferential comprehension questions need to be asked.

8. **Problem Solving:** Students need more exposure to application and analysis items in order to apply these higher order thinking skills when problem solving. Students need to be encouraged to understand that problems may have more than one entry point and there are many strategies to solve a problem. Students need to continue to work on translating between and among representations when problem solving. They do not seem to realize that they can use varied representations when asked to solve a word problem. These other representations may support their problem solving and their reasoning.

Key Messages

The following key messages should be considered when using this document to inform classroom instruction and assessment.

The Nova Scotia mathematics curriculum is based upon several key assumptions or beliefs about mathematics learning that have grown out of research and practice. These beliefs include the following:

- Mathematics learning is an active and constructive process.
- Learning is most effective when standards of expectation are made clear with ongoing assessment and feedback.
- Learners are individuals who bring a wide range of prior knowledge and experiences and who learn via various styles and at different rates.
- Learning is most likely to occur when placed in meaningful contexts and in an environment that supports exploration, risk taking, and critical thinking and that nurtures positive attitudes and sustained effort.
- Students are curious, active learners with individual interests, abilities, and needs. They come to classrooms with varying knowledge, life experiences, and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.
- Students learn by attaching meaning to what they do and need to construct their own meaning of mathematics. This meaning is best constructed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of models and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students and enhance the formation of sound, transferable, mathematical concepts. At all levels, students benefit from working with and translating through a variety of materials, tools, and contexts when constructing meaning about new mathematical ideas. Meaningful discussions can provide essential links among concrete, pictorial, contextual, and symbolic representations of mathematics.
  (EECD, 2013b, p. 23)

Research has shown that assessment for learning practices produce significant and often substantial learning gains, close achievement gaps, and build students' ability to learn new skills (Black & Wiliam 1998; OECD 2006). Student involvement in assessment promotes learning. Timely and effective teacher feedback and student self-assessment allow students to reflect on and articulate their understanding of mathematical concepts and ideas.

- Provincial assessment results form part of the larger picture of assessment for each student and complements assessment data collected in the classroom. Ongoing assessment for learning is essential to effective teaching and learning. Assessment for learning can and should happen every day as part of classroom instruction. Assessment of learning should also occur regularly and at the end of a cycle of learning. Both forms of assessment are necessary to guide teaching, stimulate learning, and produce achievement gains.
- It is important to construct assessment activities that require students to complete tasks across the cognitive levels. While it is important for students to be able to answer factual and procedural type questions, it is also important to embed activities that require strategic reasoning and problem-solving.

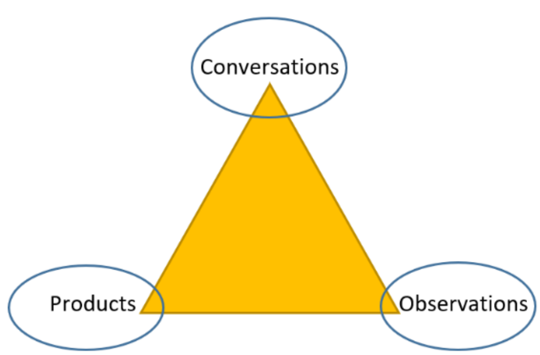
- Ongoing assessment for learning involves the teacher focusing on how learning is progressing during the lesson and the unit, determining where improvements can be made and identifying the next steps. "*Such assessment becomes 'formative assessment' when the evidence is actually used to adapt the teaching to meet learning needs," (*Black, Harrison, Lee, Marshall & Wiliam, 2003, p.2). Effective strategies of assessment for learning during a lesson include strategic questioning, observing, conversing (conferring with students to "hear their thinking"), analyzing student's work (product), engaging students in reviewing their progress, as well as providing opportunities for peer and self-assessment.
- Assessment of learning involves the process of collecting and interpreting evidence for the purpose of summarizing learning at a given point in time and making judgments about the quality of student learning on the basis of established criteria. The information gathered may be used to communicate the student's achievement to students, parents, and others.
- All forms of assessment should be planned with the end in mind, thinking about the following questions:
    - What do I want students to learn? (identifying clear learning targets)
    - What does the learning look like? (identifying clear criteria for success)
    - How will I know they are learning?
    - How will I design the learning so that all will learn?
- Before planning for instruction using the suggestions for instruction and assessment, it is important that teachers review individual student results in conjunction with current mathematics assessment information. A variety of current classroom assessments should be analyzed to determine specific strengths and areas for continued instructional focus or support.

Balanced Assessment in Mathematics: Effective ways to gather information about a student's mathematical understanding

- Conversations/Conferences/Interviews: Individual, Group, Teacher-initiated, Child-initiated
- Products/Work Samples: Mathematics journals, Portfolios, Drawings, Charts, Tables, Graphs, Individual and classroom assessment, Pencil-and-paper tests, Surveys, Self-assessment
- Observations: Planned (formal), Unplanned (informal), Read-aloud (literature with mathematics focus), Shared and guided mathematics activities, Performance tasks, Individual conferences, Anecdotal records, Checklists, Interactive activities

Department of Education and Early Childhood Development (EECD), Province of Nova Scotia (2013a). *Mathematics 1 curriculum guide, implementation draft*. Halifax, NS: Author.
(EECD, 2013a, p. 4)

"Triangulation increases the reliability and validity of student learning assessment and facilitates the implementation of pedagogical differentiation. Using triangulation, we take into account all learning styles and we engage all students, including those who have difficulty expressing themselves in writing and those who do not have the ability to undertake a written assessment task to demonstrate their learning." — Anne Davies (*Free Translation)*



Mathematics in Grade 3 Lesson Learned 1
Translating Between and Among Representations

Students did well when translating and moving flexibly between and among all representations of a concept. Students still need to be encouraged to translate between words, pictures, or symbols. If asked to represent a question using representations, (words, symbols, or pictures), most students provide words, pictures, or symbols. Many students realize that they can use varied representations (words, pictures, or symbols) when solving a question.

Mathematics should be viewed as an integrated whole rather than as the study of separate strands or units. Connections must also be made between and among the different representational modes – contextual, concrete, pictorial, linguistic/verbal, and symbolic. The process of making connections, in turn, facilitates a child's learning.

"Children who have difficulty translating a concept from one representation to another are the same children who have difficulty solving problems and understanding computations. Strengthening the ability to move between and among these representations improves the growth of children's concepts" (Van De Walle, John A. 2001, *Elementary and Middle School Mathematics*, Fourth Edition, p. 34).

One way to encourage children to use multiple representations is to explicitly ask for them.

Ask questions such as

- How many ways can you show the number 20 using words, pictures, and numbers?
- How many ways can you represent 75?
- Can you represent a rectangle as a combination of other shapes?
- Can you represent this line plot as a bar graph?
- Can you use an equation to represent how you thought about this story problem?

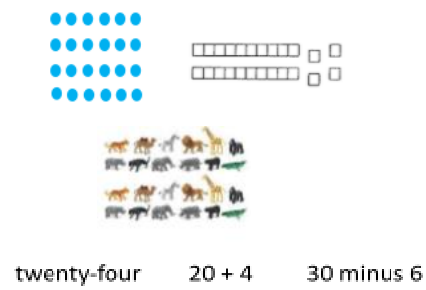
A. What conclusions can be drawn from the Nova Scotia Assessment: Mathematics in Grade 3?

Historically, our students were challenged when asked to translate between and among representations of a concept. Students now have translating between and among representation well under control. We found that students have a good understanding of basic facts and procedures, but when given application items, they appear to want to rush to symbolic. For example, when problem solving, students are able to understand the context of the question, but many are not able to translate between the representations (translating from words to pictures or symbolic to pictures, etc.).

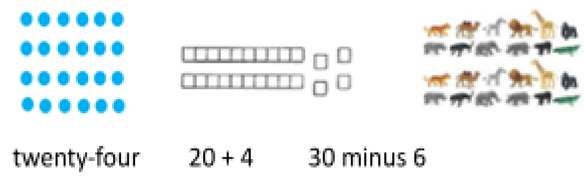
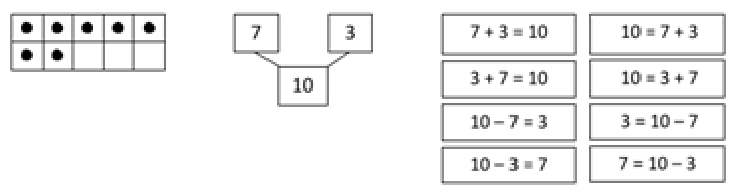
B. Do students have any misconceptions or errors in their thinking?

It is important to distinguish between misconceptions and errors. Misconceptions are, in effect, the misunderstandings about mathematical ideas which students entertain, and which usually lead to errors occurring. These are the most serious kinds of errors which require urgent action on the part of the teacher. Errors may occur for a variety of underlying reasons, ranging from careless mistakes to errors resulting from misconceptions. – John SanGiovanni – (*Mine The Gap For Mathematical Understanding Grades K-2*)

Results of the 2018–2019 Nova Scotia Assessment Mathematics in Grade 3
● most students answered questions correctly between and among representations

Common Misconceptions/Errors	What it Looks Like
Translating Between and Among Representations	
If asked to represent a question using words, symbols, or pictures, students usually only provided symbols.	Students do not recognize the various representations they can use when solving a problem. 
Students have difficulty with application questions concerning the perimeter of a shape when the shape is not pictured.	Some students are uncertain about how many sides the named shape would have in a question such as: *The perimeter of a hexagon is 24 cm. How long is each side?*
Students should be encouraged to translate between the name of the shape (words) and a picture. Students could draw the picture and check how many sides the named shape has.	An example of this type of problem could be: I am a 3-D object. I have 5 faces. I have 5 vertices. I have 8 edges. What shape am I?

C. What are the next steps in instruction for the class and for individual students?

	How to Support
Representations to Communicate Mathematical Ideas The five representations of a concept are contextual, concrete (two-sided counters, base-ten materials, etc., pictorial (drawing, number line, ten frame), symbolic, and verbal (written/oral) Activity shown is taken from *All About Ten* (Teaching Children Mathematics. NCTM, August 2010, p. 44)	● Provide opportunities for students to use multiple representations to communicate mathematical ideas. ● For example, the number 24 can be represented in many ways:  ● **Instructional Strategies** Present problems that require students to translate between parts of representations.  ● Provide experiences in which students are selecting, applying, modeling, and translating among mathematical representations to solve problems. ● Implement instructional strategies that support students' development of representational competency. These include: ○ Engaging in dialogue about the explicit connections between representations ○ Alternating directionality of the connections made among representations. The **directionality** of the connections made between the representations and the problem situation is another important feature of representational competence. ○ Encouraging purposeful selection of representation. (Teaching Children Mathematics. NCTM, August 2010, p. 40).

D. What are the most appropriate methods and activities for assessing student learning?

Questions from the strands will be used to represent some of the appropriate methods and activities for assessing student learning.

Encourage students to share their thinking, their strategies, and their solutions. Rich discussion fosters the development of alternative problem-solving strategies.

<table>
<tr><th colspan="2">Cognitive Level of Questions Examples
Translating Between and Among Representations</th></tr>
<tr><td>1. Knowledge (3M03)

Peter found a stick that was more than 1 m long.

How many centimetres long could the stick be?

A. 80 cm
B. 90 cm
C. 100 cm
D. 115 cm

Show what you know using words, pictures, or symbols.

Correct Answer: D</td><td>2. Application (3M05)

The perimeter of a pentagon is 20 cm.

How long is each side?

A. 4 cm
B. 5 cm
C. 20 cm
D. 25 cm

Show what you know using words, pictures, or symbols.

Correct Answer: A</td></tr>
<tr><td>3. Analysis (3G01)

I am a 3-D object.
I have 5 faces.
I have 5 vertices.
I have 8 edges.

Which shape am I?

A. cube
B. sphere
C. square-based pyramid
D. triangular-based pyramid

Show what you know using words, pictures, or symbols.

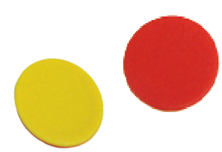
Correct Answer: C</td><td>4. Application (2N09)

You have 29 counters.

You give your friend 14 counters.

How many counters do you have now?

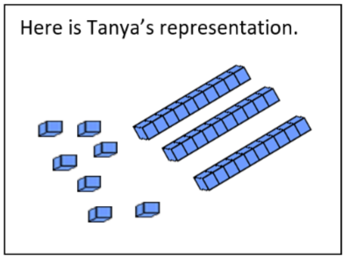
Show what you know using words, pictures, or symbols.

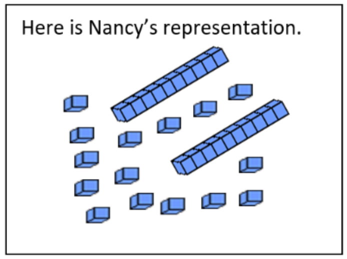


Correct Answer: 15 counters</td></tr>
</table>

5. Application: (2PR04/PR03)

Tanya and Nancy used base-ten blocks to represent numbers.





Write number sentences for each of the representations in the pictures shown. Then compare the sum of the numbers using the equal sign (=) or the not equal sign (≠).

Show what you know using words, pictures, or symbols.

Correct Answer: 30 + 7 ≠ 20 + 15

6. Application (a, b, c)/Analysis (d) (2N04)

Show what you know using pictures, words, or symbols for the following questions:

Materials required for this question: one full piece of paper, 20 cubes (cube-a-links)

Questions for the student:

a) Write the number 17 on a piece of paper. Now turn/flip your paper over.

b) On the blank side of your paper that you turned over, show the number 17 using your cubes in two parts.

c) Now draw a picture of your arrangement of the cubes used to represent 17 on the blank piece of paper.

d) Can you show the number 17 represented another way using the cubes?

Correct Answers:

a) 17

b) 11 cubes + 6 cubes (many combinations that add up to 17)

c) 12 cubes + 5 cubes (many combinations that add up to 17)

d) many combinations that add up to 17

Mathematics in Grade 3 Lesson Learned 2
Representing and Partitioning Whole Numbers

Students have improved when asked to apply their knowledge of basic facts and skills and to represent a situation or the steps in a procedure when given application questions. Students were successful when translating between representations of a concept (contextually, concretely, pictorially, symbolically, and verbally). When students are working to partition whole numbers and to perform operations, it is very important for students to understand that numbers can be broken down into two or more parts in many ways.

A. What conclusions can be drawn from the Nova Scotia Assessment: Mathematics in Grade 3?

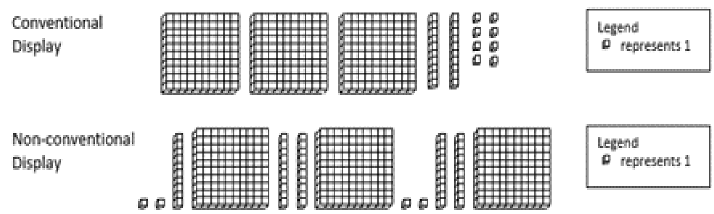
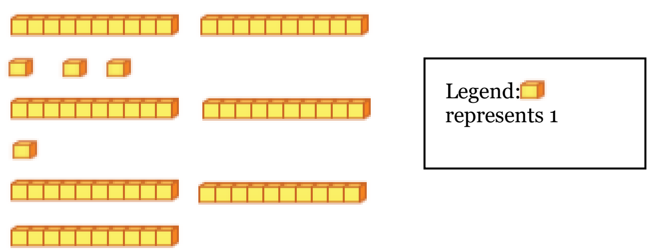
Students were able to do well when explicitly given all the information needed to do the question. Students performed well on knowledge questions that required them to use basic skills, symbolic procedures, and factual knowledge. For example, when asked to choose the number that is equal to thirty-one tens (310), most students were able to find the correct answer. Students were successful problem solvers and performed well on questions that required analysis and non-routine problem solving.

However, our assessment information also shows that many students experienced challenges with application questions. Our students were challenged when asked to apply their knowledge of basic facts and skills to a context. They also struggled with translating between representations of a concept (contextually, concretely, pictorially, symbolically, and verbally) when asked to solve a story problem.

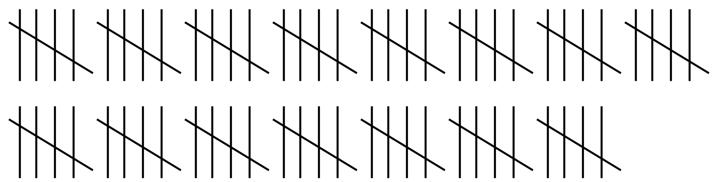
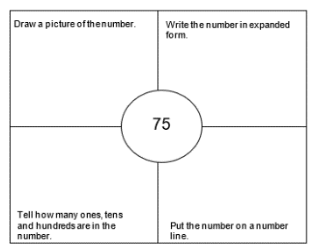
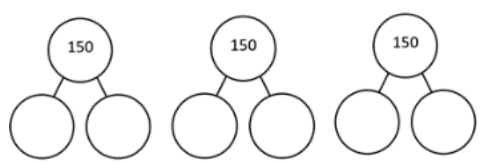
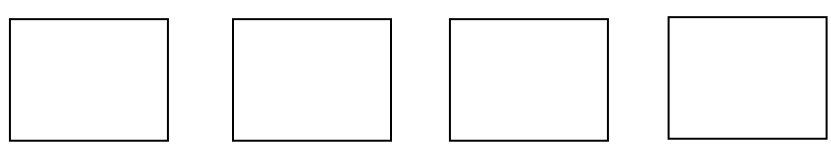
B. Do students have any misconceptions or errors in their thinking?

It is important to distinguish between misconceptions and errors. Misconceptions are, in effect, the misunderstandings about mathematical ideas which students entertain, and which usually lead to errors occurring. These are the most serious kind of errors which require urgent action on the part of the teacher. Errors may occur for a variety of underlying reasons, ranging from careless mistakes to errors resulting from misconceptions. - John SanGiovanni – (*Mine The Gap For Mathematical Understanding Grades K-2*)

Results of the 2018–2019 Nova Scotia Assessment Mathematics in Grade 3
● most students answered questions correctly when representing and partitioning whole numbers, for example, students represented a number using base-ten blocks in a conventional display.
● 53% of the students had difficulty using place-value strategies to represent symbolically a number written in words

Common Misconception/Errors	What it Looks Like
Representing and Partitioning Whole Numbers	
Conventional and non-conventional displays of base-ten blocks • It is important that students have opportunities to view and create numbers using conventional and non-conventional displays of base-ten blocks.	
Representing Numbers non-conventionally	 Students may not recognize that this represents 74.
Partitioning Numbers using a Variety of Ways and Different Expressions • Many students have the misconception that these are expressions that have an answer of 150, and do not understand that these also represent four ways of writing 150. An expression names a number. Sometimes an expression is a number such as 150. Sometimes an expression shows an arithmetic expression but may also be represented by its partitions. Numbers can also be represented by a different expression.	**Partition the number 150 in a variety of ways:** **150 = 100 + 50 (traditional expanded notation)** **However, many students may not use a variety of partitions, as shown, and may not recognize these as equivalent expressions.** **150 is:** **80 + 70** **100 + 50** **50 + 50 + 50** **200 – 50**

C. What are the next steps in instruction for the class and for individual students?

	How to Support
Representing Numbers Unconventionally We say that students have concept attainment when they are able to translate between and among all five representations of a concept (contextual, concrete, pictorial, symbolic and verbal). Students need numerous experiences representing numbers to 1000 and translating between and among these representations of a concept to strengthen their knowledge. They need many experiences with base-ten materials, pictures such as number lines and tallies, ten-frames, words, and contexts to conceptualize a number being made up of two or more parts.	• ***Display base ten blocks in a variety of ways to represent a number.***  • ***What number is represented by the following tallies?***  • ***Use the number in the oval to complete the sections.***  **(Support Questions 1 – 3: Mine the Gap K-2 pp 45 -46)**
Partitioning Numbers using a Variety of Ways Different Expressions Partitioning numbers using the models shown supports students' ability to recognize that any number can be partitioned into two or more parts. It also helps students develop part-part-whole thinking. Although it is important for students to experience a variety of partitions including traditional expanded notation (150 = 100 + 50), they should also continue to experience partitions such as 90 + 60, 75 + 75…). This is the most important understanding that can be developed about number relationships.	• **Break apart the number in the circle in three different ways.**  • **A farmer has 150 kg of food to put in 4 big bins. The farmer wants to put food in each bin, but each bin does not need the same amount. How much food could the farmer put in each bin?** 

Develop critical thinking by asking students to explain how the representations are alike and why they are different.	**a) Mike started to break apart each number. Help him finish.** **150 = ___60___+ ____________ +_________** **150 = ___________ + ____________** **b) How are these two ways to break apart 150 the same?** **c) Why are they different?** **(Support Questions 4 – 6: *Mine the Gap K-2* pp 105-107)**

D. What are the most appropriate methods and activities for assessing student learning?

Some sample questions related to representing and partitioning whole numbers which will be used to represent some of the appropriate methods and activities for assessing student learning are shown.

