Nova Scotia Assessment: Mathematics in Grade 6

Lessons Learned

"For learners to succeed, teachers must assess students' individual abilities and characteristics and choose appropriate and effective instructional strategies accordingly."

– Helene J. Sherman

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NOVA SCOTIA

Contents

Purpose of this document

This *Lessons Learned* document was developed based on an analysis of the Item Description Reports for the Nova Scotia Assessment: Mathematics in Grade 6 (2012–2013, 2013–2014, 2014–2015, 2015–2016, 2016–2017, 2017–2018, 2018–2019 and 2019–2020). It is intended to support all elementary classroom teachers (in particular grades 3–6) and administrators at the school, region, and provincial levels, in using the information gained from this assessment to inform next steps for numeracy focus. The analysis of these items forms the basis of this document, which was developed to support teachers as they further explore these areas through classroom-based instruction and assessment across a variety of mathematical concepts.

After the results for each mathematics assessment become available, an Item Description Report is developed to describe each item of the assessment in relation to the curriculum outcomes and cognitive processes involved with each mathematical strand. The percentage of students across the province who answered each item correctly is also connected to each item. Item Description reports for mathematics are made available to school regions for distribution to schools, and they include provincial, regional, and individual school data. Schools and regions should examine their own data in relation to the provincial data for continued discussions, explorations, and support for mathematics focus at the classroom and school regions.

This document specifically addresses areas that students across the province found challenging based on provincial assessment evidence. It is essential that teachers consider assessment evidence from a variety of sources to inform the next steps most appropriate for their students. Effective classroom instruction and assessment strategies are responsive to the individual learners within a classroom.

The M6 assessment generates information that is useful in guiding classroom-based instruction and assessment in mathematics. This document provides an overview of the mathematics tasks included in the assessment, information about this year’s mathematics assessment results, and a series of Lessons Learned for mathematics. Suggested next steps for classroom instruction and assessment are included for each Lesson Learned.

Overview of the Nova Scotia Assessment: Mathematics in Grade 6

Nova Scotia Assessments are large-scale assessments that provide reliable data about how well all students in the province are learning the mathematics curricula. It is different from many standardized tests in that all questions are written by Nova Scotia teachers to align with curriculum outcomes and the results reflect a snapshot of how well students are learning these outcomes. These results can be counted on to provide a good picture of how well students are learning curriculum outcomes within schools, regions, and in the province as a whole. Since the assessments are based on the Nova Scotia curriculum, and are developed by Nova Scotia teachers, results can be used to determine whether the curriculum, approaches to teaching and allocation of resources are effective. Furthermore, because individual student results are available, these, in conjunction with other classroom assessment evidence, help classroom teachers understand what each student has under control and identify next steps to inform instruction.

The assessment provides information about mathematics for each student and complements assessment data collected in the classroom. This assessment is administered at the beginning of Grade 6. It is designed to provide detailed information for every student in the province regarding his or her progress in achieving a selection of mathematics curriculum outcomes at the end of Grade 5. Information from this assessment can be used by teachers to inform their instruction and next steps in providing support and intervention for their students.

The design of the assessment includes the following:

- mathematical tasks that reflect a selection of outcomes aligned with the curriculum from the end of grade 2 to the end of grade 5 from across all strands of the mathematics curriculum
- all items are in selected response format
- all items are designed to provide a broad range of challenge, thereby providing information about a range of individual student performance.

Table 1: Specific Curriculum Outcomes Assessed in 2019–2020 by Strand

Strand	**Specific Curriculum Outcomes**
Number (N)	3N09, 4N06, 4N08, 4N09, 4N11, 5N01, 5N02, 5N05, 5N06, 5N07, 5N08, 5N09, 5N10, 5N11
Patterns and Relations (PR)	2PR01, 3PR01, 3PR02, 4PR03, 4PR04, 4PR05, 5PR01, 5PR02
Measurement (M)	3M03, 4M01, 4M03, 5M01, 5M05
Geometry (G)	4G01, 5G01, 5G02, 5G04,
Statistics and Probability (SP)	3SP01, 4SP01, 4SP02, 5SP02, 5SP03, 5SP04

Table 2: Specific Curriculum Outcomes Assessed in 2019–2020 by Grade Level

SCO Assessed		
2PR01	Grade 2	3.12%
3N09 - 3PR01 - 3M03 - 3SP01	Grade 3	12.50%
4N06 - 4N08 - 4N09 - 4N11 - 4PR03 - 4PR05 - 4M03 - 4G01 - 4SP02	Grade 4	28.13%
5N01 - 5N02 - 5N05 - 5N06 - 5N07 - 5N08 - 5N09 - 5N10 - 5N11 - 5PR01 - 5PR02 - 5M01 - 5M05 - 5G01- 5G02- 5SP02 - 5SP03 - 5SP04	Grade 5	56.25%
Total = 32		100%

Cognitive levels of questions in mathematics are defined as:

- *Knowledge questions* may require students to recall or recognize information, names, definitions, or steps in a procedure.
- *Application questions* may require students to make connections, represent a situation in more than one way (translate between representations), or solve contextual problems.
- *Analysis questions* may require students to go beyond comprehension and application to higher order thinking skills, such as generalizations and non-routine problem-solving.

These are the percentages of the questions on the Nova Scotia provincial assessments for M6:

- Knowledge 20–30%
- Application 50–60%
- Analysis 10–20%

These percentages are also recommended for classroom-based assessments.

Please refer to Appendix A at the end of this document for further information about cognitive levels of questioning. The Nova Scotia Assessment: Mathematics 6 includes 70 items distributed over two days for a duration of 60 minutes each day; 35 items on Day 1 and 35 items on Day 2. The chart below shows the distribution of items each day, by mathematical strand and cognitive level.

Table 3: Number of Items by Strand and Cognitive Level

Number of Items Day 1				
	Knowledge	Application	Analysis	
Number	4	11	2	Total 17
Patterns and Relations	1	2	1	Total 4
Measurement	1	3	1	Total 5
Geometry	1	3	1	Total 5
Statistics and Probability	1	2	1	Total 4
Number of Items Day 2				
	Knowledge	Application	Analysis	
Number	4	11	2	Total 17
Patterns and Relations	1	2	1	Total 4
Measurement	1	3	1	Total 5
Geometry	1	3	1	Total 5
Statistics and Probability	1	2	1	Total 4
Total	16	42	12	70 Questions
Cognitve Level 2019-2020%	22.9%	60%	17.14%	100%
Table of Specifications	(20-30%)	(50-60%)	(10-20%)	

Performance Levels

Below are the Nova Scotia Assessment: Mathematics in Grade 6 Performance Levels.

Level 1: Students at Level 1 can generally solve problems when they are simple and clearly stated or where the method to solve the problem is suggested to them. They can do some basic operations (+, –, x, ÷), but may not understand when each operation should be used. They can recognize some mathematical terms and symbols, mainly from earlier grades.

Level 2: Students at Level 2 can generally solve problems similar to problems they have seen before. They depend on a few familiar methods to solve problems. They can usually do basic operations (+, –, x, ÷) and can usually understand where they should be used. They can understand and use some mathematical terms and symbols, especially those from earlier grades.

Level 3: Students at Level 3 can generally solve problems that involve several steps and may solve new and novel problems. They can apply number operations (+, –, x, ÷) correctly and can judge whether an answer makes sense. They can understand and use many mathematical terms and symbols, including those at grade level.

Level 4: Students at Level 4 can solve new, novel, and complex problems. They can apply number operations (+, –, x, ÷) with ease. They can think carefully about whether an answer makes sense. They find mathematical terms and symbols easy to use and to understand.

Assessment Results

The Nova Scotia Assessment: Reading, Writing, and Mathematics in Grade 6 (RWM6) has been administered since the 2012–2013 school year. The following percentage of students performed at the expectation of Level 3 or above on the mathematics portion of the assessment: 73% (2012–2013), 73% (2013–2014), 69% (2014–2015), 68% (2015–2016), 70% (2016–2017), 70% (2017-2018), and 71% (2018-2019).

Table 1: Mathematics in Grade 6: Student Performance Levels

	2014–2015 (8809 Students)	2015–2016 (7992 Students)	2016–2017 (7715 Students)	2017–2018 (7869 Students)	2018–2019 (8164 Students)
Performance Level 1	12%	13%	13%	13%	12%
Performance Level 2	19%	19%	17%	17%	17%
Performance Level 3	60%	59%	59%	59%	59%
Performance Level 4	9%	9%	11%	12%	12%

Mathematics in Grade 6 Lessons Learned

The assessment information gathered from the Nova Scotia Assessment: Mathematics in Grade 6 data has been organized into 7 **Lessons Learned**:

- Number
- Estimation
- Patterns and Relations
- Measurement
- Geometry
- Statistics and Probability
- Problem Solving

Each Lesson Learned is divided into four sections that address the following questions:

A. What conclusions can be drawn from the 2019–2020 NSA: Mathematics in Grade 6?
B. Do students have any misconceptions or errors in their thinking?
C. What are the next steps in instruction for the class and for individual students?
D. What are the most appropriate methods and activities for assessing student learning?

Lessons Learned

1. **Number**: Students had difficulty when exposed to more than knowledge items in order to apply the higher order thinking skills to do application and analysis items. Students need to experience all of the Story Structures for Addition and Subtraction, and for Multiplication and Division. Students need to concentrate on demonstrating an understanding of addition and subtraction of decimals limited to hundredths and thousandths. Students need to use benchmarks and number lines when comparing and ordering fractions.

2. **Estimation**: Students had difficulty when asked to use estimation strategies. They appear not to use any strategy for estimation and simply guess at an answer. When working with estimation questions many students want to get the right answer and feel there is no value in estimating first. Students need to use estimation strategies such as using benchmarks, rounding, front-end addition, making a friendly number, compensation, subtraction (left-to-right calculations), and clustering of compatible numbers.

3. **Patterns and Relations**: Students had difficulty when moving from the basic understanding of patterns to the generalization of a pattern rule to enable them to find any term. Students need work on continuing and extending a pattern to predict a subsequent term that is not consecutive. Students need to continue to work with representations of patterns, contextually, pictorially, symbolically, and verbally.

4. **Measurement**: Students had difficulty when using the five representations of a concept (contextual, concrete, pictorial, symbolic, verbal) in order to have concept attainment when working with area and perimeter. Students need to work with perimeter and area together in application and analysis questions. Students need to build a conceptual understanding of what it means to measure with a ruler and not approach the activity as a rote procedure.

5. **Geometry:** Students had difficulty when asked to draw upon their previous knowledge of 2-D shapes to assist them in their identification and descriptions of prisms. Students require more experiences to identify and name common attributes of both rectangular-based and triangular-based prisms. Students need to sort a given set of right rectangular-based and triangular-based prisms, using the shape of the base. Sets of 3-D objects usually include a variety of prisms. Students need to be able to identify examples of rectangular-based prisms and triangular-based prisms in their environment. Students need more exposure and experience with the Cognitive Levels of Questioning (knowledge, application and analysis) in order to apply these higher order thinking skills when working with all geometry concepts.