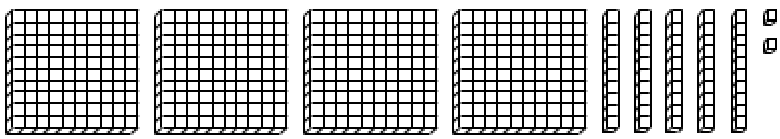
Encourage students to share their thinking, their strategies, and their solutions. Rich discussion fosters the development of alternative problem-solving strategies.

Cognitive Level of Questions Examples

Representing and Partitioning Whole Numbers

1. Application (3N02)

What number do these base-ten blocks represent?



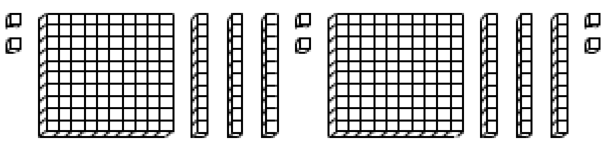
Legend
represents 1

Write the number, _________.

Correct Answer: 452

2. Application (3N02)

What number do these base-ten blocks represent?



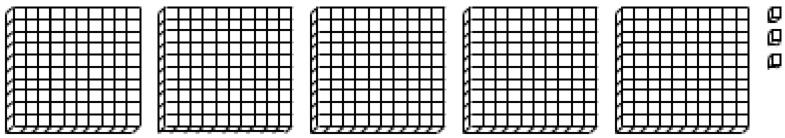
Legend
represents 1

Write the number, __________.

Correct Answer: 266

3. Application (3N02)

What number do these base-ten blocks represent?



Legend
represents 1

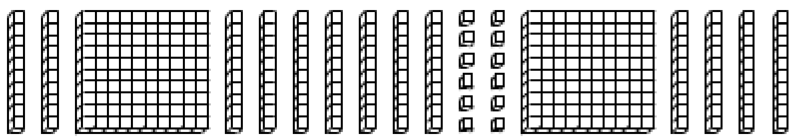
Write the number, _________.

Correct Answer: 503

4. Application (3N02)

What number do these base-ten blocks represent?

Correct Answer:



Legend
represents 1

Write the number, _________.

Correct Answer: 342

5. Analysis (3N02)

Draw a picture of base-ten blocks to show 236 in 3 different ways.

Legend
represents 1

Various representations using base-ten blocks are possible.

6. Application (3N02) The number 358 is the same as: ○ 100 + 100 + 50 + 8 + 100 ○ 300 + 5 + 8 ○ 400 – 58 ○ 3 + 5 + 8 **Correct Answer: A**	**7. Application (3N02)** Choose the number that is equal to thirty-one tens. ○ 31 ○ 301 ○ 310 ○ 3010 **Correct Answer: C**
8. Application (3N02) The number 642 is the same as: ○ 5 hundreds, 2 tens, and 14 ones ○ 64 tens and 2 ones ○ 6 tens and 42 ones ○ 6 hundreds, 20 tens and 4 ones **Correct Answer: B**	**9. Application (2N04)** Write three expressions that can be used to represent 53. 53 is the same as ______________________ 53 is the same as ______________________ 53 is the same as ______________________ **(Answer will vary)**
10. Application (3N02) Write the following numbers in words: 263 ______________________ 373 ______________________ 487 ______________________ 597 ______________________	

Mathematics in Grade 3 Lesson Learned 3
Whole Number Operations

Students were challenged when asked to apply basic skills, knowledge, and computational procedures to application and analysis questions. Students need to be able to apply the higher order thinking skills of problem solving, creativity, and reasoning to do application and analysis items. Students should have experiences with all the story structures for addition and subtraction. Students also need to be encouraged to estimate before calculating an answer to a question.

A. What conclusions can be drawn from the Nova Scotia Assessment: Mathematics in Grade 3?

Students were able to do well when explicitly given all the information needed to do the question. Students performed well on knowledge questions that required them to use basic facts and skills, symbolic procedures, and factual knowledge. For example, students were able to solve 487 – 37 when the problem was presented symbolically.

However, when students were asked to apply basic skills, knowledge, and computational procedures to application and analysis questions, they were challenged. At times, students were not sure whether they should add or subtract when questions were presented in the context of a story problem. The assessment analysis also showed that our students did not understand the relationship between addition and subtraction. Many of the students were doing addition and subtraction questions as procedures and were not making any connection between these two operations.

B. Do students have any misconceptions or errors in their thinking?

It is important to distinguish between misconceptions and errors. Misconceptions are, in effect, the misunderstandings about mathematical ideas which students entertain, and which usually lead to errors occurring. These are the most serious kinds of errors which require urgent action on the part of the teacher. Errors may occur for a variety of underlying reasons, ranging from careless mistakes to errors resulting from misconceptions. - John SanGiovanni – (*Mine The Gap For Mathematical Understanding Grades K-2*)

Results of the 2018–2019 Nova Scotia Assessment Mathematics in Grade 3
● 53% of the students had difficulty using place-value strategies, to represent a number written in words symbolically
● 52% of the students had difficulty to subtract a 3-digit whole number from a 3-digit whole number arranged vertically; one of the digits was zero (trading)
● 55% of the students had difficulty estimating the cost of objects in a context to the nearest hundred ($)
● 54% of the students had difficulty when solving a multi-step story problem involving whole number computation

Common Misconception/Errors	What it Looks Like
Place Value and Story Structures	
Place Value	• Students may have the misconception that they always subtract the smaller digit from the larger digit. 509 (minuend) - 389 (subtrahend) 280 (difference) 509 – 389 = 280 • Students may forget to regroup when adding. 145 + 247 = 3812 **145** **+ 247** **3812** • Students may misalign the digits or add using place value incorrectly 123 + 25 = 373 **123** **+ 25** **373**
Story Structures and Multi-Step Story Problems	Students may have trouble when reading and solving the different story structures. • May overgeneralize all stories to addition • May not see part-part-whole relationships • May subtract a whole from a part • May not understand the relationship between addition and subtraction

More information about developing part-part-whole relationships can be found in:

Mathematics Primary Curriculum Guide (Draft May 2013) on pages 40–44 and 89–90.

Mathematics 1 Curriculum Guide (Draft May 2013) on pages 63–64, 134–135, 42–46, and 126–127.

Mathematics 2 Curriculum Guide (Draft May 2013) on pages 42–46, 169–170, 66–72, and 178–182.

Mathematics 3 Curriculum Guide (Draft May 2013) on pages 36–40, page 171, 70–76 and 182–188.

C. What are the next steps in instruction for the class and for individual students?

Operations: Addition and Subtraction

<table>
<tr><th></th><th>How to Support</th></tr>
<tr><td>

Place Value

Place Value recording

328 + 245

300 + 20 + 8

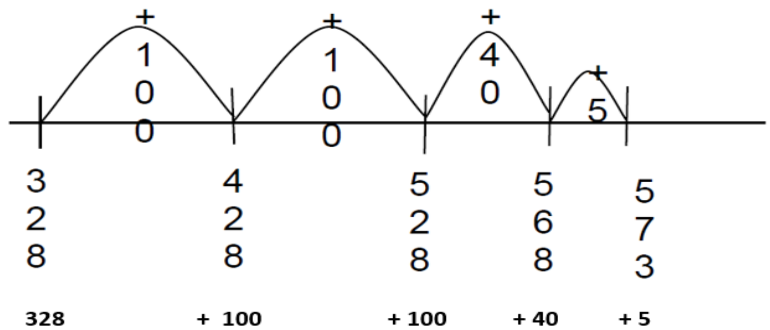
200 + 40 + 5

500 + 60 + 13

573

</td><td>

- Use an open number line to solve for addition and subtraction.



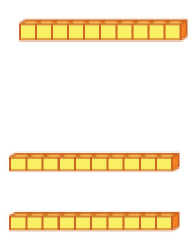
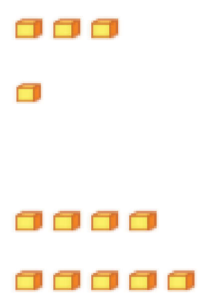
328 + 100 = 428
428 + 100 = 528
528 + 40 = 568
568 + 5 = 573

</td></tr>
<tr><td>

Place Value

</td><td>

- Model using base-ten blocks (place value blocks) and place value charts.

<table>
<tr><th>Hundreds</th><th>Tens</th><th>Ones</th></tr>
<tr><td></td><td></td><td></td></tr>
</table>

To solve 14 + 29

(Tens) 10 + 20 = 30

(Ones) 4 + 9 = 13

(Answer) 30 + 13 = 4

</td></tr>
</table>

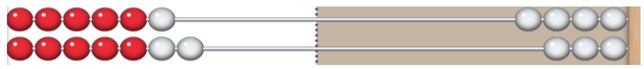
<table>
<tr>
<td>

Basic Addition and Subtraction Facts

Students will be expected to use and describe strategies to determine sums and differences using manipulatives and visual aids.

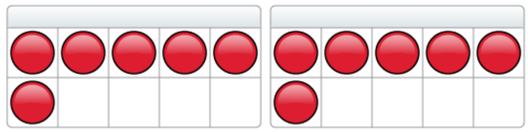
</td>
<td>

- Rekenrek (Modelling Near Doubles)



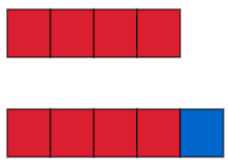
6 + 7 = 10 + 3

- Ten frames (Modelling Doubles)



6 + 6 = 10 + 2

- Linking Cubes (Modelling Near Doubles)



4 + 5 = 4 + 4 + 1

- Walk on Number lines or open number lines (Modelling One More/Counting On)

5 + 1 = 6

</td>
</tr>
<tr>
<td>

Relationship between addition and subtraction

Addition and subtraction are related as they are inverse operations.

</td>
<td>

- Model this relationship through personalized story problems and part-part whole mats (Strip Diagrams).

6 + 5 = 11, so 5 + 6 = 11   11 – 5 = 6, so 11 – 6 = 5
11 = 6 + 5, so 11 = 5 + 6   6 = 11 – 5, so 5 = 11 – 6

<table>
<tr><td colspan="2">11</td></tr>
<tr><td>5</td><td>?</td></tr>
</table>

</td>
</tr>
</table>

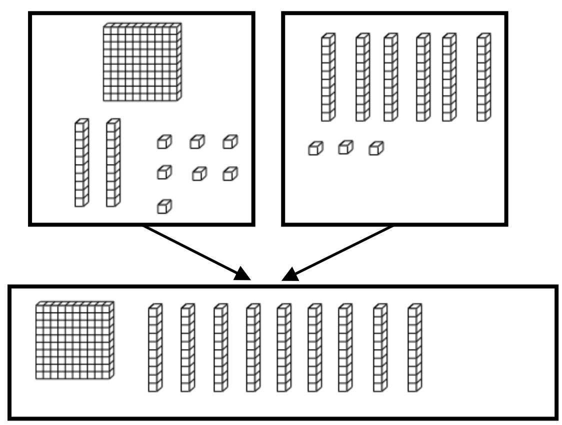
<table>
<tr>
<td>Relationship between addition and subtraction

The development of the meaning of addition and subtraction cannot be rushed.

Explore adding and subtracting situations in meaningful contexts using a variety of concrete materials and pictures to model and compare contexts.</td>
<td>The use of strip diagrams is as a strategy to help students interpret story problems:

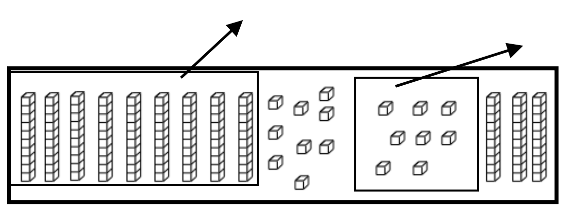
Bobby was given 63 green stamps. Bobby already had 127 stamps. How many stamps does Bobby have now? The strip diagram for this problem is as follows:

<table><tr><td>127</td><td>63</td></tr><tr><td colspan="2">?</td></tr></table>
Bobby had 98 stamps. Bobby was given more by a friend. Now Bobby has 137 stamps. How many stamps did Bobby’s friend give Bobby? The strip diagram for this problem is as follows:

<table><tr><td>98</td><td>?</td></tr><tr><td colspan="2">137</td></tr></table></td>
</tr>
<tr>
<td>When students draw pictures to illustrate joining or separating situations, they must indicate the action of joining or separating. Often, students use arrows to indicate the joining or separating of sets.</td>
<td>
Joining

127 + 63 = 190

Separating



137- 98 = 39</td>
</tr>
</table>

Story Structures:

Addition and Subtraction: [Mathematics 2 Curriculum Document] Background: Page 68

Addition and Subtraction: [Mathematics 3 Curriculum Document] Background: Page 71

<table>
<tr><th colspan="3">Join</th><th>Part-Part-Whole</th><th>Compare</th></tr>
<tr><th>Result Unknown</th><th>Change Unknown</th><th>Start Unknown</th><th>Whole Unknown</th><th>Difference Unknown</th></tr>
<tr><td>Pat has 8 marbles. Her brother gives her 4. How many does she have now?

8 + 4 = ?</td><td>Pat has 8 marbles but she would like to have 12. How many more does she need to get?

8 + ? = 12
or
12 – 8 = ?</td><td>Pat has some marbles. Her brother gave her 4 and now she has 12. How many did she have to start

? + 4 = 12
or
12 – 4 = ?</td><td>Pat has 8 blue marbles and 4 green marbles. How many does she have in all?

8 + 4 = ?</td><td>Pat has 8 blue marbles and 4 green marbles.
How many more blue marbles does she have?

8 – 4 = ?
or
4 + ? = 8</td></tr>
<tr><th colspan="3">Separate</th><th>Part-Part-Whole</th><th>Compare</th></tr>
<tr><th>Result Unknown</th><th>Change Unknown</th><th>Start Unknown</th><th>Part Unknown</th><th>Smaller or Larger Unknown</th></tr>
<tr><td>Pat has 12 marbles. She gives her brother 4 of them. How many does she have left?

12 – 4 = ?</td><td>Pat has 12 marbles. She gives her brother some.
Now she has 8. How many marbles did she give to her brother?

12 – ? = 8
or
12 – 8 = ?</td><td>Pat has some marbles. She gives her brother 4 of them. Now she has 8. How many marbles did she have to start?

? – 4 = 8
or
8 + 4 = ?</td><td>Pat has 12 marbles. Eight are blue and the rest are green. How many are green?

8 + ? = 12
or
12 – 8 = ?</td><td>Pat has 8 blue marbles and some green marbles. She has 4 more blue marbles than green ones. How many green marbles does she have?

8 – 4 = ?
or
? + 4 = 8</td></tr>
</table>

D. What are the most appropriate methods and activities for assessing student learning?

Some sample questions related to operations with whole numbers which will be used to represent some of the appropriate methods and activities for assessing student learning are shown.

Encourage students to share their thinking, their strategies, and their solutions. Rich discussion fosters the development of alternative problem-solving strategies

Cognitive Level of Questions Examples

Addition & Subtraction

1. Knowledge (3N09)

What is the sum of 363 and 25?

A. 308

B. 388

C. 618

D. 5113

Correct Answer: B

2. Knowledge (3N09)

Which of the following represents 809 - 489?

A. 320

B. 420

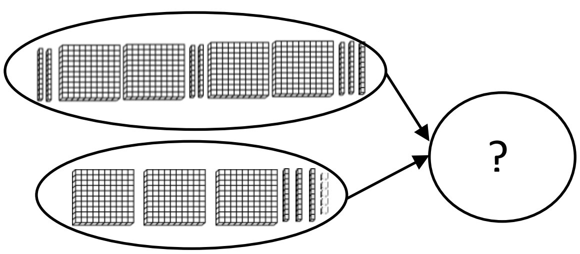
C. 480

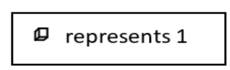
D. 1298

Correct Answer: A

3. Application (3N09)

Which sum is represented by the base-ten blocks shown?





A. 407 + 315 = 722

B. 470 + 305 = 775

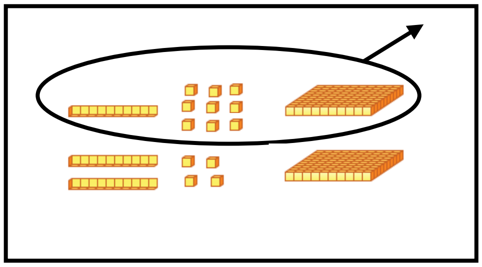
C. 470 + 315 = 715

D. 470 + 335 = 805

Correct Answer: D

4. Application (3N09)

Which equation represents the base-ten picture shown?

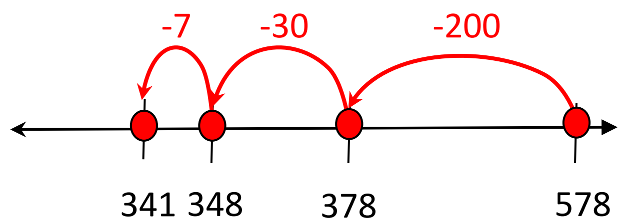


A. 124 - 119 = 5
B. 119 + 124 = 243
C. 243 - 119 = 124
D. 243 - 124 = 119

Correct Answer: C

5. Application (3N09)

Which equation is represented on the number line shown?



A. 578 – 341 = 237
B. 341 + 237 = 578
C. 341 + 578 = 919
D. 578 – 237 = 341

Correct Answer: D

6. Analysis (3N09)

Peter has 123 marbles. He gives some marbles to his friend Paul. Now Peter has 86 marbles.

How many more marbles does Peter have than Paul?

A. 37
B. 49
C. 123
D. 209

Correct Answer: B

Cognitive Level of Questions Examples

Relationship Between Addition & Subtraction

7. Knowledge (3N10)

Which expression can help solve the following equation?

$12 - 2 = \boxed{?}$

A. 2 + 10
B. 12 + 2
C. 10 - 2
D. 2 + 12

Correct Answer: A

8. Application (3N09)

Forty-two students and parents were in the gym. Twenty-six of them were students. How many were parents?

Choose the equation that shows a way to work out this problem.

26	$\triangle$
42	

A. $42 = 26 + \triangle$
B. $26 + 42 = \triangle$
C. $\triangle - 26 = 42$
D. $26 - \triangle = 42$

Correct Answer: A

9. Application (3N09)

Marley did this subtraction:

675 – 346 = 329

Which expression could help her check her work?

A. 675 + 329
B. 675 + 346
C. 329 + 346
D. 346 – 329

Correct Answer: C

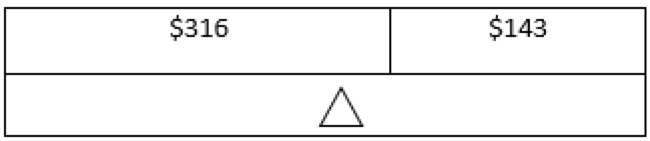
10. Application (3N09)

Kim created a word problem as shown:

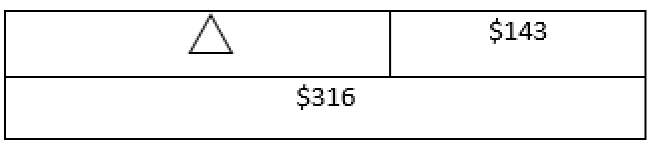
Stewart was saving money to buy a new bike. Stewart was given $143 dollars for his birthday and now has $316. How much money did Stewart have to start?

Which strip diagram represents Kim's word problem?

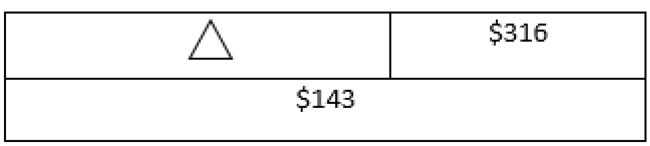
A.



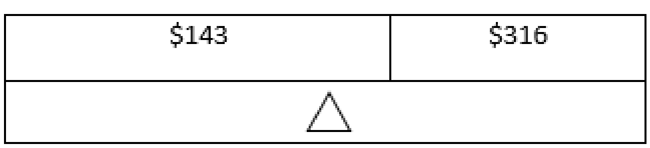
B.



C.



D.



Correct Answer: B

Cognitive Level of Questions Examples

Partitioning & Equations

11. Knowledge (3N05)

The number 605 is the same as:

A. 500 + 150 + 5
B. 400 + 100 + 15
C. 500 + 100 + 5
D. 600 + 12 + 5

Correct Answer: C

Which number is equal to seventy-one tens?

A. 71
B. 701
C. 710
D. 6011

Correct Answer: C

12. Knowledge (3PR03)

Which number is missing in this equation?

___ = 7 + 9

A. 19
B. 16
C. 13
D. 6

Correct Answer: B

Which number is missing in this equation?

3 + _______ = 12 – 6?

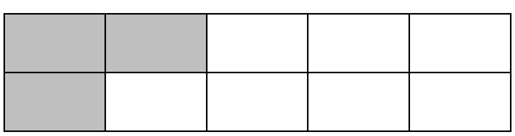
A. 1
B. 3
C. 6
D. 9

Correct Answer: B

Cognitive Level of Questions Examples - Fractions

13. Application (3N13)

Which fraction represents the shaded part?



A. $\frac{3}{10}$

B. $\frac{3}{7}$

C. $\frac{7}{3}$

D. $\frac{10}{3}$

Correct Answer: Answer: A

Mathematics in Grade 3 Lesson Learned 4
Patterns and Relations

Students were challenged when asked to transfer their visual representation of patterns to numerical patterns. Students improved when asked to describe either an increasing pattern or a decreasing pattern, and need to recognize that each term has a numeric value. They still have difficulty when asked to identify in an increasing pattern a specific element. Students seemed to forget that a pattern rule must have a starting point. Without the starting point, the pattern rule is incomplete. They need to be encouraged to provide all the information for a pattern rule. Students need to continue to work with representations of patterns, concretely, contextually, pictorially, symbolically, and verbally.

Patterns are the foundation for many mathematical concepts. Patterns should be taught throughout the year in situations that are meaningful to students. Patterns are explored in all the strands and are also developed through connections with other disciplines, such as science, social studies, English language arts, physical education, and music. Providing students with the opportunity to discover and create patterns, and then describe and extend those patterns, will result in more flexible thinking across strands and across subjects. Students should initially describe non-numerical patterns, such as shape, action, sound, and then incorporate numerical patterns by connecting them to the non-numerical patterns.

A large focus in Mathematics 3 is the introduction and development of decreasing patterns. Students use their knowledge of increasing patterns to make connections to the concept of decreasing patterns, since similar understandings are developed.

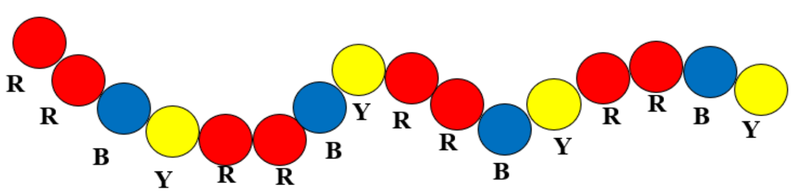
A. What conclusions can be drawn from the Nova Scotia Assessment: Mathematics in Grade 3?

Students did very well to recognize simple errors in increasing number patterns, identifying a pattern rule used to create a given increasing pattern, and identifying the next term in an increasing pictorial pattern. These types of items were either knowledge questions or application questions. Analysis questions related to patterns challenged our students. For example, students experience difficulty when asked to create an increasing pattern in which a specific element is identified (e.g., the 7th element is 56).

Students did extremely well when the patterns that they were working with were a visual representation of a pattern. See below for examples:

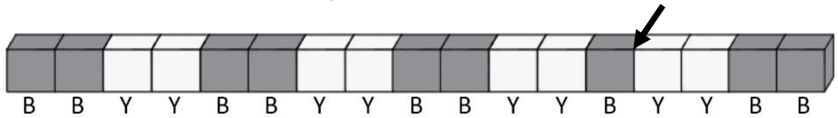
Example 1:

"The pattern for my beads is red (R), red (R), blue (B), yellow (Y)"



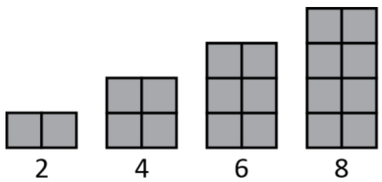
Example 2:

"I see a mistake in this block pattern. It needs another blue block here."

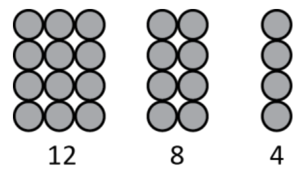


Students were challenged when they were transferring their knowledge of visual patterns to numerical patterns. Students should be able to describe an increasing pattern made of shapes but need to recognize that each term in the pattern also has a numeric value. For example,

Example 3:



increasing pattern made of shapes

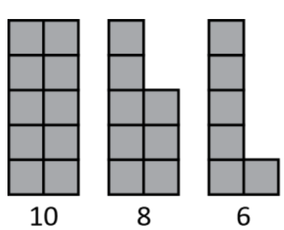
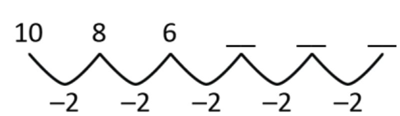


decreasing pattern made of shapes

B. Do students have any misconceptions or errors in their thinking?

It is important to distinguish between misconceptions and errors. Misconceptions are, in effect, the misunderstandings about mathematical ideas which students entertain, and which usually lead to errors occurring. These are the most serious kind of errors which require urgent action on the part of the teacher. Errors may occur for a variety of underlying reasons, ranging from careless mistakes to errors resulting from misconceptions. - John SanGiovanni – (*Mine The Gap For Mathematical Understanding Grades K-2*)

Results of the 2018–2019 Nova Scotia Assessment Mathematics in Grade 3
• Most students correctly answered all questions related to patterns and relations.
• Students did very well to recognize simple errors in increasing number patterns, identifying a pattern rule used to create a given increasing pattern, and identifying the next term in an increasing pictorial pattern. These types of items were either knowledge questions or application questions.
• Analysis questions related to patterns challenged our students. For example, students experience difficulty when asked to create an increasing pattern in which a specific element is identified (e.g., the 7th element is 56).
• Students were challenged when they were transferring their knowledge of visual patterns to numerical patterns. Students should be able to describe an increasing pattern made of shapes but need to recognize that each term in the pattern also has a numeric value.
• Students did extremely well when the patterns that they were working with were a visual representation of a pattern.

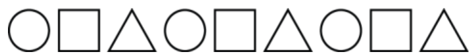
Common Misconceptions/Errors	**What it Looks Like**
Misconceptions of Increasing and Decreasing Patterns	
One of the most fundamental concepts in pattern work, but also one not clear to all students, is that, although the part of the pattern that they see is finite, when mathematicians talk about a pattern, they are talking about something that continues beyond what the student sees. (Small, 2009, p.568)	Students do not realize that there are different ways to continue a pattern if a pattern rule is not provided. For example, if given 5, 10, 15 … as the beginning of the pattern, they may only see it as a repeating problem, and will not consider that it might be a growing pattern. (Small, 2009, p. 579)
Students have difficulty extending an increasing number pattern or a decreasing number pattern.	A suggested strategy is to have students locate the numbers in the pattern on a hundred chart and place a transparent counter over each number. Have students use the visual pattern in the counters to extend the pattern. Help students relate the visual pattern to the starting point and the number added each time in the number pattern. (Pearson, 2009b, p. 14)
As students describe decreasing shape patterns, they may not recognize that each term has a numeric value. In the pattern, the pattern rule is to start with 10 squares and decrease by 2 squares each time.  	As students describe decreasing shape patterns, help them recognize that each term has a numeric value. The above pattern can be expressed as 10, 8, 6, … by counting the number of squares in each term. Students may also find it useful to record the change from one term to the next as shown below:  Remind students that a pattern rule must have a starting point, or the pattern rule is incomplete. Eg., if a student describes the pattern 10, 8, 6, ... as a decrease by 2 without indicating that the pattern starts at 10, the pattern rule is incomplete.

Pattern Rules: Teachers often ask students to describe a pattern to ensure that they understand it. Ideally, a pattern's description is a *pattern rule,* whether described verbally, pictorially, or symbolically. The rule is an unambiguous description of the pattern. **Student Response:** • *The student used a verbal pattern rule.* • Use words to describe this pattern: 3, 7, 11, 15, … • Start at 3 and keep adding 4 (Small, 2009, p. 572)	**Student Response:** • *The student has incorrectly interpreted the pattern rule as a description of a repeating pattern. This may be because the student is still more comfortable with repeating patterns.* • Make a pattern that follows this pattern rule: • Start at 4 and add 3 each time. • Write the numbers in the blanks. 4 3 4 3 4 3 **Student Response:** • *The student has applied the pattern rule incorrectly. This may be because the student is still more comfortable with arithmetic patterns that grow by a constant amount.* • Make a pattern that follows this pattern rule: • Start at 4 and add 1, then 2, then 3, and so on. • Write the numbers in the blanks. 4 14 24 34 44 54 64 … (Small, 2009, p. 580)
It is good practice to always show at least three full repetitions of the core of a pattern to make it reasonable for a student to identify it. (Small, 2009, p. 569)	• One of the reasons for this is to remove some of the inherent ambiguity. • A pattern beginning 5, 10, … could be 5, 10, 15, 20, … or 5, 10, 20, 40 … (Small, 2009, p. 569)

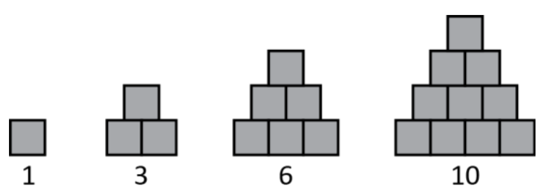
C. What are the next steps in instruction for the class and for individual students?

<table>
<tr><th></th><th>How to Support</th></tr>
<tr><td>Identify the core of a pattern</td><td>

Use appropriate patterning vocabulary, such as core (the repeating part of the pattern) and elements (the actual objects used in the pattern).
It is important to create patterns that have the core repeated at least three times.
highlight, or isolate, the core each time it repeats.
Remind students that repeating patterns can be extended in both directions.
Encourage students to reference the position of the elements of the pattern using ordinal numbers. The core of the shape pattern below is circle, square, triangle. There are three elements in this pattern, namely a circle, a square, and a triangle. The core of this three-element pattern is circle (1st element), square (2nd element), and triangle (3rd element).
There are three elements in this pattern, namely a circle, a square, and a triangle.


</td></tr>
<tr><td>Students should be able to describe an increasing pattern</td><td>

An increasing pattern is a growing pattern where the size of the term increases in a predictable way. The terms in an increasing pattern grow by either a constant amount or by an increasing amount each time. Students need sufficient time to explore increasing patterns using various manipulatives, such as cube-a-links, tiles, toothpicks, counters, pattern blocks, base-ten blocks, ten-frames, bread tags, stickers, or buttons, to realize they increase in a predictable way. As students describe increasing shape patterns, help them recognize that each term has a numeric value. For example,



A counting sequence is an increasing pattern where each number represents a term in the pattern. For example, in the counting sequence 1, 2, 3, 4, …, 1 represents the first term, 2 the second term, 3 the third term … This counting sequence can be connected to ordinal numbers where students should be able to recognize that the 34th term is 34 and that 57 is the 57th term in the sequence. These ordinal number patterns should be investigated for numbers up to 100.
Students should be able to describe a given increasing pattern by stating the pattern rule. A pattern rule tells how to make the pattern and can be used to extend an increasing pattern. Give students the first three or four terms of an increasing pattern. Ask them to state the pattern rule by identifying the term that

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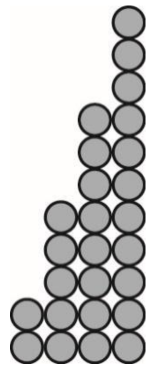
</table>

represents the starting point and describing how the pattern continues. For example, in the pattern below, the pattern rule is, “Start with 2 counters and add 3 counters each time”.



- As students describe concrete or pictorial patterns, help them recognize that each term has a numeric value. For example, the above pattern can be expressed as, 2, 5, 8, 11, … by counting the number of counters in each term. Students may also find it useful to record the change from one number to the next as shown below.

  2 5 8 11 __ __ __ …

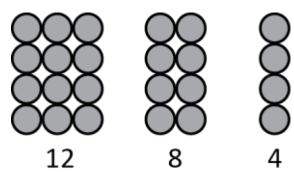
  +3 +3 +3 +3 +3 +3 +3

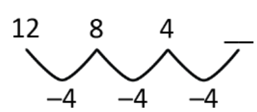
- Students need opportunities to compare numeric patterns, discussing how they are the same and how they are different. When comparing increasing patterns, compare the starting points and how each term increases. For example, one-way students may address this is by using a page with four small hundred charts. Ask them to skip count starting a 0 and shade one chart by 2s, one chart by 5s, one chart by 10s, and one chart by 25s. Then, discuss the pattern rule in each chart comparing the starting points and the amount of increases.

- Students should be able to create increasing patterns, concretely, pictorially, and symbolically, and be able to describe the pattern rule they used to create their representations. When creating increasing patterns, initially students need to choose a starting point and then decide on the amount of increase. The amount of increase may be either a constant amount or an increasing amount. Students should be able to describe their pattern by clearly explaining how it changes from one term to the next. Students may share their patterns and the strategies they used to create their pattern.

- Students should have frequent experiences using increasing patterns to solve real-world problems that interest and challenge them. They should use concrete materials or pictures to model the problem before determining the pattern rule. A variety of strategies may be used to solve the problem such as using a number line, a hundred chart, a picture, concrete materials, or skip counting.

<table>
<tr>
<td>Describe a decreasing pattern</td>
<td>

A decreasing pattern is a shrinking pattern that decreases by a constant amount each time. Students need sufficient time to explore decreasing patterns using various manipulatives, such as cube-a -links, tiles, toothpicks, counters, pattern blocks, base-ten blocks, ten-frames, bread tags, stickers, and buttons. Sometimes students are more comfortable during the exploration stage if they can experiment first, using manipulatives, then pictures, and eventually numbers.
Students should be able to identify and describe various decreasing patterns such as horizontal, vertical, and diagonal patterns found on a hundred chart. Working with decreasing patterns can be connected to skip counting in outcome N01. Provide copies of hundred charts. Ask students to begin at 100 and skip count backward by a given number, shading in the number for each count all the way to 1. Then they write a description of the pattern. For example, if they chose to skip count by 10s, the pattern results in one vertical column, regardless of the starting point.
As students begin to investigate patterns, they sometimes confuse repeating patterns with decreasing patterns. Remind them to look for a core first. If they cannot find a core, then the pattern is not a repeating pattern.
Earlier, students became familiar with assigning a numeric value to each element in an increasing pattern. This expectation also applies to decreasing patterns.
Students should be able to describe a given decreasing pattern by stating the pattern rule. A pattern rule includes a term representing a starting point and a description of how the pattern continues. A pattern rule tells how to make the pattern and can be used to extend a pattern. For example, in the pattern below, the pattern rule is to start with 12 circles and decrease by 4 circles each time.


As students describe decreasing shape patterns, help them recognize that each term has a numeric value. The above pattern can be expressed as 12, 8, 4, … by counting the number of circles in each term. Students may also find it useful to record the change from one term to the next as shown below.


Remind students that a pattern rule must have a starting point, or the pattern rule is incomplete. For example, if a student describes the pattern 12, 8, 4, ... as a decrease by 4 patterns without indicating that it starts at 12, the pattern rule is incomplete.
Students need opportunities to compare numeric patterns and to discuss how they are the same and how they are different. When comparing decreasing patterns, compare the starting points and how each term decreases using a variety of representations such as shape patterns, hundred charts, and number patterns.

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</table>

	• For example, give students a page with four small hundred charts. Ask them to skip count backward starting at 100 and shade one chart by 2s, one chart by 5s, one chart by 10s, and one chart by 25s. Then discuss the pattern rule in each chart indicating the starting point and the amount of decrease.

D. What are the most appropriate methods and activities for assessing student learning?

Some sample questions related to patterns and relations which will be used to represent some of the appropriate methods and activities for assessing student learning are shown.

Encourage students to share their thinking, their strategies, and their solutions. Rich discussion fosters the development of alternative problem-solving strategies.

Cognitive Level of Questions Examples

Patterns and Relations

1. Knowledge (2PR01)
Identify the core of this pattern.

A. 

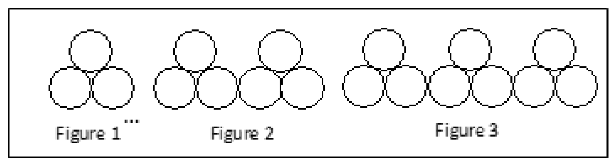
B. 

C. 

D. 

Correct Answer: C

2. Application (2PR02)
Simon created the following pattern.



How many small circles would be in the fifth figure?

A. 15 circles
B. 12 circles
C. 10 circles
D. 5 circles

Correct Answer: A

3. Application (3PR02)
Examine the following pattern:

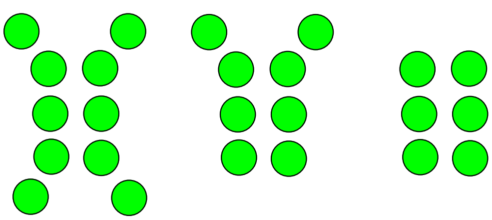


Figure 1 Figure 2 Figure 3

How many circles are there in Figure 5?