6. **Statistics and Probability:** Students had difficulty when asked to construct and interpret double bar graphs. They need more experiences when constructing and interpreting double bar graphs. Students need to develop the skill of interpreting and drawing conclusions from double bar graphs when asked questions about the data displayed. Both literal comprehension questions and inferential comprehension questions need to be asked. They need more opportunities to identify outcomes from a given probability experiment that are less likely, equally likely, or more likely to occur than other outcomes.

7. **Problem Solving:** Students had difficulty with application and analysis items when applying these higher order thinking skills when problem solving. Students need to continue to work on translating between and among representations when problem solving.

Key Messages

The following key messages should be considered when using this document to inform classroom instruction and assessment.

The Nova Scotia mathematics curriculum is based upon several key assumptions or beliefs about mathematics learning that have grown out of research and practice. These beliefs include the following:

- Instruction and assessment practices should be aligned with culturally responsive pedagogy. Culturally responsive pedagogy is teaching that connects a student's social, cultural, family, or language background to what the student is learning; it nurtures that cultural uniqueness; and it responds by creating conditions in which the student's learning is enhanced. It is critical that learning opportunities are relevant and meaningful to students, so they are responsive to students' learning needs. The content within this document may be adapted, as needed, in order to respond to students' various cultural and life contexts.
- Mathematics learning is an active and constructive process.
- Learning is most effective when standards of expectation are made clear with ongoing assessment and feedback.
- Learners are individuals who bring a wide range of prior knowledge and experiences and who learn via various styles and at different rates.
- Learning is most likely to occur when placed in meaningful contexts and in an environment, that supports exploration, risk taking, and critical thinking and that nurtures positive attitudes and sustained effort.
- Students are curious, active learners with individual interests, abilities, and needs. They come to classrooms with varying knowledge, life experiences, and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.
- Students learn by attaching meaning to what they do and need to construct their own meaning of mathematics. This meaning is best constructed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of models and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students and enhance the formation of sound, transferable, mathematical concepts. At all levels, students benefit from working with and translating through a variety of materials, tools, and contexts when constructing meaning about new mathematical ideas. Meaningful discussions can provide essential links among concrete, pictorial, contextual, and symbolic representations of mathematics. (EECD, 2014a, p. 25)

Research has shown that assessment for learning (formative assessment) practices produce significant and often substantial learning gains, close achievement gaps, and build students' ability to learn new skills (Black & Wiliam 1998; OECD, 2006). Student involvement in assessment promotes learning. Timely and effective teacher feedback and student self-assessment allow students to reflect on and articulate their understanding of mathematical concepts and ideas.

- Provincial assessment results form part of the larger picture of assessment for each student and complements assessment data collected in the classroom. Ongoing assessment for learning (formative assessment) is essential to effective teaching and learning. Assessment for learning can and should happen every day as part of classroom instruction. Assessment of learning (summative assessment) should also occur regularly and at the end of a cycle of learning. Both forms of assessment are necessary to guide teaching, stimulate learning, and produce achievement gains.
- It is important to construct assessment activities that require students to complete tasks across the cognitive levels. While it is important for students to be able to answer factual and procedural type questions, it is also important to embed activities that require strategic reasoning and problem-solving.

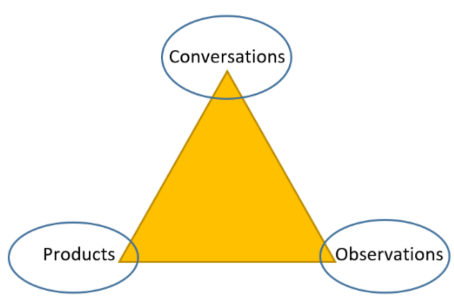
- Ongoing assessment for learning involves the teacher focusing on how learning is progressing during the lesson and the unit, determining where improvements can be made, and identifying the next steps. "*Such assessment becomes 'formative assessment' when the evidence is actually used to adapt the teaching to meet learning needs," (*Black, Harrison, Lee, Marshall & Wiliam, 2003, p.2). Effective strategies of assessment for learning during a lesson include: strategic questioning, observing, conversing (conferring with students to "hear their thinking"), analyzing student's work (product), engaging students in reviewing their progress, as well as, providing opportunities for peer and self-assessment.
- Assessment of learning involves the process of collecting and interpreting evidence for the purpose of summarizing learning at a given point in time, and making judgments about the quality of student learning on the basis of established criteria. The information gathered may be used to communicate the student's achievement to students, parents, and others. It occurs at or near the end of a learning cycle.
- All forms of assessment should be planned with the end in mind, think about the following questions:
  - What do I want students to learn? (identify clear learning targets)
  - What does the learning look like? (identify clear criteria for success)
  - How will I know they are learning?
  - How will I design the learning so that all will learn?
- Before planning for instruction using the suggestions for instruction and assessment, it is important that teachers review individual student results in conjunction with current mathematics assessment information. A variety of current classroom assessments should be analyzed to determine specific strengths and areas for continued instructional focus or support.

Balanced Assessment in Mathematics: Effective ways to gather information about a student's mathematical understanding

- Conversations/Conferences/Interviews: Individual, Group, Teacher-initiated, Child-initiated
- Products/Work Samples: Mathematics journals, Portfolios, Drawings, Charts, Tables, Graphs, Individual and classroom assessment, Pencil-and-paper tests, Surveys, Self-assessment
- Observations: Planned (formal), Unplanned (informal), Read-aloud (literature with mathematics focus), Shared and guided mathematics activities, Performance tasks, Individual conferences, Anecdotal records, Checklists, Interactive activities

(EECD, 2014b, p. 4)

Triangulation increases the fidelity and validity of student learning assessment and facilitates the implementation of pedagogical differentiation. Using triangulation, we take into account all learning styles and we engage all students, including those who have difficulty expressing themselves in writing and those who do not have the ability to undertake a written assessment task to demonstrate their learning."— Anne Davies (Free Translation)



Mathematics in Grade 6 Lesson Learned 1
Number

Students had difficulty when exposed to more than knowledge items in order to apply the higher order thinking skills to do application and analysis items. Students need to experience all the Story Structures for Addition and Subtraction, and for Multiplication and Division. Students need to concentrate on demonstrating an understanding of addition and subtraction of decimals limited to hundredths and thousandths. Students need to use benchmarks and number lines when comparing and ordering fractions. Students were not able solve a multi-step story problem involving whole number multiplication and division. More work is required for the operations of multiplication and division as many students were not able to multiply two 2-digit whole numbers vertically. Many students had difficulty when dividing a 3-digit whole number by a 1-digit whole number. Students were not able to identify a decimal number represented by base-ten blocks conventionally.

The Mathematics curriculum documents for grades Primary to 6 notes that a true sense of number goes well beyond the skills of simply counting, memorizing facts, and the situational rote use of algorithms. Number sense develops when students connect numbers to real-life experiences and use benchmarks and cultural referents. This results in students who are computationally fluent, flexible with numbers, and have intuition about numbers. The evolution of number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing rich mathematical tasks that allow students to make connections. Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects and concepts.

A. What conclusions can be drawn from the Nova Scotia Assessment: Mathematics in Grade 6?

We noticed that students were able to do well when explicitly given all the information needed to answer the number questions. Students also performed well on knowledge questions that required them to use basic facts and skills, symbolic procedures, and factual knowledge. For example, students were able to solve 1487 – 379 when the problem was presented symbolically.

However, when students were asked to apply basic skills, knowledge, and computational procedures to application and analysis questions, they were quite challenged. This appeared to be a theme throughout the assessment data. At times, students were not sure whether they should add, subtract, multiply, or divide when questions were presented in the context of a story problem. The assessment analysis also showed that our students did not understand the relationship between addition and subtraction or between multiplication and division.

Decimal computations in context were challenging for students. For example, when asked to perform basic operations with decimals, (including contexts involving money), most students were able to do knowledge questions. But when presented with application or analysis level questions, they struggled.

Students were able to represent fractions but struggled with equivalent fractions. In order for students to know how to compare and order fractions, they need to develop the concept of equivalent fractions.

B. Do students have any misconceptions or errors in their thinking?

It is important to distinguish between misconceptions and errors. Misconceptions are, in effect, the misunderstandings about mathematical ideas which students entertain, and which usually lead to errors occurring. These are the most serious kind of errors which require urgent action on the part of the teacher. Errors may occur for a variety of underlying reasons, ranging from careless mistakes to errors resulting from misconceptions. - John SanGiovanni – (*Mine The Gap For Mathematical Understanding Grades 3-5*)

Whole Number: Addition and Subtraction

When solving computation questions, many students performed better when the question was presented horizontally than vertically. Vertical representation may lead the student to using a traditional algorithm which focuses on single digits within the computation rather than thinking about the meaning of the operation and the number as a whole. For example:

6500 – 4221 = 2279

$$\begin{array}{r} 6500 \\ -\ \underline{4221} \\ 2279 \end{array}$$

Many children have the misconception that they always subtract the smaller number from the larger number. They apply this thinking regardless of whether the position of the number is in the subtrahend (a number which is to be subtracted from another number) or the minuend (a number from which another number is to be subtracted) in the question. For example:

$$\begin{array}{rl} 451 & \text{(minuend)} \\ \underline{-\ 231} & \text{(subtrahend)} \\ 220 & \text{(difference)} \end{array} \qquad \begin{array}{rl} 509 & \text{(minuend)} \\ \underline{-\ 389} & \text{(subtrahend)} \\ 280 & \text{(difference)} \end{array}$$

451 – 231 = 220

509 – 389 = 280

In the first question, when subtracting the tens, 5 – 3 = 2, students should be thinking 50 – 30 = 20 in order to complete the question correctly using his/her understanding of subtracting the smaller number from the larger number. But in the second question, the student tries to use the same method (smaller number 0 subtracted from larger number 8) and gets an answer of 8 tens, which is incorrect.

At times, students forgot to regroup when adding. They often wrote a two-digit number where there should have only been one digit. For example:

$$\begin{array}{r} 145 \\ \underline{+\ 247} \\ 3812 \end{array}$$

Some students misaligned the digits when recording their calculations and computed incorrectly. A suggested strategy for dealing with this error is for students to use grid paper or lined paper turned sideways to align the digits and to focus on the place value of the digits being added or subtracted. Another way to address these errors or misconceptions is to focus on developing personal strategies and alternative algorithms which tend to focus on the meaning of the number, rather than on individual digits.

1	2	4	3
+	3	5	2
1	5	9	5

instead of

$$\begin{array}{r} 1243 \\ +\underline{352} \\ 4763 \end{array}$$

Whole Number: Multiplication and Division

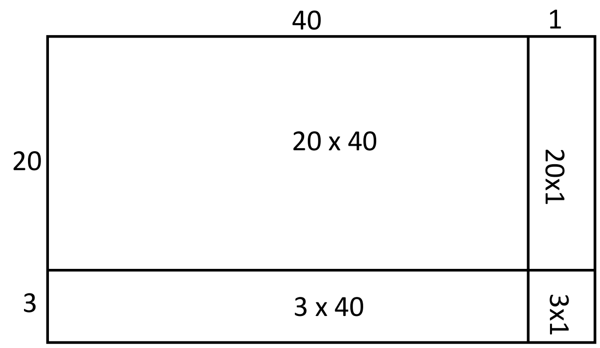
Clear and accurate language and communication are extremely important when developing multiplication and division concepts. They allow children to explain the connection between their models and the story problems they represent using verbal expressions such as “groups of,” “rows of” and “jumps of”. A misconception many students have is centered on place value language. If given a multiplication question such as 41 x 23, some students read this and say, “4 times 2” when they should be saying, “4 tens times 2 tens”. This reinforces place value thinking for operations with whole numbers. For example, some students incorrectly think:

23		
x 41		
8	(4 x 2)	4 times 2 = 8
12	(4 x 3)	4 times 3 = 12
2	(1 x 2)	1 times 2 = 2
+ 3	(1 x 3)	1 times 3 = 3
25		The answer is 25.

Students should, instead, do the following:

23		
x 41		
800	(40 x 20)	4 tens times 2 tens = 800
120	(40 x 3)	4 tens times 3 ones = 120
20	(1 x 20)	1 ten times 20 ones = 20
+ 3	(1 x 3)	1 one times 3 ones = 3
943		The answer is 943.

Or



The Area Model

Estimation should help with this type of misconception. For example, to estimate 20 x 40 = 800 so the answer should be around 800 or a bit higher. So, if the student estimates before attempting the question they should realize that 115 is a long way from 800.

Another error and misconception when multiplying whole numbers is that students ignore the zeros in a number. Such as 2 x 504. This question can be read as two groups of 504, which is the double strategy. When students are urged to always estimate before beginning a question, they will be more than likely to catch their mistakes. When they estimate, they know that 2 x 504 is just a bit more than 1000. When they estimate, they realize that 108 would be an incorrect answer. This same misconception applies when dividing whole numbers containing dividends with zeros.

504	instead of	504
x 2		x 2
108		1008

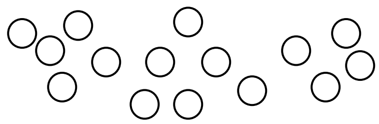
Students should be introduced to **division** through story problems. For this instruction, there are two types of situations, **equal-sharing and equal-grouping**, which need to be considered. Equal-sharing problems are those in which the number of groups is known and the number in each group needs to be found. Equal-grouping problems are those in which the number in each group is known and the number of groups needs to be found. Students should also see that dividing the class into groups of 5, giving each student 4 pencils, and placing books into stacks of 4 are all examples of equal-grouping situations.

Students should see that dividing the class into two groups, sharing 12 pieces of paper with 4 students, and sharing a large bag of candy into 3 small bags are all examples of equal-sharing situations.
Below are some sample questions that represent division through story problems.

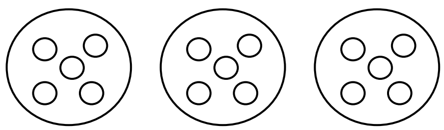
Equal-Sharing Story Problems

1. Three friends want to share 15 candies.

   How many candies will each friend get?



Each friend will get 5 candies. This may be described that when **15 is divided into 3 groups**, there are 5 in each group. (15 ÷ 3 = 5)

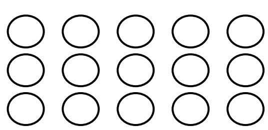


Note: If students model this problem with counters, they would most likely start with 15 counters and distribute the counters, one at a time, into three groups. After all the counters have been distributed, they see there are 5 counters in each group, indicating that each friend will get 5 candies. Some mathematics educators refer to these problems simply as equal-sharing problems.

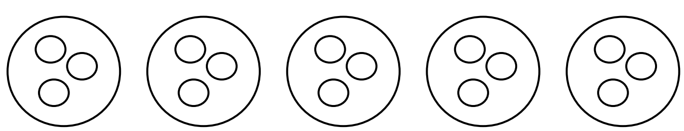
Equal-Grouping Story Problems

2. Friends want to share 15 candies by each taking 3 candies.

   How many friends will get candies?



Five friends will each get 3 candies. This may be described that when **15 is divided into groups of 3**, there are 5 groups. (15 ÷ 3 = 5)



Note: If students model this problem with counters, they would most likely start with 15 counters, take 3 of these counters to give to 1 friend, take another 3 to give to a second friend, and continue doing this until they run out of counters; thus, 5 friends will get candies. Some mathematics educators refer to these problems as equal-grouping (measurement) problems.

Students should solve many examples of both types of division problems by modelling them concretely, recording them pictorially, and describing the division in words before they are introduced to division sentences.

When dividing whole numbers, a misconception many students have is centered on place value language. When given a division question such as 412 ÷ 6, students say, “6 into 4 doesn’t work,” they are mathematically incorrect. In the question, it is 400 not 4 that is being divided by 6.

In developing the division concept, students should meet situations involving remainders. A common question that many students have is, “What do I do with a remainder?” They should understand that when solving division problems, remainders are handled differently depending on the context. Students should *recognize when a remainder is significant or not involved in the decision making in a problem-solving* context. For example, the remainder needs to be ignored when you want to know how many $2 notebooks can be bought with $11. The answer is 5. Although there is no money left over, it is not enough money to buy another (6^{th}) notebook.

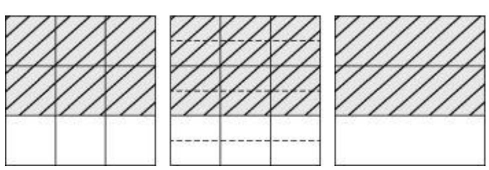
Fractions

One misconception that students have when working with fractions is understanding that the whole in fractions is very important. Fractional parts are equal shares or equal-sized portions of one whole or one unit. Fractional parts have special names that tell how many parts of that size are needed to make one whole. For example, *thirds* require three parts to make one whole. The more fractional parts used to make a one whole, the smaller the parts. For example, eighths are smaller than fifths.

A common misconception for students is to transfer previously learned whole-number concepts to fractions. Students may have a strong mindset about numbers that may cause them difficulties with the relative size of fractions. In their experience, larger numbers mean “more” and therefore, thinking seven is more than four, so sevenths should be larger than fourths. For example, in comparing fractions such as $\frac{3}{6}$ and $\frac{3}{7}$, they might think that $\frac{3}{7}$ is greater than $\frac{3}{6}$ because 7 is greater than 6. Students may, therefore, incorrectly think that a larger denominator means the fraction is larger. Students will need to spend considerable time on strategies and discussions that help them to develop number sense for fractions. The inverse relationship between number of parts and size of parts is better understood by students when they explore and discover this on their own rather using models than being told, though some students may need explicit instruction around this.

Students believe that the equivalent fraction of a given fraction does not have the same value of the given fraction. They think that it is always bigger or smaller than the original fraction.

To correct this misconception, ask students to create an equivalent fraction by drawing pictures or diagrams of the following example:



Ask students to read fractions correctly.
Six-ninths, not six over nine.
Twelve-eighteenths and not twelve over eighteen.
Two-thirds and not two over three.
The square is one whole. It’s the same whole for all the diagrams.

The square represents one whole. It is the same whole for the all three diagrams.

The first square on the left is divided into 9 equal parts. The shaded region represents $\frac{6}{9}$ of the square.

The square in the middle is divided into 18 equal parts. The shaded region represents $\frac{12}{18}$ of the square.

The last square on the right is divided into 3 equal parts. The shaded region represents $\frac{2}{3}$ of the square.

The 3 shaded regions of the square (the same whole) are equal, then $\frac{6}{9}$, $\frac{12}{18}$ and $\frac{2}{3}$ are equivalent.

Decimals

A misconception that students have is that decimals are not numbers. They think they are something very different than a whole number because they have "decimals" embedded in them. Number sense with decimals requires that students develop a conceptual understanding of decimals as numbers. To work effectively with decimals, students should demonstrate the ability to represent decimal numbers using words, models, pictures, and symbols and make to connections among various representations.

Another common misconception students have is they do not recognize the differences between decimal numbers when reading decimals numbers. Students should be encouraged to read decimals meaningfully. For example, when reading numbers, the word "and" is reserved for the decimal. 5.32 is read as five and thirty-two hundredths, not as five point three two, or five decimal thirty-two. Students should also have experience reading numbers in several ways. For example, 1.83 may be read as 1 and 83 hundredths, but might also be read as 1 and 8 tenths, 3 hundredths, or as 18 tenths, 3 hundredths.

Although teachers will model the correct reading of whole numbers and decimal numbers and will use the word "and" only for the decimal point (e.g., 16.8 is read as sixteen and eight-tenths, while 1235 is read as one thousand two hundred thirty-five), it is also important to acknowledge that in everyday use people often read numbers in ways that are not mathematically accurate, such as reading 0.34 as zero decimal thirty-four or zero point thirty-four.

Students believe that they can apply the whole number rule to compare decimal numbers. To compare two decimal numbers, students should instead first compare the whole part of the decimal number. In the following example, 13.45 < 123.45, they compare 13 and 123 to conclude that since 13 is less than 123, 13.45 < 123.45. But if the whole part of the two decimal numbers are equal, students mistakenly compare the decimal parts as if they are whole numbers. A recent study shows that students compare the decimal parts of two decimal numbers by referring to the first-place value number after the decimal as a whole number. In 13.475, the digit 4 is often read as the digit of hundreds instead of tenths. Students do not understand the place value of digits to the right of the decimal.

Many students have the misconception that when computing with decimals, you line up numbers regardless of the place value of the digits. When one of the numbers does not have a defined decimal in it, this creates all kinds of problems for students. All numbers are decimal numbers, but we do not put a decimal after each whole number. For example, 22 can also be written as 22.0. For example: 5.2 + 8.5 + 22 could also be set up like this: 5.2 + 8.5 + 22.0

5.2		5.2		5.2
8.5	instead of	8.5	or	8.5
+2 20		+2 2		+2 2
35.7		159		15.9

Ask students to read decimal numbers correctly. The correct way to read a decimal number in all contexts is: 5.2 is five and two tenths not five point 2.

5.2 + 8.5 + 0.22 = 37.7

5.2 + 8.5 + 22 = 159

If students are encouraged always to estimate before attempting this type of computation, then they would catch the error. If they estimate 5.2 to be 5 and 8.5 to be 9 and then add 22 to these numbers, they will realize the answer is around 36, not 159 as in the example above.