A. 2 circles
B. 3 circles
C. 4 circles
D. 5 circles

Correct Answer: A

Cognitive Level of Questions Examples
Patterns and Relations
(Numerical Patterns)

4. Application (3PR02)	**5. Application (3PR02)**
What are the two missing numbers in the number pattern shown? 66, 61, 56, 51, ____, 41, 36, ____, … A. 45 and 35 B. 46 and 31 C. 52 and 31 D. 52 and 37 **Correct Answer: B**	Natalie created the following decreasing pattern: 546, 536, 526, 516, 506, 496, … What is the rule for this pattern? A. Subtract 10. B. Start at 546 subtract 5 each time. C. Start at 496 and subtract 10 each time. D. Start at 546 and subtract 10 each time. **Correct Answer: D**
6. Application (3PR02) Monique created the following decreasing number pattern. 55, 50, 45, 35, 30, 25, 20, 10, 5, … Two numbers are missing in this pattern. What are the two missing numbers? A. 40 and 30 B. 15 and 25 C. 40 and 15 D. 45 and 15 **Correct Answer: C**	**7. Application (3PR01)** Which statement about the following two patterns is true? 62, 74, 86, 98, … and 62, 50, 38, 26, … A. They have the same starting point and increase in the same way. B. They have the same starting point and they are increasing patterns. C. They have the same starting point and they are decreasing patterns. D. They have the same starting point and they do not change in the same way. **Correct Answer: D**

Cognitive Level of Questions Examples
Patterns and Relations
(Numerical Patterns)

8. Application (3PR01)

Marthe created the following pattern using yellow and red counters:

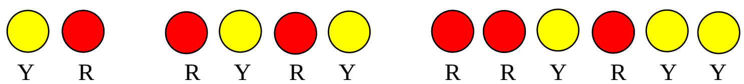


Figure 1 Figure 2 Figure 3

What is the rule for this pattern?

A. Figure 1: add one yellow counter to the left, and one red counter to the right each time.

B. Figure 1: add two red counters to the left, and two yellow counters to the right each time.

C. Figure 1: add one yellow counter, and one red counter to the left, and one yellow counter, and one red counter to the right each time.

D. Figure 1: add one red counter to the left, and one yellow counter to the right each time.

Correct Answer: D

9. Application (3PR03)

Forty-two students in the second and third grades are in the school gymnasium.
Twenty-six students are in the second grade.

What equation do you use to determine the number of third grade students who are in the gymnasium?

A. $42 = 26 + \square$

B. $26 + 42 = \square$

C. $\square - 26 = 42$

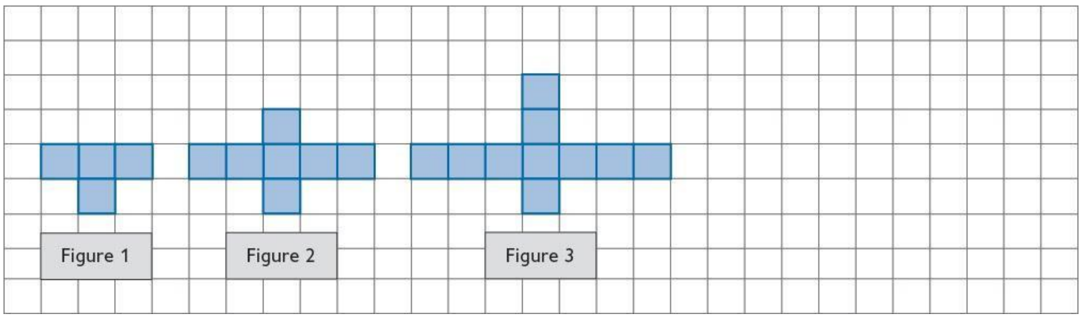
D. $26 - \square = 42$

Correct Answer: A

Cognitive Level of Questions Examples
Patterns and Relations
(Predict how many squares in each Figure)

10. Application (2PR02)

Examine the following pattern:



How many squares would be in Figure 4?

A. 14 squares

B. 13 squares

C. 12 squares

D. 11 squares

Correct Answer: B

Mathematics in Grade 3 Lesson Learned 5
Measurement

Students are expected to build conceptual understanding of what it means to measure with a ruler. Emphasis should be placed on counting the intervals between the numbers, rather than looking at the number on the ruler that is aligned with the end of the object. Students also need to use a ruler to measure the length of a pencil or other objects with and without using zero as the starting point. Students need to recognize which mass unit (gram or kilogram) is appropriate for measuring and comparing the mass of a specific item. Students need to be able to explain that perimeter is the distance around a shape or the length of the boundary of an enclosed region. Students need to find the perimeter of different regular, and composite concrete shapes, before being introduced to questions in pictorial form. Students need to work with perimeter in application and analysis questions.

A. What conclusions can be drawn from the Nova Scotia Assessment: Mathematics in Grade 3?

Students did very well when presented with questions related to direct measure, time referents, and approximate measure when using personal referents. For example, students were able to identify how many centimetres are in one metre (Knowledge question). They were also able to solve a given problem involving the number of seconds in a minute, the number of minutes in an hour, and the number of hours in a day (Knowledge question).

Students were able to use their personal referents for 1 g and 1 kg to estimate the mass of common objects, such as a bag of sugar or a paper clip (Application question). Students also did well when estimating the length or height of an object using personal referents. For example, students used the height of a doorknob from the floor as a personal referent for 1 m (Application question).

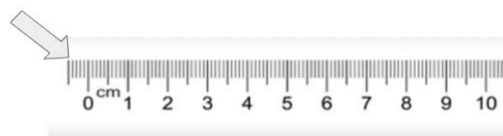
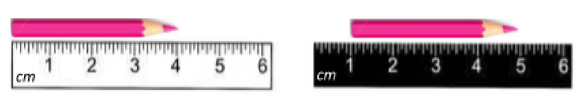
Having these personal referents helps students visualize measurements and estimate more accurately. Personal referents also help students identify the units required for specific measurements.

A big idea developed in Grade 3 is perimeter. Students appeared to not understand the concept of perimeter even though a definition was given in parentheses in a question on the assessment. Students need to be able to explain that perimeter is the distance around a shape or the length of the boundary of an enclosed region.

B. Do students have any misconceptions or errors in their thinking?

It is important to distinguish between misconceptions and errors. Misconceptions are, in effect, the misunderstandings about mathematical ideas which students entertain, and which usually lead to errors occurring. These are the most serious kind of errors which require urgent action on the part of the teacher. Errors may occur for a variety of underlying reasons, ranging from careless mistakes to errors resulting from misconceptions. - John SanGiovanni – (*Mine The Gap For Mathematical Understanding Grades K-2*)

Results of the 2018–2019 Nova Scotia Assessment Mathematics in Grade 3
• 61% of students made a mistake when measuring a pencil length using a ruler
• 51% of students had difficulty when calculating the perimeter of a regular polygon in a context

Common Misconceptions/Errors	What it Looks Like
Misconceptions of Measurement: Using a Ruler to Measure, Estimating Mass, and Perimeter	
A common error that many students make is the placement of the ruler when measuring an object. This may indicate that they do not understand that they are counting the intervals between the numbers to determine length. Students may not realize that the scale of the ruler begins at 0 cm.	• Some students do not consider the gap between the end of the ruler and the zero mark.  • Some may ignore the 0 cm mark and begin at the 1 cm mark or at other points on the ruler other than zero.  • Some students may start at 1 cm on the ruler but still use the ruler accurately by taking this into consideration.  The pencil being measured starting at the 0 cm mark measures 4 cm. The pencil being measured starting at 1 cm and ending at 5 cm, still measures 4 cm.

Estimating Mass

Students have misconceptions related to estimating mass and comparing the kilogram and the gram.

Students may not understand that mass and size are not necessarily related.

- Students have difficulty matching an item with its estimated mass.

Estimate the mass of the banana:

- ❑ **1 gram**
- ❑ **1 kilogram**
- ❑ **100 grams**
- ❑ **100 kilograms**

- Students may not recognize which mass unit (kilogram or gram) is appropriate for measuring and comparing the mass of a specific item.

Perimeter

Students have misconceptions related to how to determine the perimeter of a shape and how to draw a shape of a given perimeter.

Misconceptions often occur because students do not make a distinction between area and perimeter. Outcomes based on the concept of area are not found in grades two or three.

- When students are given a shape drawn on 1 cm paper, they may count each square around the outside of the shape instead of counting the side of each square along the perimeter.

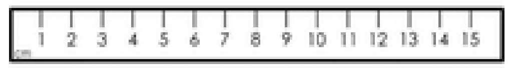
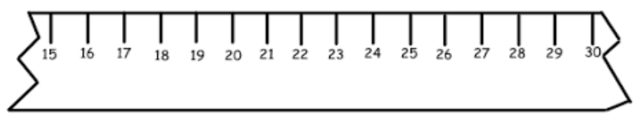
As shown in the example below, students would say that the perimeter is 12 cm.

1	2	3	4	5
12				6
11	10	9	8	7

- Students may also count all the squares shown instead of counting the units along the perimeter as shown below.
- In this example, the students would say the rectangle has a perimeter of 15 cm.

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15

C. What are the next steps in instruction for the class and for individual students?

	How to Support
Standard Units Students need to develop a familiarity with standard units and explore the relationship between them.	● Have students use simple rulers that are created by students initially. ● Move onto tools that are easy for students to read. ● Students should use rulers (or the side of the ruler) that show only numbered centimetres and not millimetres.  ● Students should identify objects from around the classroom that would be an appropriate referent for a centimetre or a metre; for example, the width of a pencil (cm), the distance from the bottom of a door to the doorknob (1m).
Measuring with a Ruler The skill of learning how to use a ruler is introduced for the first time in grade 3. Emphasis should be on counting the intervals between the numbers, rather than looking at the number on the ruler that is aligned with the end of the object.	● Give a piece of broken ruler to each student and ask them to measure items in the classroom. Observe how they attempt to measure items.  ● Encourage students to estimate measurements before verifying them using a measurement tool. ● Have students measure the length, width, or height of given 3-D objects in the classroom. ● Show students how to measure something that is longer than a ruler by marking, recording, and starting again. ● Demonstrate that the numbers on the ruler correspond to the number of small cubes by starting at 0 and lining up small cubes from base-ten materials along the ruler.
Mass It is important that students have a personal referent for a gram and a kilogram.	● Have students brainstorm items that have a mass of 1 gram. Use a small base-ten cube as a personal referent of a gram. Provide students with items such as a raisin, bean seed, jellybean, or a paper clip, to conceptualize how a gram feels. ● Provide students with materials such as sand, flour, sugar, or base-ten materials to fill a container until it exactly balances with a 1 kg mass on a balance scale to help create a referent for 1 kg.

Students should use the word mass rather than weight to say whether an object is heavier or lighter than another object. Mass and weight are two different physical quantities. Have students understand that grams are used to measure very light objects and kilograms are more appropriate units for heavier objects.	● Have students use referents for 1 g and 1 kg to estimate the mass of common objects such as an eraser, an apple, a juice box, or a textbook, and to estimate whether an object is heavier or lighter than 1 kg. ● Encourage students to use the known mass of one object to estimate the mass of another object. ● Investigate how everyday items, such as food items, are measured. Include items that are small and dense, such as a golf ball, as well as those that are large and hollow or porous, such as a beach ball. ● Model for students how 1000 g is equal to 1 kg using a balance scale: use food items of various benchmark masses, such as 2 bags of 500 g, 4 boxes of 250 g, or a pre-counted bag of 1000 jelly beans.
Perimeter Students learn perimeter best when they can make connections to real life examples. Ideas include fencing a yard, measuring the perimeter of the classroom or gym using metre sticks.	● Provide students with opportunities to find the perimeter of many different regular shapes concretely before being introduced to pictorial forms. Pentominoes may be used to illustrate measuring and recording the perimeter of a given composite shape. ● In addition to composite shapes with straight sides, it is important to expose students to other shapes, such as their handprint. String can be used to outline the shape and then measured with a ruler. ● Students should be given opportunities to explore their own methods for determining the perimeter of a shape using regular and irregular shapes with string and/or rulers. Students should not develop or follow a formula for calculating perimeter in the early grades. ● Students should be given opportunities to construct multiple shapes of a given perimeter. They should begin by drawing rectangles using centimetre grid paper and horizontal and vertical lines only. They may explore various rectangles before exploring other shapes. ● Students need to be exposed to word problems with a context about perimeter rather than simply finding the perimeter of many different regular shapes.

D. What are the most appropriate methods and activities for assessing student learning?

Some sample questions related to measurement which will be used to represent some of the appropriate methods and activities for assessing student learning are shown.

Encourage students to share their thinking, their strategies, and their solutions. Rich discussion fosters the development of alternative problem-solving strategies.

<table>
<tr><th colspan="2">Cognitive Level of Questions

Examples for Standard Units</th></tr>
<tr>
<td>1. Application (3M03)

Which object is the best referent for 1 meter?

A. the length of a hallway
B. the width of a finger
C. the height of a building
D. the width of a door

Correct Answer: D</td>
<td>2. Application (3M03)

Estimate the height of a doorknob from the floor:

A. 1 m
B. 2 m
C. 3 m
D. 4 m

Correct Answer: A</td>
</tr>
<tr><td colspan="2">3. Application (3M03)

Is this pencil 15 cm long? Explain your thinking on the lines provided.</td></tr>
</table>

____

____

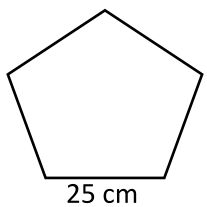
Cognitive Level of Questions Examples Measuring Mass	
4. Application (3M04) Which item would be the best referent for a gram? A. a raisin B. a sneaker C. a textbook D. a lunch box **Correct Answer: A**	**5. Application (3M04)** The mass of a bag of apples is 1000 grams. It is the same as A. 1 g B. 1 kg C. 10 kg D. 100 kg **Correct Answer: B**
6. Analysis (3M04) Which item has more mass, a golf ball or beach ball? Explain your thinking on the lines provided. ____ ____ ____ ____	

Cognitive Level of Questions Examples

Perimeter

7. Application (3M05)

Look at the shape shown.



What is the perimeter of this regular pentagon?

A. 25 cm

B. 50 cm

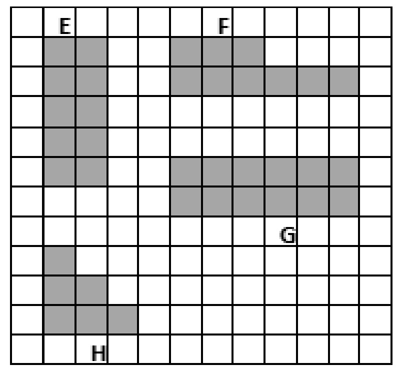
C. 100 cm

D. 125 cm

Correct Answer: D

8. Application (3M05)

Samir draws these 4 shaded shapes on grid paper.



Which shaded shapes have the same perimeter?

A. E and F

B. F and G

C. F and H

D. H and G

Correct Answer: B

9. Analysis (3M05)

Farmer Bill has 24 metres of fencing. How many different rectangular chicken coops can he make?

A. 3

B. 4

C. 6

D. 24

Correct Answer: B

Cognitive Level of Questions Measurement

10. Knowledge (2M01)

What is the best unit to choose to measure the length of a season?

A. Days

B. Weeks

C. Months

Correct Answer: C

11. Application (3M03)

The width of your thumb is about

A. 1 cm

B. 10 cm

C. 100 cm

D. 1 m

Correct Answer: B

12. Knowledge (3M04)

The mass of a box of table salt is about

A. 1 g

B. 100 g

C. 1 kg

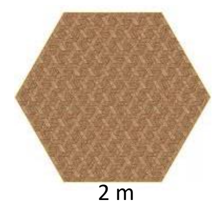
D. 100 kg

Correct Answer: C

13. Application (3M05)

A carpet is in the shape of a hexagon. Each side of the carpet is 2 m long.

What is the perimeter of this carpet?



A. 12 m

B. 10 m

C. 8 m

D. 6 m

Correct Answer: A

14. Analysis (3M05)

How many rectangles can you draw that have a perimeter of 16 cm each?

A. 4

B. 3

C. 2

D. 1

Correct Answer: A

Mathematics in Grade 3 Lesson Learned 6
Geometry

Students need to continue developing their knowledge of 2-D shapes and 3-D objects by describing and sorting them according to their geometric attributes. Students need experiences to identify the faces, edges, and vertices of 3-D objects including spheres, cones, cylinders, pyramids, cubes and other prisms. Students need to be provided with opportunities to explore these attributes through sorting and constructing activities. Students need to extend their knowledge of both regular and irregular polygons. They need to focus on comparing the number of sides as the key attribute for classifying polygons. Students need more experiences with irregular polygons, so that they begin to realize that a polygon, regardless of its dimensions, or position in space, remains the same shape.

A. What conclusions can be drawn from the Nova Scotia Assessment: Mathematics in Grade 3?

Students did very well when presented with questions related to 3-D objects, such as cylinders and spheres, found in their everyday life. They were able to identify the object being described. These were all application questions.

Students need to continue to develop their knowledge of geometry by describing and sorting 3-D objects according to their geometric attributes. Students require more experiences to identify the faces, edges, and vertices of 3-D objects including spheres, cones, cylinders, pyramids, and cubes and other prisms.

Students need to focus on comparing the number of sides as the key attribute for classifying polygons. Students need to be able to name the specific polygons including the triangle, quadrilateral, pentagon, hexagon, and octagon.

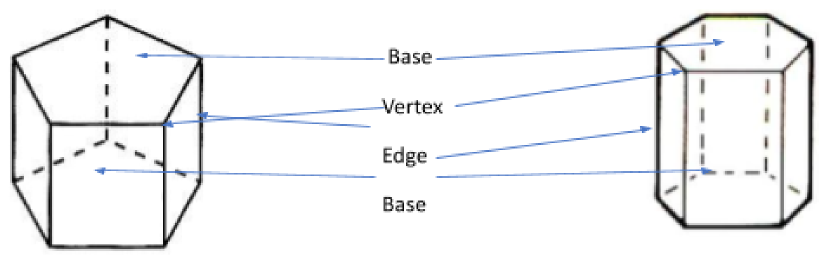
B. Do students have any misconceptions or errors in their thinking?

It is important to distinguish between misconceptions and errors. Misconceptions are, in effect, the misunderstandings about mathematical ideas which students entertain, and which usually lead to errors occurring. These are the most serious kind of errors which require urgent action on the part of the teacher. Errors may occur for a variety of underlying reasons, ranging from careless mistakes to errors resulting from misconceptions. - John SanGiovanni – (*Mine The Gap For Mathematical Understanding Grades K-2*)

Results of the 2018–2019 Nova Scotia Assessment Mathematics in Grade 3
• 63% of students had difficulty determining the rule for sorting a set of three-dimensional objects consisting of prisms and pyramids

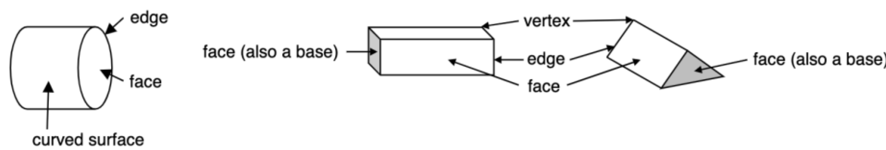
<table>
<tr><th>Common Misconceptions/Errors</th><th>What it Looks Like</th></tr>
<tr><td colspan="2">

- Students have difficulty distinguishing what determines the faces, edges, or vertices of an object.