The student results of the **Nova Scotia Assessments between 2018-2019 and 2019-2020 in Grade 6** shows the following:

2018-2019 Results

52% of the students did not know how to subtract decimal hundredths from whole number horizontally.

32% of the students did not know how to represent the addition of 2 decimal numbers using base-ten blocks.

52% of the students did not know how to subtract decimal tenths from decimal hundredths horizontally.

52% of the students did not know how to divide a 3-digit whole number by a 1-digit whole number.

64% of the students did not know how to interpret the remainder of division.

55% of the students were not able to match a fraction with its concrete representation (using pattern blocks).

58% of the students made errors when multiplying two 2-digit whole numbers.

29% of the students were not able to match a fraction with a decimal representation.

2019-2020 Results

52% of the students did not know how to subtract decimal tenths from decimal hundredths horizontally.

58% of the students did not know how to solve an everyday context word problem in which hundredths are used in a multi-step problem ($).

63% of the students were not able solve a multi-step story problem involving whole number multiplication and division.

53% of the students did not know how to divide a 3-digit whole number by a 1-digit whole number.

59% were not able to multiply two 2-digit whole numbers vertically.

76% of the students were not able to identify a decimal number represented by base-ten blocks conventionally.

52% of the students were not able to determine the approximate solution to a given problem not requiring an exact answer.

55% of the students did know how to represent a decimal number to thousandths in words.

57% of the students were not able to solve a multi-step word problem involving whole number computations.

56% of the students were not able to solve a multi-step story problem involving whole number computation.

C. What are the next steps in instruction for the class and for individual students?

Before attempting pencil-and-paper or calculator computations, students must find estimates, so they are alert to the reasonableness of those pencil-and-paper or calculator answers. Teachers should also model this process before doing any calculations in front of the class. Students should constantly be reminded to estimate before calculating.

Whole Number Addition and Subtraction

Students should have many opportunities to solve and create word problems for the purpose of answering real-life questions, preferably choosing topics of interest to them. These opportunities provide students with a chance to practice their computational skills and to clarify their mathematical thinking. Students should be presented with addition and subtraction story problems of all structures:

- Join (result, change, and start unknown)
- Separate (result, change, and start unknown)
- Part-part-whole (part and whole unknown)
- Compare (difference, smaller, and larger unknown)

Join story problems all have an action that causes an increase, while separate story problems have an action that causes a decrease. Part-part-whole story problems, on the other hand, do not involve any actions, and compare story problems involve relationships between quantities rather than actions.

More information about the types of story problems can be found in the *Mathematics 4 Curriculum Guide,* p. 190.

Students should be able to solve story problems of different types by writing the most efficient open number sentences and computing the sums or differences to find the solutions. They should be able to do this either directly upon reading the problem, or by drawing or visualizing pictures that represent the problem. Students should be encouraged to model the story problems with base-ten blocks, and write number sentences that reflect their thinking.

Pictorial representations may include student-generated pictures, such as those described on p. 67 of *Teaching Student-Centered Mathematics, Grades K–3* by Van de Walle and Lovin (2006), or strip diagrams as described below. It is important that the pictures students draw represent their thinking and should mirror their work with models.

After students have modelled and solved a number of addition and subtraction situations, they may be introduced to strip diagrams as another way to represent the situations. For example, Bobby was given 363 green stamps. He already had 2127 stamps. How many stamps does he have now? The strip diagram for this problem is as follows:

<table>
<tr><td>2127</td><td>363</td></tr>
<tr><td colspan="2">?</td></tr>
</table>

Personal strategies make sense to students and are as valid as the traditional algorithm. Therefore, emphasis should be on students' algorithms rather than on the traditional algorithm. Most important is that students can justify how and why an algorithm works. Students should be encouraged to refine their strategies to increase their efficiency, and teachers should monitor each student's symbolic recording of the strategy to ensure that the recording is accurate, mathematically correct, organized, and efficient.

Examples of personal strategies and symbolic recordings can be found in the *Mathematics 4 Curriculum Guide*, pp. 182–188.

Whole Number Multiplication and Division

By the end of Mathematics 4, students are expected to be proficient with their multiplication facts. Proficiency with their division facts is not expected until the end of grade 5. Proficiency is defined as the ability to recall the multiplication facts quickly and accurately when needed. This could be achieved through learning a series of strategies, each of which addresses a cluster of facts. Each strategy is introduced, reinforced, and assessed before being integrated with previously learned strategies. It is important that students understand the logic and reasoning of each strategy, so the introduction of each strategy is very important. As students master each cluster of facts for a strategy, it is recommended that they record these learned facts on a multiplication chart. By doing this, they visually see their progress and are aware of which facts they should be practicing.

More information about these strategies and a suggested sequence for them can be found in the *Mathematics 4 Curriculum Guide*, pp. 200–206.

Students should use a variety of concrete and pictorial models to investigate multiplication and division to help them develop an understanding of the connection between the models and the symbols. Base-ten blocks serve as a tool for understanding the operations of multiplication and division. It is important that the students use correct mathematical language as they manipulate the materials and pictorially record their work with base-ten blocks. It is important to start with a word problem and then have students use materials to model the problem and to determine the product. For example: A marching band has 24 rows with 5 players in each row. How many people are in the band?

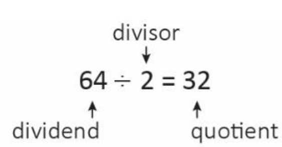
More information about the three categories of story problem structures can be found in the *Mathematics 5 Curriculum Guide*, p. 212.

It is not expected that students would be explicitly taught all possible multiplication and division algorithms. Instead, teachers should provide opportunities for students to develop their personal algorithms. Students can begin by using concrete materials to solve the problem while the teacher assists by recording the thinking symbolically and facilitating the documentation. Through these opportunities to model and record solutions to story problems, students discover the most efficient, accurate, and mathematically correct algorithms for the numbers included in a given problem.

Students should estimate products and quotients prior to exploring their own methods or procedures for finding the solution.

Providing students with base-ten materials allows them to solve the problems and discuss the concept of remainders. Teachers can work with the students to demonstrate ways of documenting their thinking. Students should understand that the remainder (the number of units left over) must be less than the divisor. Models help to clarify this idea.

In Mathematics 4, students are expected to express remainders as a digit and not as a fraction or decimal (e.g., a remainder of 7 is written as R7). Students also need to know that the answer for a division sentence is the quotient and the number to be divided is the dividend.



Students should understand that when solving division problems, remainders are handled differently depending on the context. They should recognize when a remainder is significant for decision making. For example, the remainder

- needs to be ignored (When you want to know how many $2 notebooks can be bought with $11, the answer is 5, since there is not enough money to buy 6 notebooks.)
- needs to be rounded up (When you want to know how many four-passenger cars are needed to transport 27 children, the answer is 7 since you cannot leave anyone behind.)
- must be addressed specifically (When 91 students are to be transported in 3 buses, there may be 30 students on two buses and 31 on the other because you cannot leave anyone behind.)
- is best described as a fraction (When 4 children share 9 oranges, each gets 2 oranges and $\frac{1}{4}$ of the remaining orange; remainders expressed as fractions are addressed in Mathematic 5.)

Additional information for examples of personal strategies and their symbolic recordings, can be found in the *Mathematics 4 Curriculum Guide*, pp. 218–220.

Fractions

Students construct an understanding of fractions when they begin with models. Students need to see fractions modelled using many different concrete materials. The models should include number lines, area models using pattern blocks or fraction circles, and set models using counters, money, or egg cartons. We might ask students to demonstrate different ways they may cut a square in halves, fourths, or eighths.

Presenting fractions in context will make them more meaningful to students. They could estimate fractions based on their class; for example: What fraction of the students in the class are left-handed, wear glasses, ride the bus to school, etc? Then the data could be gathered to give precise answers to the questions. It is important that students develop visual images for fractions and be able to tell “about how much” a particular fraction represents and learn common benchmarks, such as one-half.

Students should continue to use conceptual methods to compare fractions. These methods include the following:

- Comparing each to a benchmark (e.g., is $\frac{2}{5}$ more than or less than $\frac{1}{2}$?).
- Comparing the two numerators when the fractions have the same denominator.
- Comparing the two denominators when the fractions have the same numerator.
- Considerable time needs to be spent on activities and discussions to develop a strong number sense for fractions. Provide students with a variety of experiences using different models (number lines, pattern blocks, counters, fraction strips, etc.) and different representations of one whole with the same model. Students should recognize that a fraction can name part of a set as well as part of one whole and the size of these can change. Students also need to understand that fractions can only be compared if they are parts of the same whole. One-half of a watermelon cannot be compared to one-half of an orange. When comparing one-half and one-fourth, the whole is the “unit” (1).

It is important that students are able to visualize equivalent fractions as the naming of the same region or set partitioned in different ways. Students should be given opportunities to explore and develop their own strategies for creating equivalent fractions. They should be able to explain their strategy to others. Rules for multiplying numerators and denominators to form equivalent fractions should not be provided to students to follow without a conceptual understanding of why they work. Students must use concrete models to develop their understanding of sets of equivalent fractions. When formulating and verifying a rule for developing a set of equivalent fractions, students should recognize the multiplicative nature between the numerator and denominator.

Decimals
Students need experience representing decimal numbers with proportional concrete and pictorial models. These may include ten-frames, base-ten blocks, or grids. Students can begin work with decimal tenths using ten-frames with the whole ten-frame representing one and each block of the frame representing one-tenth. Students may then work with base-ten blocks and determine the value of other pieces if the rod represents one or if the flat represents one or if the large cube represents one. Varying which block represents one whole helps students develop flexibility in their thinking about decimal fractions. Later, they may model decimals using non-proportional models, such as money, and explain relationships, such as two dimes is two-tenths or twenty-hundredths of a dollar.

To work effectively with decimals, students should demonstrate the ability to represent decimal numbers using words, models, pictures, and symbols and make connections among various representations. Conceptual understanding of decimals requires that students connect decimals to whole numbers and to fractions. Decimals are shown as an extension of the whole number system by introducing a new place value, the tenth's place, to the right of the one's place. The tenth's place follows the pattern of the base-ten number system by iterating one-tenth ten times to make one whole or a unit (Wheatley & Abshire, 2002, p. 152). Similarly, the hundredth's place to the right of the tenth's place iterates one-hundredth ten times to make one-tenth.

Foster an understanding of decimals by ensuring that students read them correctly. For example, 3.4 should be read as 3 and 4 tenths, not 3 point 4, or 3 decimal 4. It is also important that students understand the relationship between fractions and decimals. For example, 12.56 is read as 12 and 56 hundredths. Saying decimal numbers correctly will assist students in gaining an understanding of how decimals relate to fractions (Specific Curriculum Outcome 5N09). By saying 12 and 56 hundredths, 56 is the numerator and 100 is the denominator. Saying the number correctly also reinforces the idea that the digits to the right of the decimal are part of the whole number. Decimals have multiple names, and students must become proficient at representing and naming them in a variety of ways (e.g., 5.67 could be read as five and sixty-seven hundredths or fifty-six tenths, 7 hundredths).