This pentagonal prism has 7 faces (5 rectangular lateral faces and 2 pentagonal faces which are the 2 bases), 15 edges and 10 vertices.

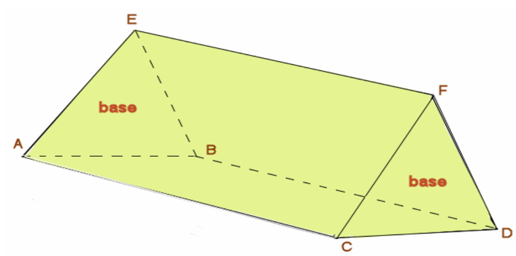
This hexagonal prism has 8 faces (6 rectangular lateral faces and 2 hexagonal faces which are the 2 bases), 18 edges and 12 vertices.



</td></tr>
<tr><td>

- Another common error or misconception that students have is understanding that the base of a geometric object is whatever face it is sitting on.

</td><td>

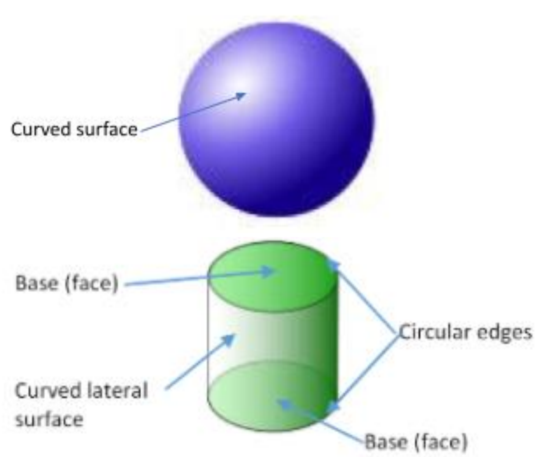


The two congruent triangles ABE and CDF are the two bases. The rectangles ABDC, ACFE and BDFE are the lateral faces.

</td></tr>
<tr><td>

- When students are working with 3-D objects such as cylinders, cones, and spheres, they consider the rounded surfaces on these 3-D objects to be faces.

</td><td>



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</table>

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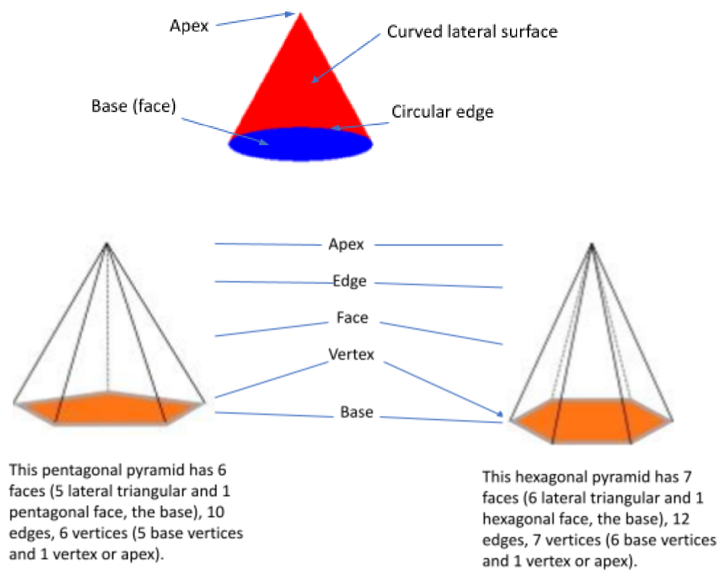
<tr>
<td>

- Another important concept for students to understand is that an edge occurs where two faces of a 3-D object meet.

  A vertex (vertices) is a point where three or more edges meet. (**Note:** On a cone and a pyramid the highest point above the base is called the **apex**. In a pyramid the apex is also a vertex, but for a cone, it is a mistake to refer to the apex as a vertex as there are no edges that meet.)

</td>
<td>



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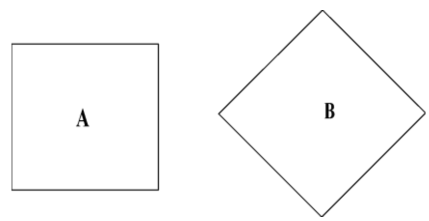
<td>

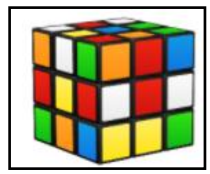
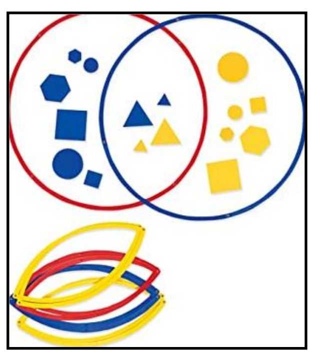
- Some students incorrectly believe that the orientation of a geometric figure, changes the figure itself. Students recognize that shape A is square but think that shape B is not a square.

</td>
<td>



</td>
</tr>
</table>

C. What are the next steps in instruction for the class and for individual students?

	How to Support
3D Objects Students should be able to determine the number of faces, edges, and vertices of a given 3-D object: ● A cylinder is a 3-D object with 2 faces, 1 curved surface, 2 edges, and 0 vertices. ● A cone is a 3-D object with 1 face, 1 curved surface, 1 edge, and 1 apex. ● A sphere is a 3-D object with 1 curved surface, 0 faces, 0 edges, and 0 vertices.	● A suggested strategy to help students identify faces, edges, and vertices is to put a small piece of modelling clay on each face, edge, or vertex as they count. This should help students describe 3-D objects according to the shape of the faces and the number of edges and vertices. ● Show students models and real-life objects of cylinders, cones, and spheres. Ask students what the difference is between these solids and the prisms and pyramids already studied. Show students the faces, edges, and vertices of each solid.     ● Brainstorm, with the students, what each term means. ● Students should compare and sort 3-D objects by observing the number of faces, edges, and vertices. A student may sort objects in various ways, such as those that have all square faces, those that have circular faces, those that have 8 vertices, or those that have straight edges. Use sorting circles to engage students in sorting activities. 

<table>
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- Students should play games with their peers in which they sort objects and ask their peers to guess the sorting rule according to the number of faces, edges, and vertices. It is essential to have a large collection of 3D solids available for students to explore. Eg. Who am I? I am an object with no flat faces, no edges, no vertices.

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<td>

Polygons

[Curriculum Document]

[Grade 3 Pages]: 152-153

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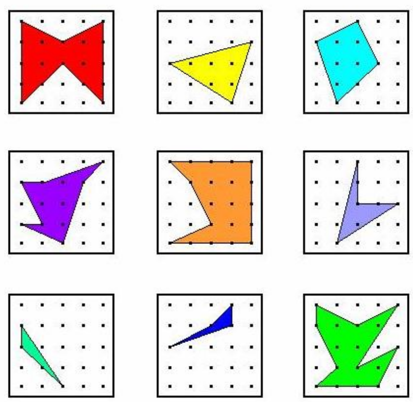
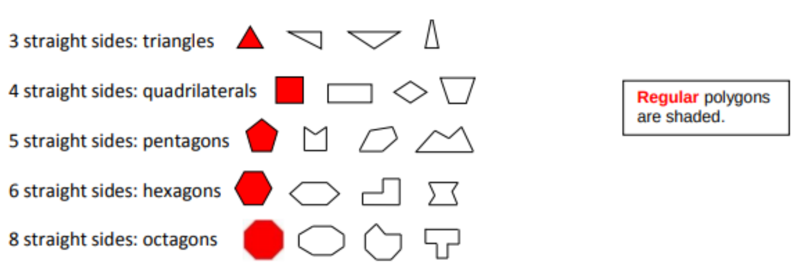
<td>

- Provide students with various sizes of a polygon. Have students count the number of sides and identify the polygon. Having a variety of these experiences with different polygons, students should begin to realize that a polygon, regardless of its dimensions, remains the same shape.
- Use geoboards to create polygons.



- Students should find examples of polygons in the world around them. Sort the shapes according to the number of sides. By sorting polygons according to the number of sides, students can learn the names for the polygons.
- Students should focus on comparing the number of sides as the key attribute for classifying polygons.
- In the following diagram, the shaded polygons are regular polygons, and all others are irregular polygons.



</td>
</tr>
</table>

D. What are the most appropriate methods and activities for assessing student learning?

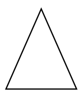
Some sample questions related to geometry which will be used to represent some of the appropriate methods and activities for assessing student learning are shown.

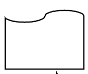
Encourage students to share their thinking, their strategies, and their solutions. Rich discussion fosters the development of alternative problem-solving strategies.

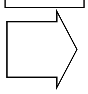
<table>
<tr><th colspan="2">Cognitive Level of Questions Examples for Geometry</th></tr>
<tr>
<td>

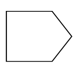
1. Knowledge (3G02)

Which figure **is not** a polygon?

A. 

B. 

C. 

D. 

Correct Answer: B

</td>
<td>

2. Analysis (3G01)

I am a 3-D object.
I have 5 faces.
I have 5 vertices.
I have 8 edges.

Which shape am I?

A. cube
B. sphere
C. square-based pyramid
D. triangular-based pyramid

Show what you know using words, pictures, or symbols.

Correct Answer: D

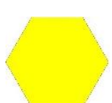
</td>
</tr>
<tr>
<td>

3. Knowledge (3G02)

Which figure is a quadrilateral?

A. 

B. 

C. 

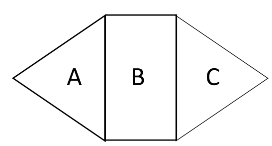
D. 

Correct Answer: B

</td>
<td>

4. Application (3G02)

Which polygon results when the geometric figures of A, B, and C are joined in this way?



A. a hexagon

B. a pentagon

C. an octagon

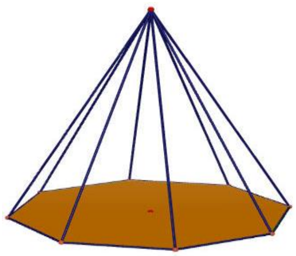
D. a quadrilateral

Correct Answer: A

</td>
</tr>
</table>

5. Analysis (3G01)

Look at the following pyramid:



Which statement is **true**?

A. The pyramid has 9 faces, 9 edges and 9 vertices.

B. The pyramid has a hexagonal base and 8 triangular faces.

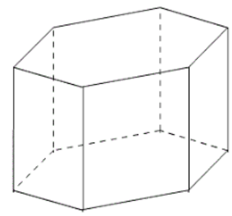
C. The pyramid has 8 triangular faces, 8 edges and 8 vertices.

D. The pyramid has an octagonal base and 16 edges.

Correct Answer: D

6. Analysis (3G01)

Look at the following prism:



Which statement is **true**?

A. The prism has 8 faces, 18 edges and 12 vertices.

B. The prism has 6 faces, 18 edges and 12 vertices.

C. The prism has a hexagonal base and 8 rectangular faces.

D. The prism has an octagonal base, 8 faces and 12 vertices.

Correct Answer: A

7. Knowledge (3G01)

What objects have no vertices?

A. the prism and the sphere

B. the cylinder and the pyramid

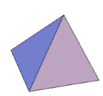
C. the prism and the pyramid

D. the cylinder and the sphere

Correct Answer: D

8. Knowledge (3G01)

Which object resembles a cone?

A. 

B. 

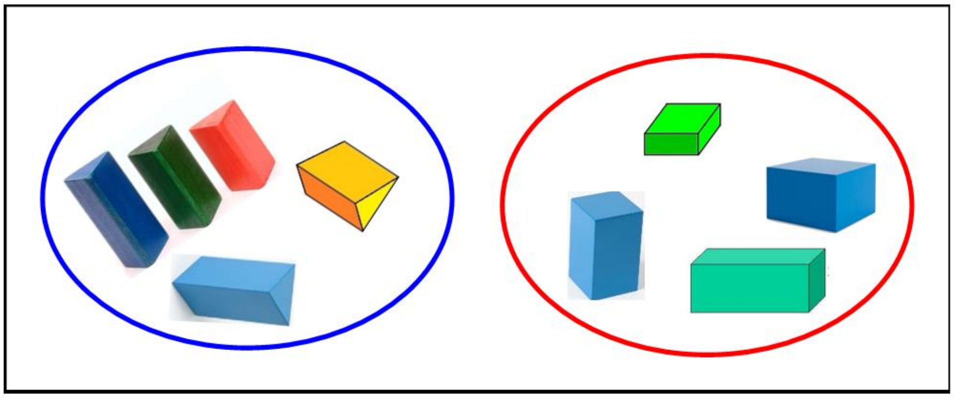
C. 

D. 

Correct Answer: C

9. Analysis (2G01)

Robert has sorted the following prisms using a Venn diagram:



Which sorting rule did he use?

A. Prisms that have 6 faces and prisms that have 8 vertices.

B. Prisms that have 6 edges and prisms that have 6 faces.

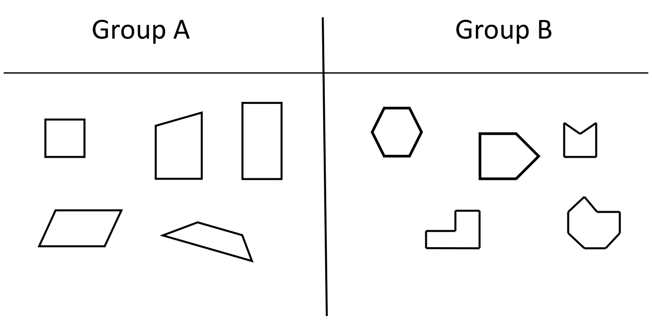
C. Prisms that have 6 vertices and prisms that have 12 edges.

D. Prisms that have 6 vertices and prisms that have 8 edges.

Correct Answer: C

10. Analysis (2G01)

Lucie sorted these polygons.
What sorting rule did Lucie use?



A. Polygons that have 4 sides and polygons that have 5 sides.

B. Polygons that have 4 sides and polygons that have more than 4 sides.

C. Polygons that have 4 sides and polygons that have 6 sides.

D. Polygons that have more than 4 sides and polygons that have 8 sides.

Correct Answer: B

Mathematics in Grade 3 Lesson Learned 7
Statistics and Probability

Students were challenged using tally marks, lists, charts, line plots, and bar graphs to organize data relevant to their everyday life. Students need opportunities and experiences to interpret information collected, organized, and displayed in tally charts, charts, line plots and bar graphs. Students need to develop the skill of interpreting graphs, answering questions, and drawing conclusions from those tally charts, line plots and bar graphs. They need to be encouraged to ask or write questions that go beyond simplistic reading of a graph. Both literal and inferential comprehension questions need to be asked.

A. What conclusions can be drawn from the Nova Scotia Assessment: Mathematics in Grade 3?

Students did very well interpreting data from line plots. Line plots contain information that can be used to answer questions using a visual comparison, counting, and reading labels. It offers students a visual comparison of the different quantities of every piece of data. The cognitive level of these questions is application and analysis.

Students did very well when interpreting the data from a vertical bar graph when asked to answer a straightforward literal comprehension question about the data. They understood that they had to add all the data together to find the correct answer. This was an application question.

Students had difficulty when asked to interpret a table showing the results of a survey using tally marks. They understood the concept of the tally table and what it represented but could not interpret what they were being asked to answer for the data displayed. They did not realize that to answer the question, they had to compare two quantities and then subtract the smallest quantity from the largest in order to determine their answer. This was a knowledge question.

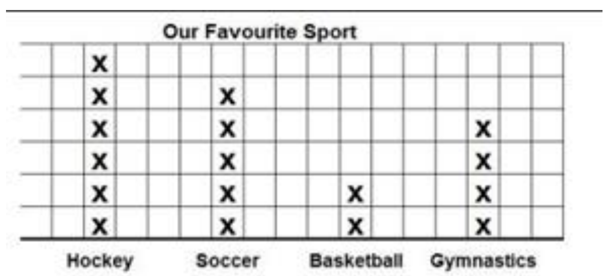
Students also had difficulty when asked to interpret a question about information displayed in a bar graph. Again, they understood the concept of the bar graph and what it represented, but they did not realize that in order to answer the question, they had to perform an operation of addition with all the information given in each bar being displayed. This was an application question.

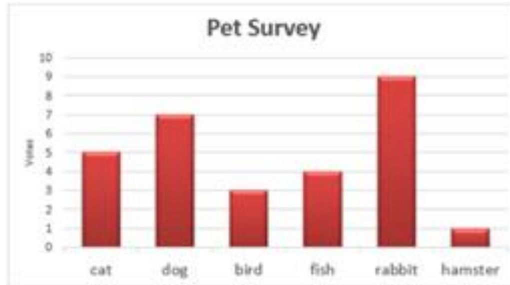
Students found it difficult to draw conclusions by comparing data represented in different types of displays. They were asked to draw a conclusion concerning the common attributes of line plots, horizontal bar graphs, pictographs, and vertical bar graphs with the same given set of data. They could not determine which data display did not represent the data correctly.

B. Do students have any misconceptions or errors in their thinking?

It is important to distinguish between misconceptions and errors. Misconceptions are, in effect, the misunderstandings about mathematical ideas which students entertain, and which usually lead to errors occurring. These are the most serious kinds of errors which require urgent action on the part of the teacher. Errors may occur for a variety of underlying reasons, ranging from careless mistakes to errors resulting from misconceptions. - John SanGiovanni – (*Mine The Gap For Mathematical Understanding Grades K-2*)

Results of the 2018–2019 Nova Scotia Assessment Mathematics in Grade 3
● 64% of students had difficulty identifying the bar graph that best represents a set of given data

Common Misconceptions/Errors	**What it Looks Like**
A common misconception or error that many students make is concerning the common attributes of line plots, horizontal bar graphs, pictographs, and vertical bar graphs with the same given set of data. They should also notice that the attributes can differ; for example, there could be different titles, different use of the horizontal axis, and different labels. Although students did well when working with line plots, they did make errors when reading or counting the Xs on a line plot.	The attributes that are common include: ● title, ● labels, ● horizontal axis, ● use of dots or crosses 

The difficulty with bar graphs, whether horizontal or vertical, is that students interpret data from the wrong bar on the graph. Therefore, when asked to answer questions concerning the bar graph data, it is not correct.	

C. What are the next steps in instruction for the class and for individual students?

<table>
<tr><th></th><th>How to Support</th></tr>
<tr><td>Statistics and Probability

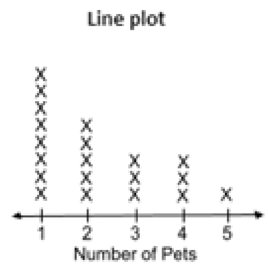
Students should be encouraged to collect, organize, and record their data using a tally system, line plots, charts, and lists to answer questions relevant to their everyday life.</td><td>

Provide students with opportunities to use tally marks and lists to keep track of information as they collect it.
A list can be made into a chart on which students would record their tally marks. Grouping the tally marks makes it easier for students to total the numbers by skip counting.

Tally System
<table>
<tr><th colspan="2">Students Who Have Pets</th></tr>
<tr><th>Number of Pets</th><th>Number of Students</th></tr>
<tr><td>1</td><td>~~IIII~~III</td></tr>
<tr><td>2</td><td>~~IIII~~</td></tr>
<tr><td>3</td><td>III</td></tr>
<tr><td>4</td><td>III</td></tr>
<tr><td>5</td><td>I</td></tr>
</table>
Chart/List
<table>
<tr><th colspan="2">Students Who Have Pets</th></tr>
<tr><th>Number of Pets</th><th>Number of Students</th></tr>
<tr><td>1</td><td>8</td></tr>
<tr><td>2</td><td>5</td></tr>
<tr><td>3</td><td>3</td></tr>
<tr><td>4</td><td>3</td></tr>
<tr><td>5</td><td>1</td></tr>
</table>
</td></tr>
</table>

<table>
<tr><td></td><td>

Ensure students include a title or heading and labels on constructed charts and graphs to inform the reader about the meaning of the data. Draw students’ attention to how difficult it is to make sense of a graph when a title is not provided, such as in the line plot pictured below.
Provide students with opportunities to organize data on a line plot(s) (first using grid paper). A line plot provides a bridge from tally charts to bar graphs.



</td></tr>
<tr><td>Students need to develop the skill of interpreting graphs, answering questions, and drawing conclusions from tally charts, bar graphs, and line plots.</td><td>

Provide opportunities for students to discuss the information obtained from a display of data. Students should be encouraged to work together to formulate questions that can be answered by other students using the data.
Present students with vertical and horizontal bar graphs that represent two different sets of data and discuss the similarities and differences found between the two bar graphs, such as title, axes, and labels for the axes, numerical scale, and bars.
Have students draw conclusions from the information presented in graphs. They should be encouraged to ask questions that go beyond simplistic reading of a graph.
Teachers should ask both literal and inferential comprehension questions, such as, What can you tell about _____ by looking at this graph? How many more/less than …? Based on the information presented in the graph, what other conclusions can you make? Why do you think _____?

</td></tr>
</table>

D. What are the most appropriate methods and activities for assessing student learning?

Some sample questions related to statistics and probability which will be used to represent some of the appropriate methods and activities for assessing student learning are shown.

Encourage students to share their thinking, their strategies, and their solutions. Rich discussion fosters the development of alternative problem-solving strategies.

Cognitive Level of Questions Examples for Statistics and Probability

1. Knowledge (2SP01)

Emma asked her classmates a question to collect data. She recorded their answers in the following list:

Favourite Type of Film

Drama	3
Science-Fiction	4
Comedy	6
Action	2

Which question did Emma ask her classmates?

A. Do you prefer films about the police?

B. Do you prefer comedy more than action films?

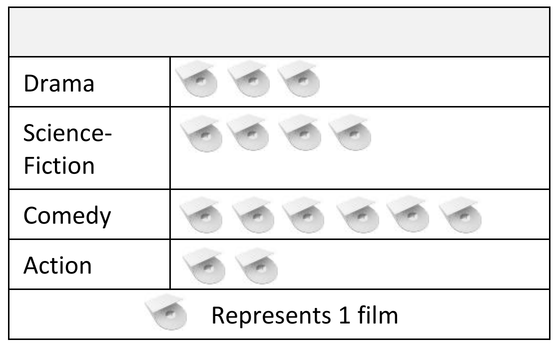
C. What is your favourite type of film?

D. Do you prefer drama more than action films?

Correct Answer: C

2. Application (2SP02)

Emma constructed the following pictograph to present the collected data about her classmates. She made some errors.



What error did Emma make?

A. She forgot the title.

B. The number of comedy films is not equal to the number recorded in the list above.

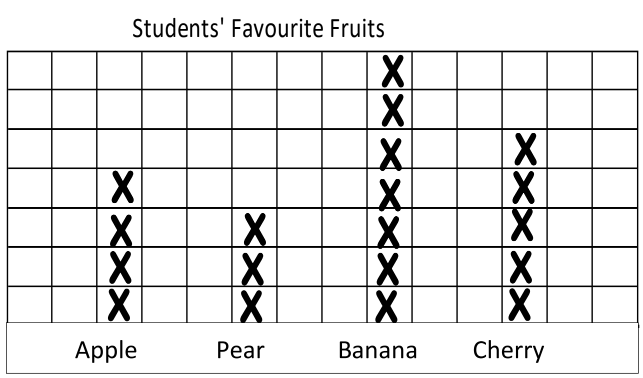
C. She forgot the legend.

D. She forgot the title and the legend.

Correct Answer: A

3. Application (3SP01)

Lee surveyed the Grade 2 students about their favourite fruits. The line plot shows the results of this survey.



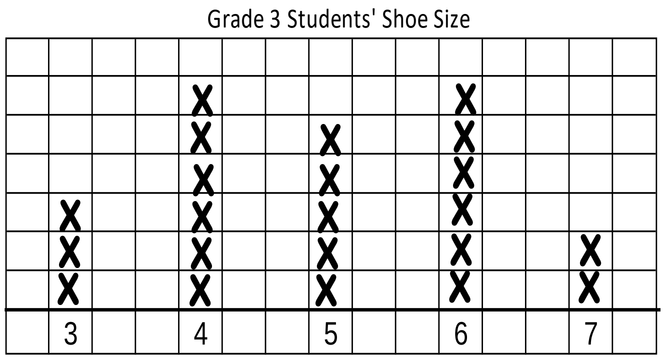
Which statement is **true**?

A. The students prefer apples to bananas.
B. The students prefer pears to cherries.
C. The apple is the most popular fruit.
D. The pear is the least popular fruit.

Correct Answer: D

4. Application: (3SP01)

This line plot shows the shoe size of Grade 3 students:



What conclusion can you draw from this line plot?

A. There are more students with size 5 shoes, then size 4 shoes.

B. There are more students with size 7 shoes, then size 3 shoes.

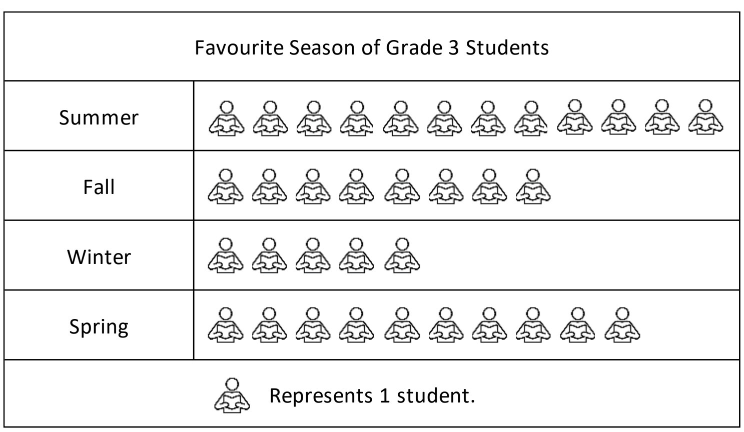
C. There are fewer students with size 6 shoes, then size 7 shoes.

D. There are just as many students with size 4 shoes, as students with size 6 shoes.

Correct Answer: D

5. Application (2SP02)

Tony asked the Grade 3 students about their favourite season. This pictograph shows the results of Tony's survey.



Which statement is **true**?

A. Spring is the most popular season.

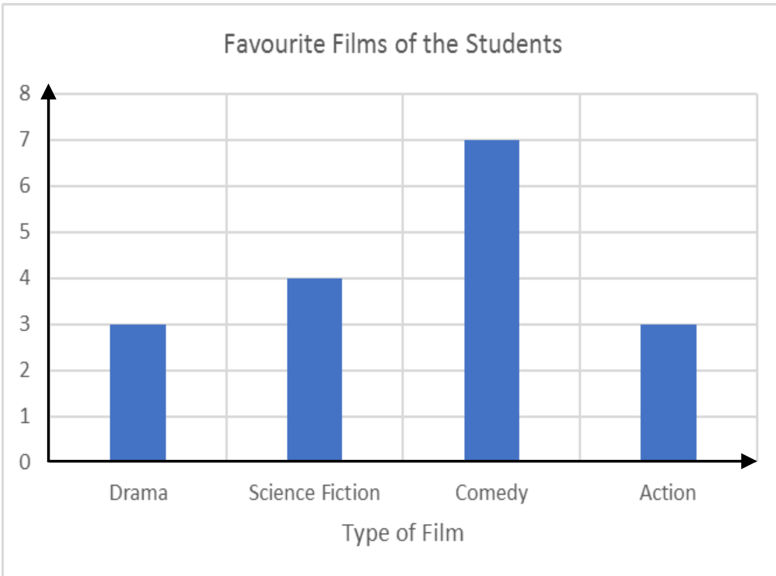
B. Fall is the least popular season.

C. Fewer students prefer fall to winter.

D. Summer is the most popular season.

Correct Answer: D

6. Analysis (3SP02)
Tanya used the data collected by Emma to construct the following bar graph:



Examine the bar graph. What errors did Tanya make?

A. The axes labels are missing.

B. The vertical axis label is missing.

C. The vertical axis label and the bar for comedy are missing.

Correct Answer: B

7. Application (3SP01)
This table shows the results of a survey of students favourite snacks.

Snacks	**Tally Number**
Cheese	𝍸 𝍸 𝍸
Granola Bars	𝍸 𝍸 |
Yogurt	𝍸 𝍸 𝍸 ||||
Vegetables	𝍸 𝍸 𝍸 ||

How many more students prefer vegetables than cheese?

A. 2

B. 4

C. 6

D. 13

Correct Answer: A

Mathematics in Grade 3 Lesson Learned 8
Problem Solving

Students need more exposure to application and analysis items in order to apply these higher order thinking skills when problem solving. Students need to be encouraged to understand that problems may have more than one entry point and there are many strategies to solve a problem. Students need to continue to work on translating between and among representations when problem solving. They do not seem to realize that they can use varied representations when asked to solve a word problem. These other representations may support their problem solving and their reasoning.

Learning through problem solving should be the focus of mathematics at all grade levels. Problem solving is one of the critical mathematical processes that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics. When students encounter new situations, and respond to questions such as, How would you …? or How could you …? the problem-solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.

In order for an activity to be problem-solving based, it must appropriately challenge students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but is simply practice. A true problem-solving activity requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement, perseverance, and collaboration. Problem solving is also a powerful teaching tool that fosters multiple, creative, and innovative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident mathematical risk takers.

Students need to be able to explore a wide variety of methods for solving and verifying problems in all areas of mathematics. They must be challenged to find multiple solutions for problems and be given opportunities to create and solve their own problems.

A. What conclusions can be drawn from the Nova Scotia Assessment: Mathematics in Grade 3?

We found that students have a good understanding of basic facts and procedures, and are successful when explicitly given all the information needed to do a knowledge question. But when given application and analysis items, they are not able to apply higher order thinking skills when problem solving. For example, students were not sure whether they should add or subtract when questions were presented in the context of a story problem. Overall, our students are experiencing challenges with problem solving across all mathematical strands. This appeared to be a theme throughout the assessment data when students were asked to "solve a word problem" or "solve a multi-step problem". The data also showed that, across all mathematics strands, students struggle with items that require translating among representations in problem-solving contexts.

B. Do students have any misconceptions or errors in their thinking?

It is important to distinguish between misconceptions and errors. Misconceptions are, in effect, the misunderstandings about mathematical ideas which students entertain, and which usually lead to errors occurring. These are the most serious kinds of errors which require urgent action on the part of the teacher. Errors may occur for a variety of underlying reasons, ranging from careless mistakes to errors resulting from misconceptions. - John SanGiovanni – (*Mine The Gap For Mathematical Understanding Grades K-2*)

Results of the 2018–2019 Nova Scotia Assessment Mathematics in Grade 3
• 53% of the students had difficulty using place-value strategies, to represent a number written in words symbolically
• 54% of the students had difficulty estimating the cost of objects in a context to the nearest hundred ($)

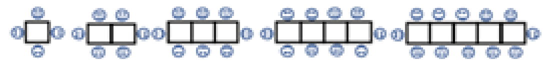
Common Misconceptions/Errors	What it Looks Like
Students often associate solving a mathematical problem to doing routine calculations without attending to the meaning of the context and to the credibility of the answers. Many students have the misconception that a word problem (a problem in a context) is too hard for them to attempt. Putting words around the numbers seems to obstruct their ability to think about the question.	• Some students will take numbers from a story problem and add or subtract using the two numbers without understanding the action taking place in the problem. They do not question whether their solution is reasonable to the given problem. • **Problem:** Some friends are coming to your birthday party. A square table with 4 chairs can seat 4 of your friends. If 2 square tables are put together, you can seat 6 of your friends. How many friends can you seat with 5 square tables put together?
Students appeared to have a limited repertoire of problem-solving strategies to help them attempt a problem. There are many strategies and ways to solve the problem. Some students find it difficult to find an entry point to begin to solve the problem. Another misconception students have is that there is only one way to solve a word problem.	For example, when given an analysis problem in the context of an increasing pattern, students struggle with answering “What strategy could I use to solve this problem”?

Students also tend to forget that they may translate between and among representations to help them solve a problem.

If asked to solve a word problem using words, symbols or pictures, most students only provide symbols.

Other representations support their problem solving.

Solution 1:



Solution 2:

Tables	Chairs	Number of Friends
1	4	4
2	6	6
3	8	8
4	10	10
5	?	?

- Students could have used concrete materials/manipulatives such as coloured tiles, two-sided counters, cube-a-link blocks, and pictures or numbers to solve this question. Showing how the problem could be represented more abstractly with tiles would help make the connection to increasing patterns.

C. What are the next steps in instruction for the class and for individual students?

	How to Support
Problem Solving The problem-solving process consists of four steps – understand the problem, devise a plan, carry out the plan, and look back to determine the reasonableness of an answer.	● Teach lessons through a problem-solving approach when appropriate. Some lessons which require certain skills for conceptual understanding will require practice before having students apply their understanding within a problem-solving context. ● Provide a context or reason for the learning by beginning the lesson with a problem to be solved. ● After students have had the opportunity to think about the problem and work through the solution, teachers can draw the procedure used by students from their work rather than provisioning students with procedures to use when solving a given set of problems (Small, 2005, p. 154).
Problem Solving **Story Structures** Students need to learn an important aspect of problem solving in grades 1–3 is that addition and subtraction problems can be categorized based on the kinds of relationships they represent.	● Present and develop all story problem structures from students' experiences. ● Initial work with the story structures will focus on join and separate types of problems because students associate the actions in these problems with the operations. However, students must also experience addition and subtraction in part-part-whole and comparison situations.
Manipulatives Students need to learn that manipulatives can and should be used to model the strategies and the story structures.	● Provide students with manipulative such as: ○ two-sided counters ○ linking cubes ○ number cubes ○ Ten-frames ○ walk-on number line ○ base-ten blocks ○ Rekenrek

<table>
<tr>
<td>A Problem-Solving Approach

A problem-solving approach is embedded and expected throughout our curriculum, grades Primary to 12, in all strands.

Problem-solving is a key instructional strategy that enables students to take risks, secure in the knowledge that their thoughts, queries, and ideas are valued.</td>
<td>Provide direct instruction on problem-solving strategies as students share their own solutions and findings. Use students’ methods to guide instruction.
Elaborate on the methods used by students to solve and justify.
Encourage students to comment and ask questions of their peers.</td>
</tr>
<tr>
<td>Problem-Solving Strategies

Students are already drawing on personal strategies for problem solving, in many of the activities they undertake.</td>
<td>Expand students’ personal repertoires through explicit instruction on specific strategies including:
Make a chart or table
Draw a picture
Work backward
Make an organized list
Use a model
Solve a simpler problem
Guess and test
Use a pattern
The Strategies Toolkit lessons listed above can be found in the Pearson Math Makes Sense Series.</td>
</tr>
<tr>
<td>Problem-Solving

Three-Part Lesson

Van de Walle and Lovin (2006b), in their resource, Teaching Student-Centered Mathematics Grades 3–5, suggest a three-part lesson format for teaching through problem-solving. This same approach is used in our core resource, Math Makes Sense (Pearson, 2009b, pp. 13–14).</td>
<td>Before

Before students begin:
Prepare meaningful problem scenarios for students that are sufficiently challenging.
Ensure the problem is understood by all.
Explain the expectations for the process and the product.
During

As students work through the problem:
Let students approach the problem in a way that makes sense to them.
Listen to the conversations to observe thinking.
Assess student understanding of their solution.
Provide hints or suggestions if students are on the wrong path.
Encourage students to test their ideas.
Ask questions to stimulate ideas.</td>
</tr>
</table>

	After After students, have solved their problem: ● Gather for a group meeting to reflect and share. ● Make the mathematics explicit through discussion. ● Highlight the variety of answers and methods. ● Encourage students to justify their solutions. ● Encourage students to comment positively or ask questions regarding their peer's solutions.
Assessing Problem Solving Use the suggested strategies found in the opposite column with your students when assessing problem solving.	● Concepts and skills should be connected to everyday situations and other curricular areas. ● Encourage students to make connections to make mathematics come alive through math-to-world, math-to-math, and math-to-self connections. ● Develop students' mathematical vocabulary, initiate effective ways to navigate informational text, and encourage students to reflect on what they have learned. ● Embed strategies/tools such as the Frayer Model, Concept Circles, Three-Read Strategy, Exit Cards, etc. to assess student learning. Please refer to the Appendix B found at the end of this document for strategies with illustrative examples.

D. What are the most appropriate methods and activities for assessing student learning?

Some sample questions related to problem-solving which will be used to represent some of the appropriate methods and activities for assessing student learning are shown.

Encourage students to share their thinking, their strategies, and their solutions. Rich discussion fosters the development of alternative problem-solving strategies.

For more information related to the sample questions below, please refer to Appendix E.

Cognitive Level of Questions Examples for Problem Solving	
1. Application (2N09) Natasha has 4 T-shirts and 2 pairs of pants. How many different outfits can Natasha make? Show how you solved the problem and explain your strategy. **Strand: Number/Stats & Probability** ***Possible Strategy*:** *Make a chart* *Make a table* *Draw a picture* **Correct Answer: 8 outfits**	**2. Application (3N13)** Megan and Danielle each ordered the same size pizza. Megan asked to have her pizza cut into fourths. Danielle asked to have her pizza cut into sixths. Who has the larger pieces of pizza? Show how you solved the problem and explain your strategy. **Strand: Number** ***Possible Strategy*:** *Use a model* *Draw a picture* **Correct Answer: Megan**
3. Application (3N09) Peter has 123 marbles. He gives some marbles to his friend Paul. Now Peter has 86 marbles. How many marbles did Peter give to Paul? Show how you solved the problem and explain your strategy. **Strand: Number** ***Possible Strategy*:** *Use a model* **Correct Answer: 37 marbles**	**4. Analysis (3N12)** Sebastian and his sister have bicycles and tricycles. The bicycles and tricycles have 21 wheels altogether. If they have 3 tricycles, then how many bicycles do they have? Show how you solved the problem and explain your strategy. **Strand: Number** Show how you solved the problem and explain your strategy. ***Possible Strategy*:** *Work backward* **Correct Answer: 6 bicycles**

5. Analysis (3PR01)

Monette has 23 apples.
She eats three apples every day.

How many apples did Susan have left after seven days?

Show how you solved the problem and explain your strategy.

Strand: Patterns and Relations

***Possible Strategy*:**
Use a pattern
Make a list
Use a model

Correct Answer: 2 apples

6. Application (3N02)

Marbles come in packages of 10, 25, and 50.
You need 160 marbles.

Find 3 ways you could buy the marbles.

Show how you solved the problem and explain your strategy.