Decimals are fractional parts, and therefore, it is essential that the relationship between decimals and fractions be regularly addressed. The connection between decimals and fractions is developed conceptually when the students read decimals as fractions and represents them using the same visuals. For example, 0.8 is read as eight-tenths and can be represented using fraction strips or decimal strips (Wheatley & Abshire, 2002). Students should use a variety of materials to model and interpret decimal tenths and hundredths.

It is essential that students recognize that all of the properties and developed strategies for the addition and subtraction of whole numbers also apply to decimals. For example, adding or subtracting tenths (e.g., 3 tenths and 4 tenths are 7 tenths) is similar to adding or subtracting quantities of other items (e.g., 3 apples and 4 apples are 7 apples). This could be extended to addition with tenths that total more than one whole (e.g., 7 tenths and 4 tenths are 11 tenths or 1 and 1 tenth). The same is true with hundredths and thousandths. Rather than telling students to line up decimals vertically, or suggesting that they add zeroes, direct students to think about what each digit represents and what parts go together. For example, to add 1.625 and 0.34, a student might think using front-end addition, 1 whole, 9 (6 + 3) tenths, and 6 (2 + 4) hundredths, and 5 thousandths or 1.965.

D. What are the most appropriate methods and activities for assessing student learning?

Below are some sample questions related to number which will be used to represent some of the appropriate methods and activities for assessing student learning.

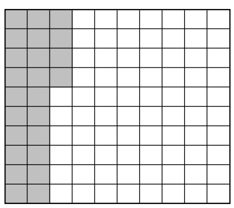
Encourage students to share their thinking, their strategies, and their solutions. Rich discussion fosters the development of alternative problem-solving strategies.

Examples:

1. Which set of decimal numbers contains numbers in decreasing order?

○	0.05	0.2	0.030	0.004
○	0.2	0.05	0.004	0.030
○	0.030	0.004	0.2	0.05
○	0.2	0.05	0.030	0.004

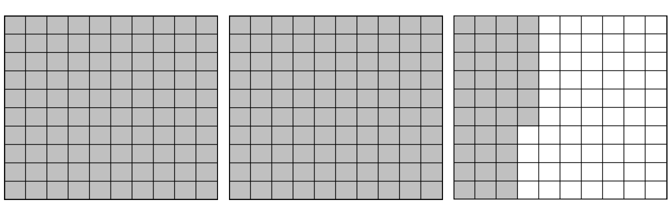
2. Look at the hundredths grid below. This grid represents one whole or 1.



Which decimal number does the shaded part represent?

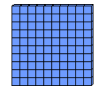
○ 0.024
○ 0.24
○ 2.4
○ 24.0

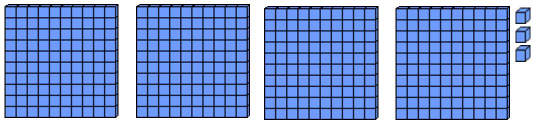
3. Each hundredths grid represents a whole or 1.



Which decimal number do the shaded parts of these grids represent?

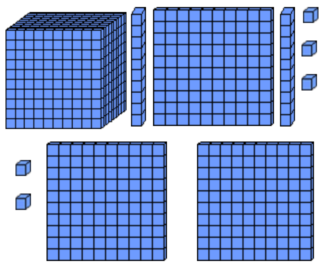
○ 0.236
○ 2.36
○ 23.6
○ 233

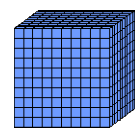
4. Which decimal number does this set of base-ten blocks represent if the flat  represents 1?



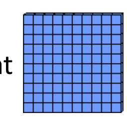
- ○ 0.403
- ○ 0.43
- ○ 4.03
- ○ 4.3

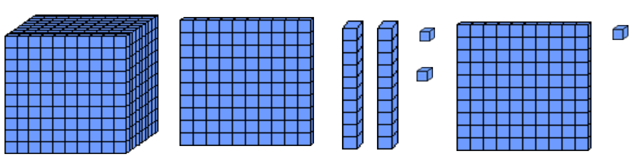
5. Leanne represents a decimal number using all the following base-ten blocks.



Which decimal number does the set of base-ten blocks above represent if the large cube  represents 1?

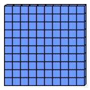
- ○ 0.1325
- ○ 1.325
- ○ 13.25
- ○ 1325

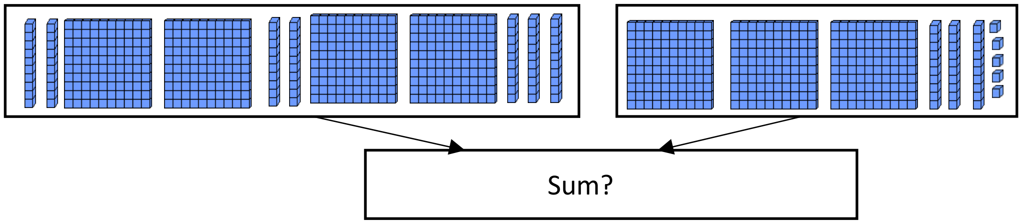
6. Which decimal number does this set of base-ten blocks represent if the flat  represents 1?



- ○ 0.1223
- ○ 1.223
- ○ 12.23
- ○ 122.3

7. Two decimal numbers are represented below using base-ten blocks.

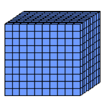
If the flat  represents 1, what will be the sum of these two numbers?

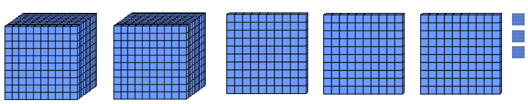
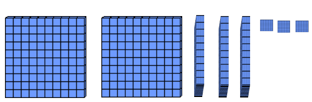
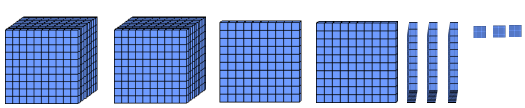
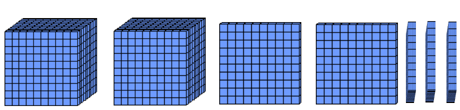


- ○ 4.07 + 3.15 = 7.22
- ○ 4.70 + 3.05 = 7.75
- ○ 4.70 + 3.15 = 7.85
- ○ 4.70 + 3.35 = 8.05

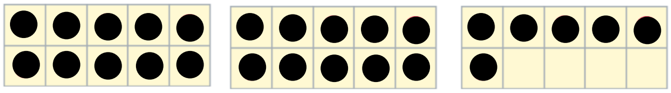
8. Which fraction is equivalent to the decimal number 0.004?

- ○ $\frac{4}{10\,000}$
- ○ $\frac{4}{1000}$
- ○ $\frac{4}{100}$
- ○ $\frac{4}{10}$

9. The large cube  represents 1. Which set of base-ten blocks represents the decimal number 0.233?

- ○ 
- ○ 
- ○ 
- ○ 

10. If a ten-frame represents one whole or 1, which decimal number will be represented by the following illustration?



- ○ 0.026
- ○ 0.206
- ○ 0.260
- ○ 2.6

11. Choose the appropriate symbol >, =, or < to compare $\frac{2}{5}$ to $\frac{4}{5}$.

○ $\frac{2}{5} > \frac{4}{5}$ ○ $\frac{2}{5} = \frac{4}{5}$ ○ $\frac{2}{5} < \frac{4}{5}$

12. Choose the appropriate symbol >, =, or < to compare 36.09 to 36.090.

○ 36.09 > 36.090 ○ 36.09 = 36.090 ○ 36.09 < 36.090

13. Here is a set of fractions: $\frac{2}{4}$ $\frac{15}{30}$ $\frac{30}{60}$

Which fraction below belongs to this set?

- ○ $\frac{1}{5}$
- ○ $\frac{1}{4}$
- ○ $\frac{1}{3}$
- ○ $\frac{1}{2}$

14. Which number is 2 hundredths more than 4.89?

- ○ 4.91
- ○ 5.81
- ○ 5.91
- ○ 6.89

15. Which statement represents the number 6.803 in words?

- ○ six thousand eight hundred three
- ○ six and eight hundred three tenths
- ○ six and eight hundred three hundredths
- ○ six and eight hundred three thousandths

16. Which statement represents the place value of each digit of 14.352?

- ○ 10 tens + 4 ones + 3 tenths + 52 hundredths
- ○ 14 tens + 4 ones + 3 tenths + 5 hundredths + 2 thousandths
- ○ 14 ones + 35 tenths + 2 hundredths
- ○ 1 ten + 4 ones + 3 tenths + 5 hundredths + 2 thousandths

17. Emma bought vegetables and fruits for $23.45.
She gave $30.00 to the cashier.
How much money did the cashier return to Emma?

- ○ $6.45
- ○ $6.55
- ○ $7.45
- ○ $7.55

18. Dorothy bought a rope 5.8 m long.
She cut the rope into 2 pieces.
What could be the length of the two pieces of rope?

- ○ 1.6 m and 42 m
- ○ 1.9 m and 3.9 m
- ○ 2.4 m and 0.34 m
- ○ 4.9 m and 2.9 m

19. Natalie bought the following items:

- toaster: $89.97
- television: $309.99
- computer: $413.49

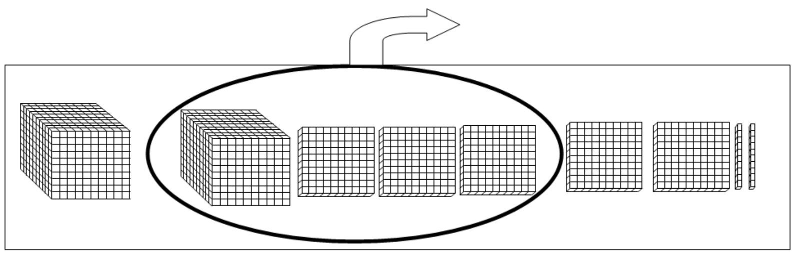
Estimate the amount of money paid by Natalie, by rounding off the price of each item to the nearest ten.

- ○ $400
- ○ $500
- ○ $720
- ○ $810

20. Mrs. Smith organizes a visit to the museum for her 21 students.
Mrs. Smith and the students want to travel in taxis to go to the museum.
Each taxi can hold 4 people, the driver is not included.
How many taxis do Mrs. Smith and her students need to go to the museum?