Strand: Number

***Possible Strategy*:**
Make an organized list

Correct answer: three possible combinations

7. Analysis (3N12)

For James birthday, his mother wants to cover each table using a 4 m long piece of paper tablecloth. How many tables can she cover using a roll of paper tablecloth which is 20 m long?

Show how you solved the problem and explain your strategy.

Strand: Number

***Possible Strategy*:**
Make an organized list
Work backward

Correct Answer: 5 tables

8. Analysis (3M03/3PR03)

Marie's height is 3 cm more than Norman's height.
Norman's height is 2 cm more than Jessica's height.

If Marie's height is 126 cm, what is Jessica's height?

Show how you solved the problem and explain your strategy.

Strand: Measurement

***Possible Strategy*:**
Draw a picture
Work backwards

Correct Answer: 121 cm

9. Analysis (3N02)

Lucy has some money. She has some 5 cent coins, some 10 cent coins and 25 cent coins.
She buys a used book for 45 cents.
She used all her money to buy this book and now has no money left.

How many ways could Lucy pay for this book using all of her money?

Show how you solved the problem and explain your strategy
Strand: Number
***Possible Strategy*:**
Draw a picture
Draw a chart

Correct Answer: 8 combinations

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Appendix A: Cognitive Levels of Questioning

Knowledge

Facility in using mathematics, or reasoning about mathematical situations, depends on mathematical knowledge and familiarity with mathematical concepts. Knowledge of a wide range of mathematical terminology, number properties, geometric properties, basic facts, and mathematical procedures open the door to the development of a deeper mathematical understanding and purposeful mathematical thinking.

Knowledge questions require students to recall or recognize information, names, definitions, or steps in a procedure.

Knowledge questions, items, or tasks require learners to:

- rely on recall and recognition of facts, terms, concepts, or properties
- recognize an equivalent representation within the same form, for example, from symbolic to symbolic
- perform a specified procedure or a learned method; for example, calculate a sum, difference, product, or quotient
- carry out routine procedures requiring algebraic thinking
- draw or measure simple geometric figures
- read information from a graph, table, or figure

Possible verbs: *identify, calculate, recall, recognize, find, evaluate, solve, list, define, name, use,* and *measure*

Application

Application questions challenge students to go beyond basic knowledge of mathematics to applying skills or reasoning to solve a typical problem. Application questions focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas. Prior or new knowledge is often used to complete a task when solving application questions. Students are expected to understand the problem, as well as identify and use an appropriate (personal) strategy to solve the problem.

Application questions require students to make connections, represent a situation in more than one way (translating between representations) or solve contextual problems.

Application questions, items, or tasks require learners to:

- show deeper mathematical understanding
- choose and apply personal strategies and reasoning to solve a problem
- represent a situation mathematically, using or translating between appropriate representations, for a particular purpose or within a given context
- involve more flexibility of thinking
- interpret and solve a word or story problem
- consolidate skills and knowledge from multiple concepts or strands
- make connections between facts, terms, properties, or operations
- compare figures or statements
- explain and provide the steps in a solution process
- identify or extend a pattern
- use information from a graph, table, or figure to solve a problem
- model a routine problem using an appropriate mathematical representation
- solve a routine problem requiring multiple steps
- interpret a simple argument
- apply a variety of skills and knowledge from prior learning to solve problems
- examine solutions to routine problems to identify the correct solution or identify errors in a given solution
- implement and execute a set of mathematical instructions (i.e., given a set of specifications, draw figures and shapes)
- make correct decisions about set membership

Possible verbs: *sort, organize, estimate, interpret, predict, translate, summarize, solve, describe, classify, sort, extend, verify, show, represent, model, correct, apply, compare,* and *explain*

Analysis

Analysis questions provide opportunity for students to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop further understanding. Extending their thinking and reasoning to solve analysis level questions challenges students to formulate a plan and monitor their own processes. Students must draw on prior knowledge to solve more complex problems and there is not a predictable, well-rehearsed approach or pathway explicitly suggested for these types of questions.

Analysis questions require students to go beyond comprehension and application to higher order thinking skills, such as generalizations and non-routine problem-solving.

Analysis questions, items, or tasks require learners to:

- use complex and non-algorithmic thinking
- problem solve, reason, plan, analyze, judge, explore, and employ creative thought
- think in abstract, creative, and sophisticated ways
- analyze a mathematical situation by determining or describing the relationships between mathematical arguments or objects
- describe how different representations can be used for different purposes
- analyze similarities and differences between procedures and concepts
- generalize results or patterns, sometimes to make them more widely applicable
- solve a novel, a multi-step, or a multiple decision point problem
- solve a problem with more than one strategy (including personal strategies)
- justify a solution to a problem or an assumption made in a mathematical model
- making connections between related mathematical ideas and other content areas
- describe, compare, and contrast solution methods
- justify the truth or falsity of a statement by referencing mathematical results or properties through deductive reasoning
- create mathematical models for a complex situation
- interpret the significance of findings in relation to a given concept
- create a problem using given data and conditions
- solve open-ended problems, questions with more than one solution

Possible verbs: *analyze, investigate, formulate, explain, describe, prove, generalize, compare, contrast, relate, connect, create, reflect, infer, justify,* and *examine*

Distribution of cognitive levels

Below are the percentages of knowledge, application, and analysis questions in the Nova Scotia provincial assessments for Mathematics in Grade 3:

- Knowledge 20–30%
- Application 50–60%
- Analysis 10–20%

These percentages are also recommended for classroom-based assessments.

Appendix B: From Reading Strategies to Mathematics Strategies

The following table illustrates when strategies are to be used, and during what part of the three-part lesson format (as referenced in Lessons Learned 1).

Name of Strategy	Before	During	After	Assessment
1. Concept Circles	X	X	X	X
2. Frayer Model	X	X	X	X
3. Concept Definition Map	X	X	X	X
4. Word Wall	X	X	X	
5. Three-Read		X	X	
6. Graphic Organizer	X	X	X	X
7. K-W-L	X		X	X
8. Think-Pair-Share	X	X	X	X
9. Think-Aloud	X	X		X
10. Academic Journal-Mathematics	X	X	X	X
11. Exit Cards			X	X

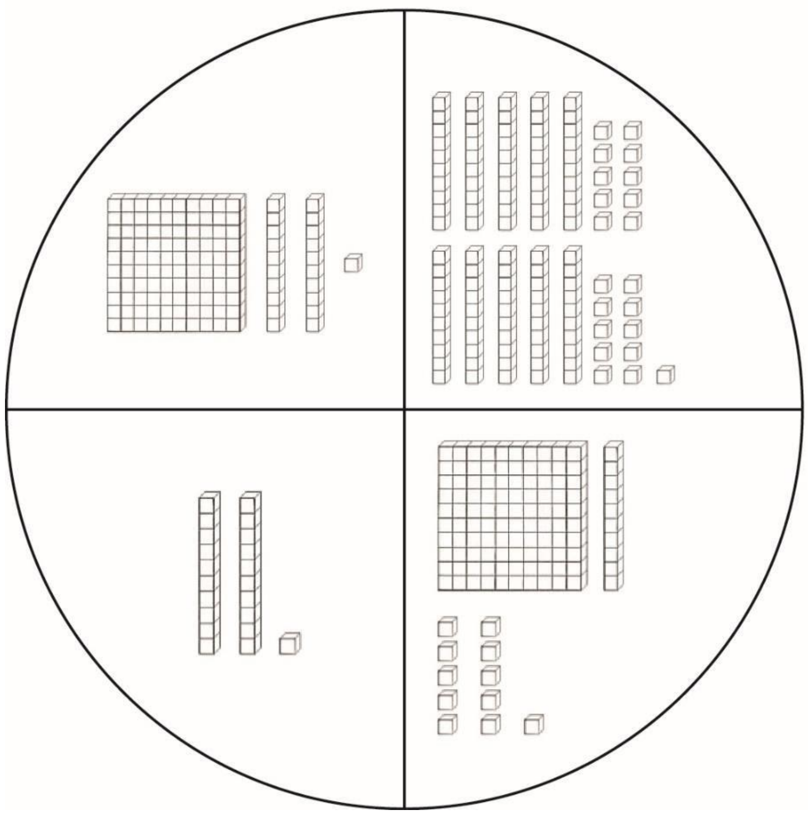
1. **Concept Circle**
   A concept circle is a way for students to conceptually relate words, terms, expressions, etc. As a "before" activity, it allows students to predict or discover relationships. As a "during" or "after" activity, students can determine the missing concept or attribute or identify an attribute that does not belong.

   The following steps illustrate how the organizer can be used:
   - Draw a circle with the number of sections needed.
   - Choose the common attributes and place them in the sections of the circle.
   - Have students identify the common concepts to the attributes.

   This activity can be approached in other ways.
   - Supply the concept and some of the attributes and have students apply the missing attributes.
   - Insert an attribute that is not an example of the concept and have students find the one that does not belong and justify their reasoning.

Example of a Concept Circle

Concept: Which pictures of base-ten blocks represent 121?

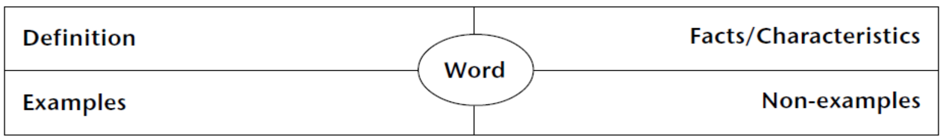


Frayer Model

The Frayer Model is a graphic organizer used to categorize a word and build vocabulary. It prompts students to think about and describe the meaning of a word by

- giving a definition
- describing main characteristics
- providing examples and non-examples of the word or concept

It is especially helpful to use with a concept that might be confusing because of its close connections to another concept.

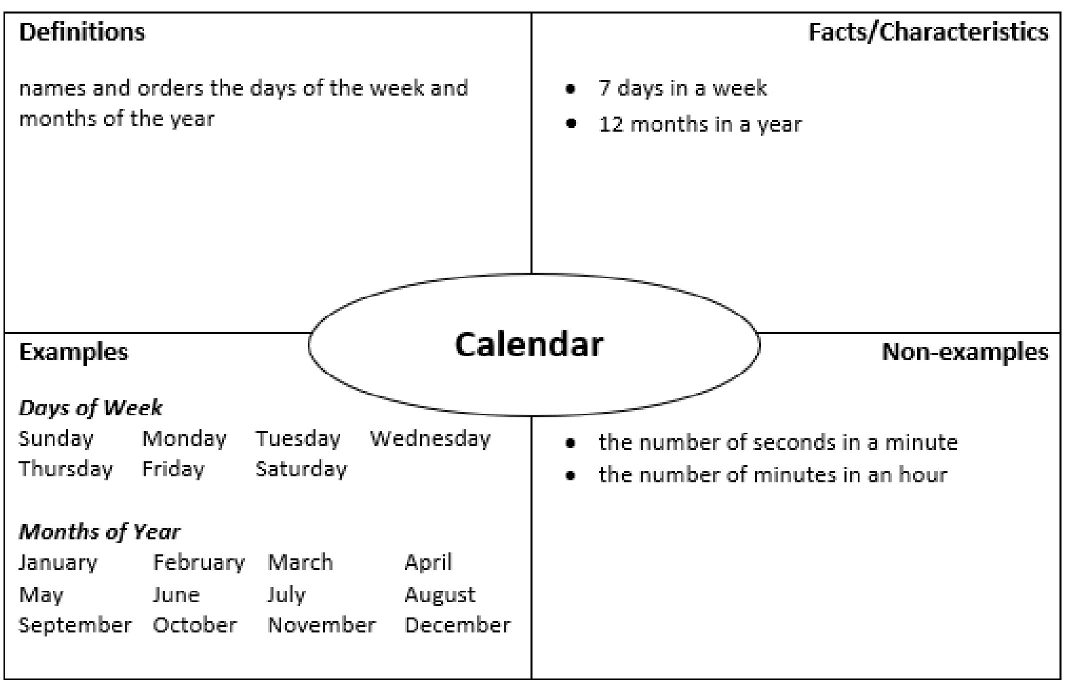


The following steps illustrate how the organizer can be used.

1. Display the template for the Frayer model and discuss the various headings and what is being sought.
2. Model how to use this example by using a common word or concept. Give students explicit instructions on the quality of work that is expected.
3. Establish the groupings (e.g. pairs) to be used and assign the concept(s) or word(s).
4. Have students share their work with the entire class.

This is an excellent activity to do in poster form to display in class. Each group might do the same word or concept, or different words or concepts could be assigned.

Example of a Frayer Model



3. Concept Definition Map

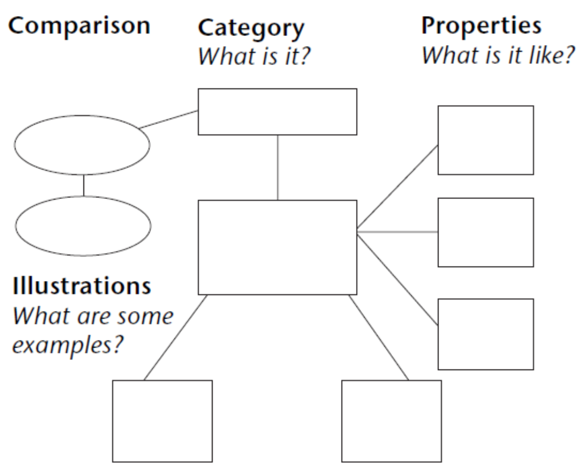
The purpose of a concept definition map is to prompt students to identify the main components of a concept, show the interrelatedness, and build vocabulary. Information is placed into logical categories, allowing students to identify properties, characteristics, and examples of the concept.

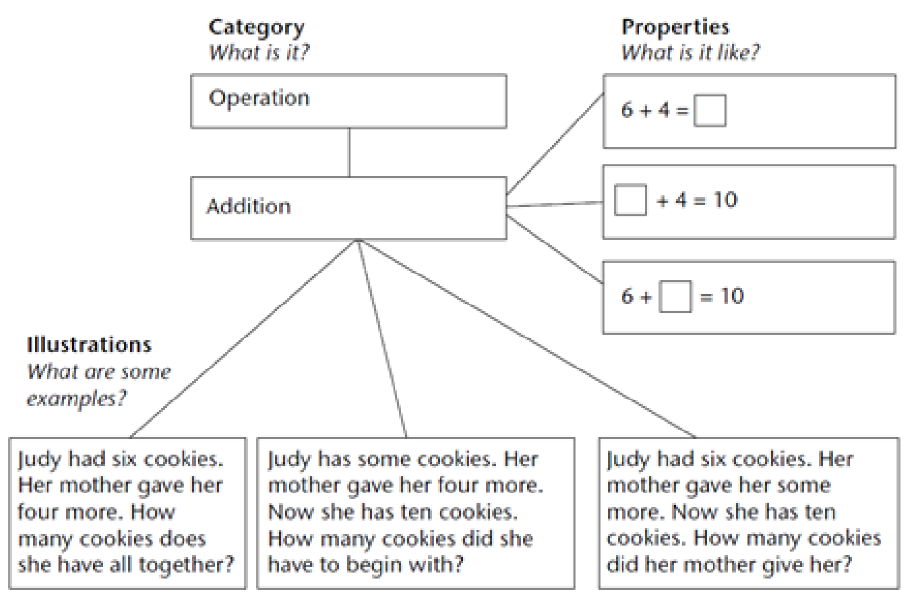
The following steps illustrate how this organizer can be used.

1. Display the template for the concept definition map.
2. Discuss the different headings, what is being sought, and the quality of work that is expected.
3. Model how to use this map by using a common concept.
4. Establish the concept(s) to be developed.
5. Establish the groupings (e.g. pairs) and materials to be used to complete the task.
6. Complete the activity by having the students write a complete definition of the concept.

Encourage students to refine their map as more information becomes available.

Example of a Concept Definition Map





4. Word Wall

A mathematics word wall is based upon the same principle as a reading word wall, found in many classrooms. It is an organized collection of words that is prominently displayed in the classroom and helps students learn the language of mathematics. A word wall can be dedicated to a concept, big idea, or unit in the mathematics curriculum. Words are printed in bold block letters on cards and then posted on the wall or bulletin region.

Illustrations placed next to the word on the word wall can add to the students' understanding. Students may also elaborate on the word in their journals by illustrating, showing an example, and using the word in a meaningful sentence or short paragraph. Students can be assigned a word and its illustration to display on the word wall. Room should be left to add more words and diagrams as the unit or term progresses.

As a new mathematical term is introduced to the class, students can define and categorize the word in their mathematics journal under an appropriate unit of study. Then the word can be added to the mathematics word wall so students may refer to it as needed. Students will be surprised at how many words fall under each category and how many new words they learn to use in mathematics.
Note: The word wall is developed one word at a time as new terminology is encountered.

Use the following steps to set up a word wall.

1. Determine the key words that students need to know or will encounter in the topic or unit.
2. Print each word in large block letters and add the appropriate illustrations.
3. Display cards when appropriate.
4. Regularly review the words as a warm-up or refresher activity.

5. Three-Read Strategy

Using this strategy, found in *Toward a Coherent Mathematics Program: A Study Document for Educators* (Nova Scotia Department of Education 2002), the teacher encourages students to read a problem three times before they attempt to solve it. There are specific purposes for each reading.

First Read

The students try to visualize the problem to get an impression of its overall context. They do not need specific details at this stage, only a general idea so they can describe the problem in broad terms.

Second Read

The students begin to gather facts about the problem to make a more complete mental image of it. As they listen for more detail, they focus on the information to determine and clarify the question.

Third Read

The students check each fact and detail in the problem to verify the accuracy of their mental image and to complete their understanding of the question.

During the Three-Read strategy, the students discuss the problem, including any information needed to solve it. Reading becomes an active process that involves oral communication among students and teachers; it also involves written communication as teachers encourage students to record information and details from their reading and to represent what they read in other ways with pictures, symbols, or charts. The teacher facilitates the process by posing questions that ask students to justify their reasoning, support their thinking, and clarify their solutions.

To teach the Three-Read Strategy, teachers should exaggerate each step as they model it. When they have students practice the strategy, teachers should ask questions that stimulate the kinds of questions that students should be asking themselves in their internal conversations. In every classroom, an ongoing discussion of this Three-Read strategy must be conducted, and many students will need to be reminded to use this strategy.

6. Graphic Organizer

A graphic organizer can be of many forms: web, chart, diagram, etc. Graphic organizers use visual representations as effective tools to do such things as

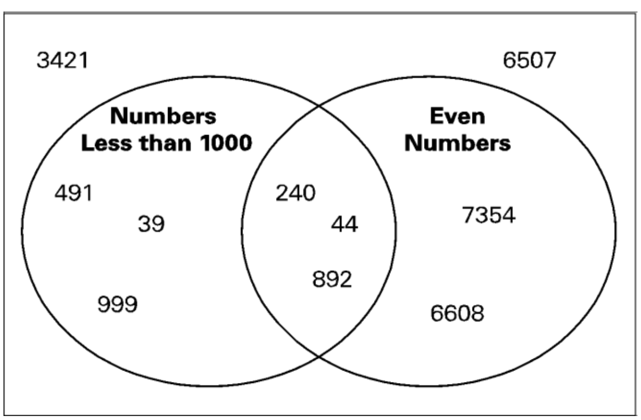
- activate prior knowledge
- analyze
- compare and contrast
- make connections
- organize
- summarize

The following steps illustrate how the organizer can be used.

1. Present a template of the organizer and explain its features.
2. Model how to use the organizer, being explicit about the quality of work that is expected.
3. Present various opportunities for students to use graphic organizers in the classroom.

Students should be encouraged to use graphic organizers on their own as ways of organizing their ideas and work. If the graphic organizer being used is a Venn diagram, it is important to draw a rectangle around Venn diagrams to represent the entire group that is being sorted. This will show the items that do not fit the attributes of the circle(s) outside of them, but within the rectangle. Therefore, elements of the set that do belong to Attribute A or Attribute B are shown within the rectangle but not within the circles in the rectangles.

Example of a Venn diagram



7. K-W-L (Know/Want to Know/Learned)

K-W-L is an instructional strategy that guides students through a text or mathematics word problem and uses a three-column organizer to consolidate the important ideas. Students brainstorm what they know about the topic and record it in the K column. They then record what they want to know in the W column. During and after the reading, students' record what they have learned in the L column.

The K-W-L strategy has several purposes:

- to illustrate a student's prior knowledge of a topic
- to give a purpose to the reading
- to help a student monitor their comprehension

Example: 3-D Objects

Know	Want to Know	Learned
A prism has vertices, faces, and edges. *The faces of the prism are rectangles.* *A milk box is a prism.* *A sphere is round.*	*Are there any other 3-D objects besides the prism?*	*A prism is named after its base.* *A prism has two bases.* *Other than the bases, faces of a prism are rectangles.* *There are the pyramids.* *A pyramid has only one base.* *We name a pyramid after its base.* *Other than the base, the faces of a pyramid are triangles.* *There is the cylinder.* *A cylinder has 2 bases which are circles.*

The following steps illustrate how the K-W-L can be used.

1. Present a template of the organizer to students, explain its features, and be explicit about the quality of work that is expected.
2. Ask them to fill out the first two sections (what they know and what they want to know before proceeding).
3. Check the first section for any misconceptions in thinking or weakness in vocabulary.
4. Have the students read the text, and taking notes as they look for answers to the questions they posed.
5. Have students complete the last column to include the answers to their questions and other pertinent information.
6. Discuss this new information with the class, and address any questions that were not answered.

8. Think-Pair-Share

Think-Pair-Share is a learning strategy designed to encourage students to participate in class and keep them on task. It focusses students' thinking on specific topics and provides them with an opportunity to collaborate and have meaningful discussion about mathematics.

- First, teachers ask students to think individually about a newly introduced topic, concept, or problem. This provides essential time for each student to collect his or her thoughts and focus on his or her thinking.
- Second, each student pairs with another student, and together the partners discuss each other's ideas and points of view. Students are more willing to participate because they do not feel the peer pressure that is involved when responding in front of the class. Teachers ensure that sufficient time is allowed for each student to voice his or her views and opinions. Students use this time to talk about personal strategies, compare solutions, or test ideas with their partners. This helps students to make sense of the problem in terms of their prior knowledge.
- Third, each pair of students shares with the other pairs of students in large-group discussion. In this way, each student has the opportunity to listen to all of the ideas and concerns discussed by the other pairs of students. Teachers point out similarities, overlapping ideas, or discrepancies among the pairs of students and facilitate an open discussion to expand upon any key points or arguments they wish to pursue.

9. Think-Aloud

Think-aloud is a self-analysis strategy that allows students to gain insight into the thinking process of a skilled reader as he or she works through a piece of text. Thoughts are verbalized, and meaning is constructed around vocabulary and comprehension. It is a useful tool for such things as brainstorming, exploring text features, and constructing meaning when solving problems. When used in mathematics, it can reveal to teachers the strategies that are part of a student's experience and those that are not. This is helpful in identifying where a student's understanding may break down and may need additional support.

The think-aloud process will encourage students to use the following strategies as they approach a piece of text.

- Connect new information to prior knowledge.
- Develop a mental image.
- Make predictions and analogies.
- Self-question.
- Revise and fix up as comprehension increases.

The following steps illustrate how to use the think-aloud strategy.

1. Explain that reading in mathematics is important and requires students to be thinking and trying to make sense of what they are reading.
2. Identify a comprehension problem or piece of text that may be challenging to students; then read it aloud and have students read it quietly.
3. While reading, model the process verbalizing what you are thinking, what questions you have, and how you would approach a problem.
4. Then model this process a second time, but have a student read the problem and do the verbalizing.
5. Once students are comfortable with this process, a student should take a leadership role.

10. Academic Mathematics Journal

An academic journal in mathematics is an excellent way for students to keep personal work and other materials that they have identified as being important for their personal achievement in mathematics. The types of materials that students would put in their journals would include:

- strategic lessons – lessons that they would identify as being pivotal as they attempt to understand mathematics
- examples of problem-solving strategies
- important vocabulary

Teachers are encouraged to allow students to use these journals as a form of assessment. This will emphasize to the student that the material that is to be placed in his or her journal has a purpose. Mark these journals only based on how students are using them and whether they have appropriate entries.

The goal of writing in mathematics is to provide students with opportunities to explain their thinking about mathematical ideas and then to re-examine their thoughts by reviewing their writing. Writing will enhance students' understanding of math as they learn to articulate their thought processes in solving math problems and learning mathematics concepts.

11. Exit Card

Exit cards are quick tools for teachers to become better aware of a students' mathematics understanding. They are written student responses to questions that have been posed in class or solutions to problem-solving situations. They can be used at the end of a day, week, lesson, or unit. An index card is given to each student (with a question that promotes understanding on it), and the student must complete the assignment before they can "exit" the classroom. The time limit should not exceed 5 to 10 minutes, and the student drops the card into some sort of container on the way out. The teacher now has a quick assessment of a concept that will help in planning instruction.

Samples:

Name

Red, Yellow, Green

Circle One.

- Red - You lost me.
- Yellow - I'm struggling a bit. Please go slower.
- Green - I've got it!

3-2-1 Activity

3 Things You Learned	2 Important Details	1 Question You Still Have
1. 2. 3.	1. 2.	1.

3-2-1 Activity

3 Things You Learned	2 Important Details	1 Question You Still Have
1. 2. 3.	1. 2.	1.

Appendix C: Cognitive Levels of Sample Questions

Translating Between and Among Representations		Representing and Partitioning Whole Numbers		Whole Number Operations		Patterns and Relations	
Question	What type of question?	Question	What type of question?	Question	What type of question?	Question	What type of question?
1	Knowledge	1	Application	1	Knowledge	1	Knowledge
2	Application	2	Application	2	Knowledge	2	Application
3	Analysis	3	Application	3	Application	3	Application
4	Application	4	Application	4	Application	4	Application
5	Application	5	Analysis	5	Application	5	Application
6	Application & Analysis	6	Application	6	Analysis	6	Application
		7	Application	7	Knowledge	7	Application
		8	Application	8	Application	8	Application
		9	Application	9	Application	9	Application
		10	Application	10	Application	10	Application
				11	Knowledge		
				12	Knowledge		
				13	Application		

Measurement		Geometry		Statistics and Probability		Problem Solving	
Question	What type of question?	Question	What type of question?	Question	What type of question?	Question	What type of question?
1	Application	1	Knowledge	1	Knowledge	1	Application
2	Application	2	Analysis	2	Application	2	Application
3	Application	3	Knowledge	3	Application	3	Application
4	Application	4	Application	4	Application	4	Analysis
5	Application	5	Analysis	5	Application	5	Analysis
6	Analysis	6	Analysis	6	Analysis	6	Application
7	Application	7	Knowledge	7	Application	7	Analysis
8	Application	8	Knowledge			8	Analysis
9	Analysis	9	Analysis			9	Analysis
10	Knowledge	10	Analysis				
11	Application						
12	Knowledge						
13	Application						
14	Analysis						

Appendix D: Answers to the Sample Questions

Translating Between and Among Representations		Representing and Partitioning Whole Numbers		Whole Number Operations		Patterns and Relations	
Question	**Answer**	**Question**	**Answer**	**Question**	**Answer**	**Question**	**Answer**
1	D	1	452	1	B	1	C
2	A	2	266	2	A	2	A
3	C	3	503	3	D	3	A
4	15 counters	4	342	4	C	4	B
5	30 + 7 ≠ 30 + 5	5	Answers will vary	5	D	5	D
6	a) 17	6	A	6	B	6	C
	b) 11 cubes + 6 cubes (many combinations that add up to 17)	7	C	7	A	7	D
	c) 12 cubes + 5 cubes (many combinations that add up to 17)	8	B	8	A	8	D
		9	Answers will vary	9	C	9	A
		10	263, 373, 487 and 597 written in words	10	B	10	B
				11	C/C		
				12	B/B		
				13	A		

Measurement		Geometry		Statistics and Probability		Problem Solving	
Question	**Answer**	**Question**	**Answer**	**Question**	**Answer**	**Question**	**Answer**
1	D	1	B	1	C	1	8 outfits
2	A	2	D	2	A	2	Megan
3	Answers vary	3	B	3	D	3	37 marbles
4	A	4	A	4	D	4	6 bicycles
5	B	5	D	5	D	5	2 apples
6	Answers vary	6	A	6	B	6	3 combinations
7	D	7	D	7	A	7	5 tables
8	B	8	C			8	121 cm
9	B	9	C			9	8 combinations
10	C	10	B				
11	B						
12	C						
13	A						
14	A						

Appendix E: Problem-Solving Sample Question Strategies (Pages 75-76)

1. Natasha has 4 T-shirts and 2 pairs of pants.
How many different outfits can Natasha make?

Show how you solved the problem and explain your strategy.

Make a chart

	Jeans (J1)	Jeans (J2)
T-shirt (T1)	T1J1	T1J2
T-Shirt (T2)	T2J1	T2J2
T-shirt (T3)	T3J1	T3J2
T-shirt (T4)	T4J1	T4J2

Natasha can make eight outfits.

Draw a picture

Natasha   can make eight outfits.

2. Megan and Danielle each ordered the same size pizza.
Megan asked to have her pizza cut into fourths. Danielle asked to have her pizza cut into sixths.
Who has the larger pieces of pizza? **(Megan)**

Show how you solved the problem and explain your strategy.

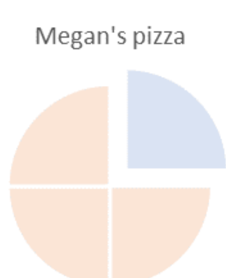
Use a model

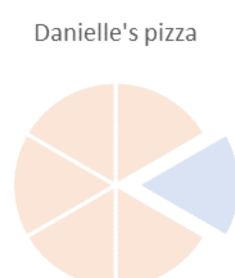
The model can be: two fractional circles, one with 4 sections and one with 6 sections

- Cuisenaire rods
- fraction strips

Draw a picture

The drawing shows that Megan has the larger pieces of pizza.





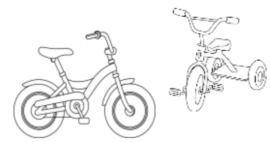
3. Peter has 123 marbles. He gives some marbles to his friend Paul. Now Peter has 86 marbles. How many marbles did Peter give to Paul?

Show how you solved the problem and explain your strategy.

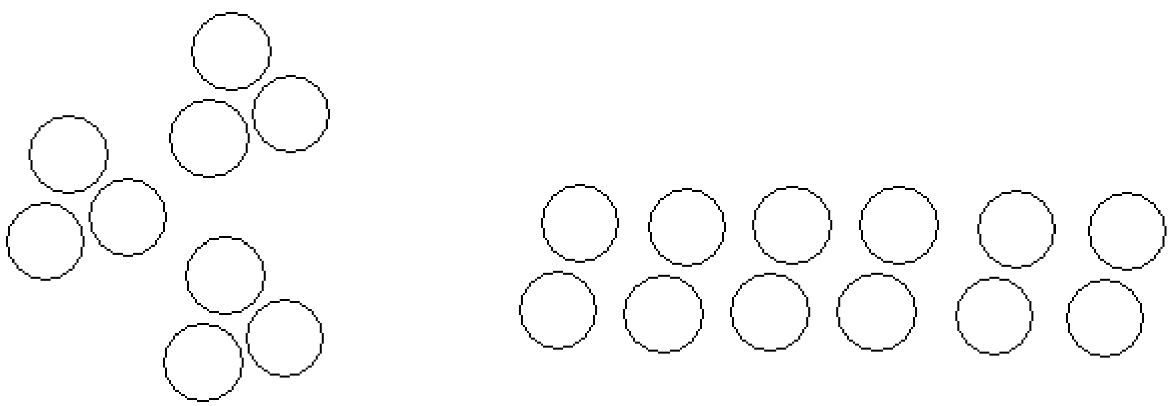
<u>Use a model</u>
Students can use coloured tokens, tiles or other concrete material to represent this subtraction situation. Students can use an equation such as:
123 – ☐ = 86, where ☐ represents the number of marbles that Peter gave Paul. **(37 marbles)**

<u>Draw a picture</u>
Students can draw a set of 123 small circles and then separate a set of 86 small circles by drawing arrows to show the action of separating and count the number of circles that remain in the starting set.

4. Sebastian and his sister have bicycles and tricycles.
The bicycles and tricycles have 21 wheels altogether.
If they have 3 tricycles, then how many bicycles do they have?
Show how you solved the problem and explain your strategy.

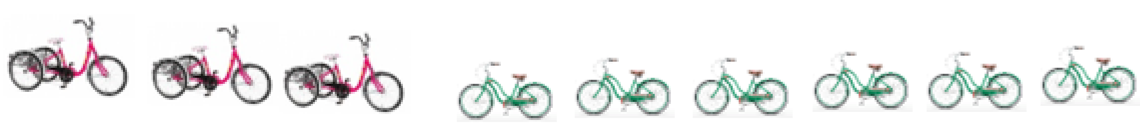


<u>Use a model</u>
Students can use coloured tokens, tiles or other concrete materials to represent this situation.



The 3 tricycles have 9 wheels. There are 6 bicycles with 12 wheels.

<u>Draw a picture</u>



There are 3 tricycles and 6 bicycles.

5. Monette has 23 apples.
She eats three apples every day.
How many apples will Monette have left after seven days?

Show your reasoning and explain your strategy.

Make a pattern

$$23 \xrightarrow{-3} 20 \xrightarrow{-3} 17 \xrightarrow{-3} 14 \xrightarrow{-3} 11 \xrightarrow{-3} 8 \xrightarrow{-3} 5 \xrightarrow{-3} 2$$

Day 1 Day 2 Day 3 Day 4 Day 5 Day 6 Day 7

The number of apples decreases according to a pattern whose rule is: Start at 23, subtract 3 each day. After 7 days, there are 2 apples left.

Make a chart

Number of Apples Eaten	
Number of Days	Number of Apples
Day 1	23 – 3 = 20
Day 2	20 – 3 = 17
Day 3	17 – 3 = 14
Day 4	14 – 3 = 11
Day 5	11 – 3 = 8
Day 6	8 – 3 = 5
Day 7	5 – 3 = 2 apples remain

After seven days, there are two apples left.

Use a model

Students can use a set of 23 two-coloured counters, 23 coloured tiles, or any other appropriate manipulative.

Day 1: Remove 3 chips from the set of 23 chips. There are 20 chips left.
Day 2: Remove 3 chips from these 20 chips. There are 17 chips left.
Day 3: Remove 3 chips from these 17 chips. There are 14 chips left.
Day 4: Remove 3 chips from these 14 chips. There are 11 chips left.
Day 5: Remove 3 chips from these 11 chips. There are 8 chips left.
Day 6: Remove 3 chips from these 8 chips. There are 5 chips left.
Day 7: Removes 3 chips from these 5 chips. There are 2 chips left.

After seven days, there are two apples left.

6. Marbles come in packages of 10, 25, and 50.
You need 160 marbles.
Find 5 ways you could buy the marbles.

Show how you solved the problem and explain your strategy.

Make a table

The possible combinations are:

Package of 10 marbles	**Package of 25 marbles**	**Package of 50 marbles**	**Total**
6 packages	2 packages	1 package	6 x 10 + 2 x 25 + 1 x 50 = 160 marbles
1 package	2 packages	2 packages	1 x 10 + 2 x 25 + 2 x 50 = 160 marbles
1 package	4 packages	1 package	1 x 10 + 4 x 25 + 1 x 50 = 160 marbles

There are three possible combinations.

Use a model

Students may use coloured tokens, tiles, or any other available manipulative.

7. For James birthday, his mother wants to cover each table using paper tablecloth which is 4 m long. How many tables can she cover using a roll of tablecloth which is 20 m long?

Show how you solved the problem and explain your strategy.

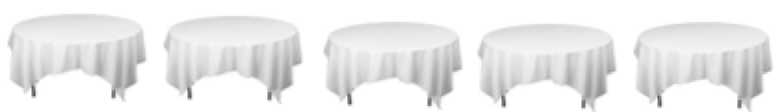
<u>Make a table</u>

Number of Tables to be Covered		
Strips of Paper Tablecloth	**Required Length (m)**	**Number of Tables**
1	4	1
2	8	2
3	12	3
4	16	4
5	20	5

The table shows that five tables can be covered with a roll of tablecloth that is 20 m long.

Jacques' mother can cover five tables.

<u>Draw a picture</u>

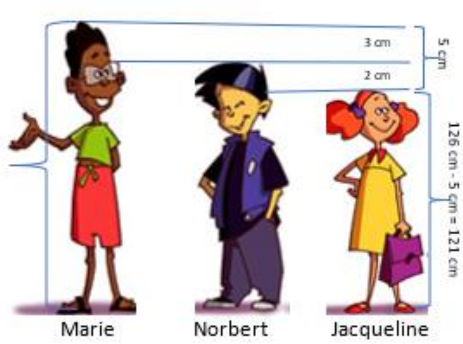


4 m 4 m 4 m 4 m 4 m.

There are 3 m of paper tablecloth left. The paper tablecloth can cover 5 tables.

8. Marie's height is 3 cm more than Norman's height.
Norman's height is 2 cm more than Jacqueline's height.
If Marie's height is 126 cm, what is Jacqueline's height?

Show how you solved the problem and explain your strategy.
<u>Draw a picture</u>



Jacqueline's height is 121 cm.

<u>Work Backwards</u>

Norman is 2 cm taller than Jacqueline. Marie is 3 cm taller than Norman.
Marie is three inches taller than Jacqueline.
Jacqueline measures 126 cm – 5 cm = 121 cm.

9. Lucy has some money. Lucy has some 5 cent coins, some 10 cent coins and 25 cent coins.
Lucy buys a used book for 45 cents.
Lucy used all of the money to buy this book and now has no money left.
How many different ways could Lucy pay for this book using all of her money?

Show how you solved the problem and explain your strategy.

<u>Make a Table</u>

5 cent coins	10 cent coins	25 cent coins	Possible Combinations
9 coins	0 coins	0 coins	$9 \times 5 = 45$ cents
7 coins	1 coins	0 coins	$7 \times 5 + 1 \times 10 = 45$ cents
5 coins	2 coins	0 coins	$5 \times 5 + 2 \times 10 = 45$ cents
3 coins	3 coins	0 coins	$3 \times 5 + 3 \times 10 = 45$ cents
1 coins	4 coins	0 coins	$1 \times 5 + 4 \times 10 = 45$ cents
4 coins	0 coins	1 coins	$4 \times 5 + 1 \times 25 = 45$ cents
2 coins	1 coins	1 coins	$2 \times 5 + 1 \times 10 + 1 \times 25 = 45$ cents
0 coins	2 coins	1 coins	$2 \times 10 + 1 \times 25 = 45$ cents

Lucy could have paid for the novel in eight different ways.

<u>Use a model</u>
Note: Students can use play money to determine possible combinations.