- ○ 6
- ○ 5
- ○ 4
- ○ 3

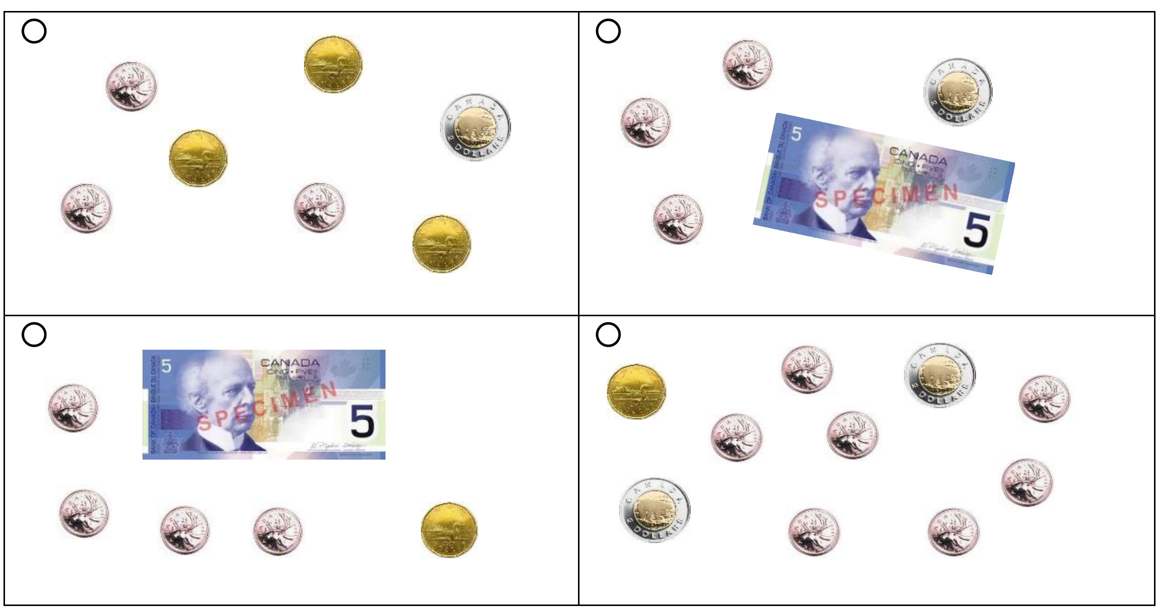
21. Which operation is represented by the following base-ten blocs?



The small cube represents 1

- ○ 122 – 13 = 109
- ○ 122 + 13 = 135
- ○ 2520 – 1300 = 1220
- ○ 2520 + 1300 = 1220

22. Eric has $6.75 in his pocket.
Which set represents Eric's money?



23. David has 145 marbles. He keeps 5 marbles in his pocket.
He distributes the rest of the marbles to his three friends Roger, Albert and Denis.
He gives 80 marbles to Roger.
He gives Albert double the number of marbles that he gives to Denis.
How many marbles does Denis get?

- ○ 85
- ○ 80
- ○ 40
- ○ 20

Mathematics in Grade 6 Lesson Learned 2
Estimation

Students seemed to improve their knowledge of how and when to use estimation strategies. They appear to now use strategies for estimation and not simply guess at an answer. When working with estimation questions many students want to get the right answer and feel there is no value in estimating first. Students need to use estimation strategies such as using benchmarks, rounding, front-end addition, making a friendly number, compensation, subtraction (left-to-right calculations), and clustering of compatible numbers.

A. What conclusions can be drawn from the Nova Scotia Assessment: Mathematics in Grade 6?

Overall, we discovered that student are not competent estimators. Students do not like to use estimation strategies, unless specifically asked to do so in a question. Even then, they appear not to use any strategy for estimation and simply guess at an answer. Many students when working with estimation questions, just want to get the “exact” answer and feel there is no value in estimating first.

The ability to estimate computations is a major goal of any modern computational program. For most people in their everyday lives, an estimate is all that is needed to make decisions, and to be alert to the reasonableness of numerical claims and answers generated by others and with technology. The ability to estimate rests on a strong and flexible command of facts and mental calculation strategies.

B. Do students have any misconceptions or errors in their thinking?

It is important to distinguish between misconceptions and errors. Misconceptions are, in effect, the misunderstandings about mathematical ideas which students entertain, and which usually lead to errors occurring. These are the most serious kind of errors which require urgent action on the part of the teacher. Errors may occur for a variety of underlying reasons, ranging from careless mistakes to errors resulting from misconceptions. - John SanGiovanni – (*Mine The Gap For Mathematical Understanding Grades 3-5*)

A misconception that students have when asked to estimate is that they think that an estimate is a random guess. A strategy to help students overcome this misconception is to discuss the reasons for estimating and as well what strategies to use. “Students should understand that an estimate does not need to be accurate, but it should be close to the actual value” (Pearson, 2007a, p.17).

Some students believe that estimates are either right or wrong. Discuss when estimates are needed and why they may be needed to be close to exact answers. Together, think of and list such situations. Share different methods by which to arrive at reasonable estimates for these situations. Encourage students to focus on arriving at estimates that are appropriate for the problem. For example, discuss an estimate for the number of people attending a concert: Is 5000 reasonable, or 5005? Emphasize that if the exact number attending was 5007, both estimates are “right”. (Pearson, 2008a, p.17)

Place value often plays a role in misconceptions and errors in student thinking.

When estimating the difference of a 3-digit number subtracted from a 4-digit number using front-end rounding, some students confuse place value and subtract a number in the thousands rather than hundreds. To help focus their attention on place value, encourage students to write out their numbers in vertical format and to read them aloud before making their estimates. (Pearson, 2008a, p.26)

Our students have many misconceptions when estimating products and quotients to solve problems. Students do not understand that rounding both factors up or down gives an estimate that is not as close as rounding one factor up and the other factor down. Encourage student to estimate the product of 28 x 9 by changing only one factor to a compatible number. They might choose 30 x 9 = 270 or 28 x 10 = 280. Then have students estimate by rounding both factors up to 30 x 10 = 300. Have them use a calculator to find 28 x 9 = 252 and recognize that the first 2 estimates are closer than the third estimate. (Pearson, 2008b, p.16)

A common error in student thinking that we discovered on the provincial assessment was when students were asked to estimate the area of a geometric figure, they appeared to lose track of their count when counting whole and part squares.

Encourage students to make the whole squares and part squares with different symbols or colours, as they count. Another common misconception or error in thinking, is students ignore the part squares when finding the area of a shape. Point out to students that including the part squares produces a more accurate measure. (Pearson, 2007b, p.36)

The student results of the **Nova Scotia Assessments between 2018-2019 and 2019-2020 in Grade 6** shows the following:

- students continue to make progress in the area of number estimation in problem solving contexts

Example 1: Estimate a difference
Students estimate the difference of 4145 – 284 incorrectly.
They correctly round off each number according to the first digit from left to right.
4145 ⟶ 4000
284 ⟶ 200
But then, ignore place value. 4000 –200 = 2000, instead of 3800.

In this example, the error made is that the hundreds digit, 2, was incorrectly subtracted from the thousands digit 4.

Often, this error occurs because the student is unable to see the digits of each number in their correct place values. To eliminate this misconception and to avoid this error, encourage the students to focus on the place value by aligning the digits of the two numbers vertically using a grid paper or a lined paper.

$$\begin{array}{r} 4000 \\ -300 \\ \hline 3700 \end{array}$$

or

4	0	0	0
–	3	0	0
3	7	0	0

Note: Students could also practice reading numbers differently. Students could otherwise read the numbers by subtracting. For example, 40 hundreds minus 3 hundreds equals 37 hundreds or 3700.
(Gives strong link to base-ten and visual representation.)

Another misconception appears in students' reasoning when they are asked to estimate products or quotients. The students do not understand that rounding up or rounding down the two factors at once gives an estimate which is not close to that obtained if only one factor is rounded off.

Example 2: Estimate a product
Estimate the product of 28 x 9.
To estimate this product, we can use compatible numbers which are close to the exact numbers, but easy to use. In this case 10 and its multiples are easy to use.

If we round off both factors, we obtain	30 x 10 = 300.
If we round off one factor, that is 28, we obtain	30 x 9 = 270
If we round off only 9, we obtain	28 x 10 = 280
The exact value of this product is	28 x 9 = 252

We notice that the first estimation gives an answer (300) further away from the exact value (252) than those provided by the other two estimations

Example 3: Estimate a quotient in a context
There are 123 apples in a box.
Danielle wants to make baskets of fruit. Each basket contains 4 apples.
About how many baskets of fruit can Danielle make?

Solution:
The term "About" infers an estimation.
We round off 123 to the nearest hundred according to the first digit from left to right.
$123 \div 4$ is rounded to $100 \div 4 = 25$ approximately
We obtain about 25 baskets. Thus, this estimation is low.
To obtain a closer estimation, we round off 123 to the nearest ten, that is 120.
$120 \div 4 = 30$
Thus, $123 \div 4$ is approximately $120 \div 4 = 30$.
Thus, Danielle can make approximately 30 baskets of fruit.

C. What are the next steps in instruction for the class and for individual students?

Estimation strategies need to be taught to students as they are not inherent in their thinking. When teachers model estimation strategies for their students and encourage them to use these strategies, students become very good estimators.

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know how, when, and which strategy to use when estimating. Estimation is used to make mathematical judgments and develop useful, efficient strategies for dealing with situations in daily life.

Van de Walle, in *Elementary and Middle School Mathematics Teaching Developmentally* (2001), suggests that students should be exposed to real examples of estimation in daily life:

Discuss situations where computational estimations are used in real life. Some simple examples include figuring gas mileage (km), dealing with grocery store situations (doing comparative shopping, determine if there is enough to pay the bill), adding up distances in planning a trip, determining approximate yearly or monthly totals of all sort of things (school supplies, haircuts, lawn-mowing income, time watching TV), and figuring the cost of going to a sporting event or show including transportation, tickets, and snacks. Help children see how each of these involves a computation (in contrast to measurement estimates). ... Discuss why exact answers are not necessary in some instances and why they are necessary in others. (p. 199)

Before attempting pencil-and-paper or calculator computations, students must find estimates, so they are alert to the reasonableness of those pencil-and-paper or calculator answers. Teachers should also model this process before doing any calculations in front of the class. Students should constantly be reminded to estimate before calculating.

While teaching estimation strategies, it is important to use the language of estimation. Some of the common words and phrases are *about*, *just about*, *between*, *a little more than*, *a little less than*, *close*, *close to*, and *near*. It is also important for students to hear and see a variety of contexts for each estimation strategy, so they can transfer the use of estimation strategies to situations found in their daily lives.
The context, the numbers, and the operations involved affect the estimation strategy chosen. Students will be expected to use the following estimation strategies:

- Front-End Estimation
- Adjusted Front-End Estimation
- Rounding
- Compatible numbers in Problem-Solving Contexts

The following examples present real situations in which estimates are used. Teachers should present and discuss each situation with the whole class by asking the students questions such as:

- On what is this estimate based?
- What terms show that we are in a situation which requires an estimation?
- Why an exact value is not necessary in such a situation?

Example 1:
When driving on the highway 102, from the airport toward Truro, Kim estimates the distance between exit 6 and exit 13 to be **about** 60 km.

Example 2:
Going by car from Halifax to Yarmouth, Michel estimates that he needs **less than** 35 L of gasoline.

Example 3:
A taxi driver estimates that it will take **approximately** 35 minutes to go from Halifax to the airport, by driving on highway 102.

Example 4:
A Radio-Canada journalist estimated the crowd participating in a climate march **between** 25 000 and 30 000 people.

Example 5:
A motorist from Dartmouth estimated that it took him **more than** 2 hours to get to Halifax in a snowstorm.

Example 6:
David said that it took him **less than** 90 minutes to complete his mathematics test.

Once the discussion ends, ask students to focus on the following points:

- The estimation is a way to obtain and use a value that is near the exact value.
- The quality of a good estimation depends on the context, on the experience that we have from similar situations, on the strategy used, and on the estimator.

Students and teachers should note that multiplication and division estimations are typically further from the actual value because of the nature of the operations involved. Students should have opportunities to decide what might be the closest estimate, whether an estimate is over or under the exact answer, and to justify their thinking.

Students should make use of place-value understandings in estimating or calculating and should be encouraged to talk about place-value concepts while explaining reasoning. Teachers should model the use of place-value language and encourage students to use place-value language when discussing their reasoning. Students should be encouraged to think about when pencil-and-paper or mental algorithms will work efficiently and when it may be more appropriate to use technology. Students should be encouraged to apply mental mathematics strategies when appropriate to develop their number and operation sense.

D. What are the most appropriate methods and activities for assessing student learning?

Below are some sample questions related to estimation which will be used to represent some of the appropriate methods and activities for assessing student learning.

Encourage students to share their thinking, their strategies, and their solutions. Rich discussion fosters the development of alternative problem-solving strategies.

Examples:

1. You drink 285 mL of milk on the first day, 325 mL of milk the second day, and 512 mL of milk on the third day.
About how many millilitres of milk did you drink during these three days?
Choose the best estimate for the answer.

- ○ 900 mL
- ○ 1000 mL
- ○ 1080 mL
- ○ 1100 mL

2. Jeff has 5800 cans of soup. He wants to collect 13 250 cans for the food bank.
About how many more does he need to collect?
Choose the best estimate for the answer.

- ○ 6800
- ○ 7000
- ○ 7400
- ○ 7600

3. In December, I had $783.00 in my bank account.
Now I only have $420.00.
About how much money have I spent?

- ○ $300
- ○ $360
- ○ $480
- ○ $1020

4. During the summer, Marcie travelled 7185 km while Jimmy travelled 4205 km.
About how much further did Marcie travel than Jimmy?

○ 2900 km
○ 2980 km
○ 2990 km
○ 3000 km

5. Ms. Harris is buying pencils for her school. There are 537 students in her class.
Pencils come in packages of 6.
How many packages of pencils should Ms. Harris buy?

○ 95
○ 90
○ 85
○ 80

6. Tony has $18.26. His Father gives him $11.78. Tony buys two cartons of milk. Each carton of milk costs $2.29.
About how much money does Tony have now?

○ $35
○ $34
○ $28
○ $25

7. Julie bought 25 boxes of Christmas decorations. Each box contains 52 ornaments.
Sherene said that Julie bought about 1000 ornaments.
Lisa said that Julie bought about 1500 ornaments.
Tom said that Julie bought about 1250 ornaments.
Joe said Julie bought a little bit less than 1400 ornaments.
Which one of the four people made the most accurate estimate?

○ Sherene
○ Lisa
○ Tom
○ Joe

Mathematics in Grade 6 Lesson Learned 3
Patterns and Relations

Students had difficulty when moving from the basic understanding of patterns to the generalization of a pattern rule to enable them to find any term. Students need work on continuing and extending a pattern to predict a subsequent term that is not consecutive. Students need to continue to work with representations of patterns, contextually, pictorially, symbolically, and verbally.

Mathematics is about recognizing, describing, and working with numerical and non-numerical patterns. Patterns exist in all strands, and it is important that connections are made among strands. Working with patterns enables students to make connections within and beyond mathematics. These skills contribute to students' interaction with and understanding of their environment. Patterns allow students to make predictions and justify their reasoning when solving problems. Learning to work with patterns in the early grades helps develop students' algebraic thinking, which is foundational for working with more abstract mathematics in higher grades. Patterns may be represented in concrete, visual, or symbolic form. Students should develop fluency in moving from one representation to another. Students must learn to recognize, extend, create, and use mathematical patterns.

A. What conclusions can be drawn from the Nova Scotia Assessment: Mathematics in Grade 6?

We noticed that students could extend a pattern when given a pattern rule to find the next term. For example, students were successful when asked to find the next term in a pattern such as 5, 10, 15, 20, ... (Application question). But they were challenged when asked to continue and extend a pattern to predict a subsequent term that was not consecutive. For example, use the pattern rule to determine the 50^{th} term in that same pattern of 5, 10, 15, 20, ... (Analysis question). Many students experienced challenges when working with patterns and relationships. They had difficulty moving from the basic understanding of patterns to the generalization of a pattern rule to enable them to find any term.

Provincial assessment information also shows that many students experienced challenges when asked to represent a word problem with an equation containing an unknown. For example: There are now 17 students in the classroom. Eight students went have already gone to gym class. How many students are usually in the classroom? Students might represent this problem as 17 = n – 8 or n = 17 + 8. Students continue to struggle with representing patterns, contextually, concretely, pictorially, symbolically, and verbally.

B. Do students have any misconceptions or errors in their thinking?

It is important to distinguish between misconceptions and errors. Misconceptions are, in effect, the misunderstandings about mathematical ideas which students entertain, and which usually lead to errors occurring. These are the most serious kind of errors which require urgent action on the part of the teacher. Errors may occur for a variety of underlying reasons, ranging from careless mistakes to errors resulting from misconceptions. - John SanGiovanni – (*Mine The Gap For Mathematical Understanding Grades 3-5*)

Patterns represent identified regularities based on the rules describing the patterns' terms. Unless a pattern rule is provided there is no single way to extend a pattern, since there would be an unlimited number of possible patterns. The common error or misconception that students have is they do not recognize that there are different ways to continue a pattern if a pattern rule is not described. For example:
If given 5, 10, 15 . . . in the beginning of the pattern, they may only see it as a repeating pattern and will not consider that it might be an increasing pattern. A suggested strategy for students who only relate to repeating patterns is to provide many opportunities to work with increasing patterns using concrete materials and pictures before working with numbers. For example: Provide blocks for students to make an increasing pattern of 1 row of 5 blocks, then 2 rows of 5 blocks, and then 3 rows of 5 blocks. Discuss how

there can be more than one pattern. For example: 5, 10, 15, 20, 25, 30,… or 5, 10, 15, 25, 35, 50, 65, … (Small, 2009, p. 579).

Another error that students sometimes have, is that they omit important information when describing a pattern rule. For example, if describing the pattern 4, 6, 9, 13, 18 . . ., a student might state the rule as "Just keep adding 1 more." A suggested strategy is to follow students' rules literally to show them the incomplete nature of their rules. For example, with the rule "Just keep adding 1 more," you might continue the pattern by writing down 4, 6, 9, 13, 18, 19, 20, 21, … literally adding 1 more. This should help the students see the need for a clearer and more complete rule such as, "Start with 4 and add 2, then add 3, then add 4, each time adding a number that is 1 greater than the time before."

The student results of the **Nova Scotia Assessments between 2018-2019 and 2019-2020 in Grade 6** shows the following:

- patterns and relations show a slight decrease in the percentage of students who have chosen the correct answers

C. What are the next steps in instruction for the class and for individual students?

Students will continue to work with, and expand on, the many patterns found in different tables and charts. The hundred charts and addition tables (up to 9 + 9) should be familiar to students, as they have worked extensively with them in Mathematics 2 and Mathematics 3. Students should be encouraged to identify and explain patterns that can be found in these familiar tables and charts. Students in Mathematics 3 began representing basic multiplications facts (up to 5 × 5) concretely, contextually, and pictorially, but have not worked extensively with a multiplication table or chart (9 × 9). Therefore, students should be encouraged to identify and explain the patterns in the multiplication table or chart. The patterns found in the addition and multiplication tables can then be used to help students determine an unknown sum, difference, product, or quotient. Students should also be encouraged to find and explain place-value patterns.

Students should use a variety of vocabulary, including vertical, horizontal, diagonal, row, column, starting point, increasing, decreasing, and repeating, to help describe the patterns that they find in charts and tables. Students should begin by representing a pattern with concrete materials and/or pictures. Then, they should represent the same pattern in a table or chart. Once a table or chart is developed, students have two representations of a pattern: the one created with the drawing or materials and the numeric version that is in the table or chart. They can then explain how these patterns are mathematically alike, that is, why the same relationship exists between the pattern in a table and its concrete representation.

Students should also be given opportunities to reproduce a pattern using concrete materials when presented with a pattern displayed in a table or chart. Students should also be asked to describe what is happening as the pattern increases (or decreases) and how the next step is related to the previous one. It is helpful for students to think of a pattern rule and apply it when analyzing tables or charts for errors.

For additional information and teaching suggestions related to patterning, please refer to:

Curriculum Guide	Outcome(s)	Pages
Grade 2	PR01, PR02	81–90
Grade 3	PR01, PR02	103–111
Grade 4	PR01, PR02, PR03, PR04	94–116
Grade 5	PR01	102–108

Department of Education and Early Childhood Development (EECD), Mathematics Curriculum Guides.

Algebraic Reasoning and Inquiry
"Patterns are key factors in understanding mathematical concepts. The ability to create, recognize, and extend patterns is essential for making generalizations, seeing relationships, and understanding the order and logic of mathematics," (Burns, 2007, p. 144). These skills provide the groundwork for algebraic reasoning and inquiry.

Algebra is a system that allows us to represent and explain mathematical relationships. Exploring patterns leads to algebraic thinking. Students are thinking algebraically when they solve open number sentences like 5 + ☐ = 13, first using boxes or open frames, then using letters, $5 + n = 13$. Students usually progress from the use of open frame to letters. When letters are used in mathematics, they are called variables. It is useful for students to think of variables as numbers that can be operated on and manipulated like other numbers.

Through the use of balance scales and concrete representations of equations, students will see the equal sign means "is the same as," and acts as the midpoint or balance, with the quantity on the left of the equal sign being the same as the quantity on the right. When the quantities balance, there is equality. When there is an imbalance, there is inequality.

Students need to be able to write a problem for a given equation such as $n \div 7 = 16$ or $16 = n \div 7$. The word problem must make sense of the equation. For the above equation, a student might write, there are sixteen weeks until we have holidays. How many days are there before holidays begin? Students are also asked to write an equation for a situation such as, Kate is four times as old as Anna. If Kate is 12 years old, how old is Anna?

D. What are the most appropriate methods and activities for assessing student learning?

Below are some sample questions related to patterns and relations which will be used to represent some of the appropriate methods and activities for assessing student learning.

Encourage students to share their thinking, their strategies, and their solutions. Rich discussion fosters the development of alternative problem-solving strategies.

Examples:

1. The rule for a number pattern is:
Starting at 92, subtract 6 each time.
Which is the pattern?

- ○ 92, 86, 80, 74, 68, …
- ○ 92, 98, 104, 110, 116, …
- ○ 92, 86, 85, 79, 78, …
- ○ 92, 86, 79, 73, 67, …

2. Which of the following patterns is described by the rule: **Start at 8, and add 3 each time?**

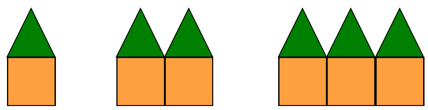
- ○ 8, 11, 14, 17, 18, …
- ○ 8, 11, 10, 13, 14, …
- ○ 8, 11, 12, 15, 16, …
- ○ 8, 11, 14, 17, 20, …

3. What are the next three terms in this counting pattern?

5, 8, 7, 10, 9, 12, 11, ____, ____, ____, ….

- ○ 13, 14, 15,
- ○ 14, 17, 16
- ○ 13, 16, 15
- ○ 14, 13, 16

4. Look at the picture below.



Term 1 Term 2 Term 3

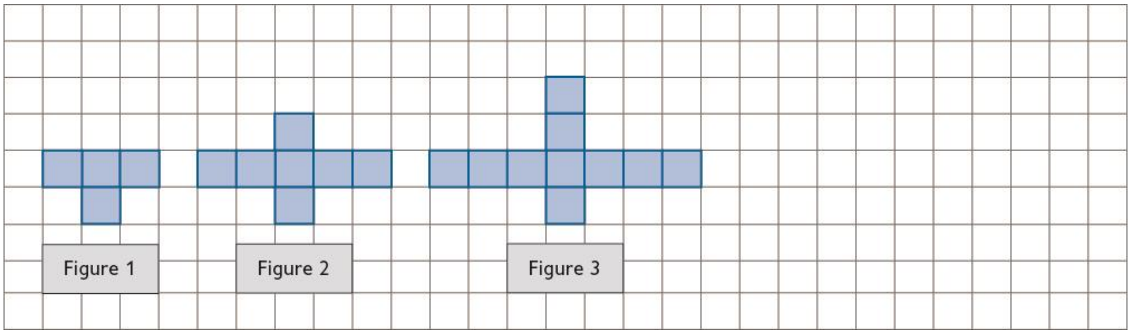
The first term is made up of two pattern blocks.
The second term is made up of four pattern blocks, and the third term is made up of six pattern blocks.
Predict the number of pattern blocks in the eighth term.

- ○ 10
- ○ 12
- ○ 14
- ○ 16

5. Harold has a bag of marbles.
He gave 5 marbles to his brother. Now Harold has 12 marbles.
How many marbles did Harold have in the bag?
Which equation can help you to solve this problem?

- ○ 12 + 5 = ☐
- ○ 12 + ☐ = 5
- ○ ☐ - 5 = 12
- ○ ☐ = 12 – 5

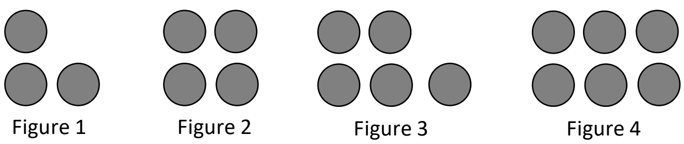
6. Examine the following pattern consisting of small squares found in the figures.



How many small squares would be found in Figure 10?

- ○ 31
- ○ 30
- ○ 21
- ○ 20

7. Examine the following pattern consisting of small circles found in the figures.



How many circles will be in the 50th figure?

- ○ 51
- ○ 52
- ○ 53
- ○ 54

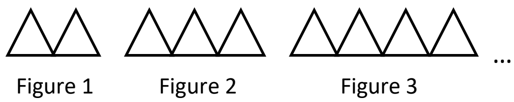
8. Albert and Nabil have 72 marbles.
Albert has 48 marbles.
How many marbles does Nabil have?
Which equation allows you to calculate the number of marbles Nabil has?

- ○ $b = 72 + 48$
- ○ $72 + b = 120$
- ○ $b - 48 = 72$
- ○ $48 + b = 72$

9. Jessica constructs squares with 32 toothpicks.
The squares do not touch.
Which equation can help you to determine the number of squares that Jessica constructs?

- ○ $32 \times 4 = \square$
- ○ $32 \div 4 = \square$
- ○ $32 + 4 = \square$
- ○ $32 - 4 = \square$

10. Fang draws this geometric pattern using triangles.



Let ***f*** represent the figure number.

What expression represents the number of triangles in Figure number ***f***?

- ○ $2f$
- ○ $f + 2$
- ○ $f - 1$
- ○ $f + 1$

11. A plant grows every day.

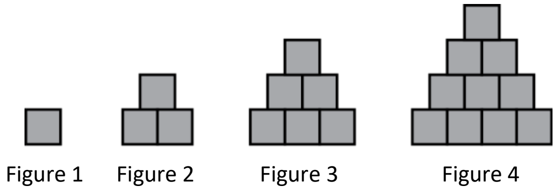
The following table represents the height, h in cm, of the plant in terms of the number, n, of days.

Number of days	Height of the Plants (cm)
1	4
2	5
3	6
4	7
5	8

Which equation represents the relationship between the height of the plant and the number of days?

- ○ $h = 4n$
- ○ $h = 5 - n$
- ○ $h = 2n$
- ○ $h = 3 + n$

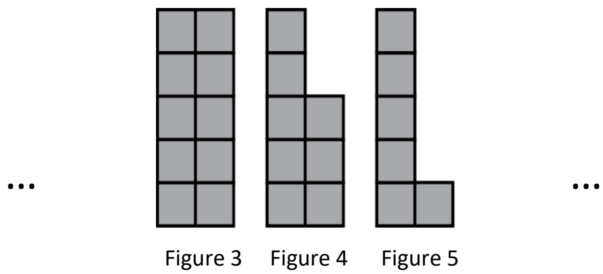
12. Observe the following increasing pattern:



How many small squares are there in Figure 10?

- O 15
- O 35
- O 55
- O 6

13. Examine the following pattern of Figure 3, Figure 4, and Figure 5 created using small squares:



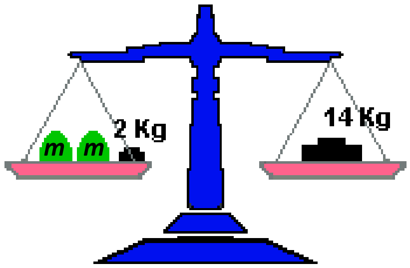
How many small squares are there in Figure 1?

- O 1
- O 4
- O 12
- O 14

14. What is the value of X on the balance below which will make the left side equal to the right side?

- O 1
- O 2
- O 3
- O 4

15. There are 2 identical objects on the left side of the balance. Let “m” be the mass of each object.



Which equation is represented by this balance?

- ○ $m + 2 = 14$
- ○ $m - 2 = 14$
- ○ $2m + 2 = 14$
- ○ $2m - 2 = 14$

Mathematics in Grade 6 Lesson Learned 4
Measurement

Students had difficulty when using the five representations of a concept (contextual, concrete, pictorial, symbolic, verbal) in order to have concept attainment when working with area and perimeter. Students need to work with perimeter and area together in application and analysis questions. Students need to build a conceptual understanding of what it means to measure with a ruler and not approach the activity as a rote procedure. Many students had difficulty when measuring the length of an object using a graduated ruler*.*

Children are naturally curious about measurement. They are interested in how tall, how big, how heavy, how long, how hot, or how cold things are. Initially, they accept answers that describe comparisons: "An elephant is about twice as tall as my teacher." Gradually, they come to understand that measurement is a tool that can help them answer questions more precisely. (Small, 2009, p.364)

A. What conclusions can be drawn from the Nova Scotia Assessment: Mathematics in Grade 6?

We noticed that students did very well when presented with questions related to time. For example, they were successful when reading time on an analog clock, and expressing it numerically (Knowledge questions). They also did well when solving a word problem involving elapsed time (Application questions). When students were asked to estimate length and capacity for real objects using benchmarks, they were able to do this with confidence. They did seem to struggle with questions involving mass and volume.

Two big ideas in measurement are perimeter and area. Students were able to determine the area of regular geometric shapes. When asked to find the area of irregular geometric shapes, they were challenged. They also struggled when asked to work with perimeter and area together in application and analysis questions. We noticed that students, when given a word problem concerning the relationship between perimeter and area, showed little or no understanding. They were unable to predict the impact on the perimeter or on the area of a two-dimensional figure when the shape of this figure changes, while maintaining the same area or the same perimeter.

For example, students struggled to recognize that shapes with the same area can have different perimeters. Word problems involving perimeter were a challenge for students as well.

Many students may make the connection between a number line and a ruler when they first begin using the ruler when measuring.

Measuring with a ruler appeared to be a rote procedure for our students with no evidence of conceptual understanding. Children who are simply reading the mark on the ruler do not understand how a ruler is a representation of a row of units. (Van de Walle, 2001, p. 234)

When students first learn to use a centimetre ruler, some will mistakenly begin measuring from points other than the 0 cm mark. Beginning partway along the ruler can indicate that the student has not yet learned that the measurement represents the whole length of the object, from where it begins to where it ends, rather than just the endpoint. Beginning from the 1 cm mark may indicate that the student does not realize that the scale on the ruler actually begins at 0 cm, or simply that the student assumes you always start at 1. This problem often occurs if the 0 cm end is not labelled. As well, students sometimes begin from the opposite end of the ruler, rather than at the 0 mark. (Small, 2009, p.383)

B. Do students have any misconceptions or errors in their thinking?

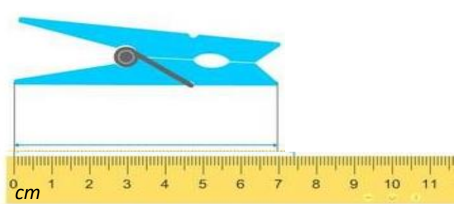
It is important to distinguish between misconceptions and errors. Misconceptions are, in effect, the misunderstandings about mathematical ideas which students entertain, and which usually lead to errors occurring. These are the most serious kind of errors which require urgent action on the part of the teacher. Errors may occur for a variety of underlying reasons, ranging from careless mistakes to errors resulting from misconceptions. - John SanGiovanni – (*Mine The Gap For Mathematical Understanding Grades 3-5*)

A common error with perimeter is that sometimes students forget to include the measures of unlabeled sides. Encourage students to label all sides before finding the perimeter. Through structured exploration activities, students should conclude that squares with the same area have the same perimeter and squares with the same perimeter have the same area. However, rectangles with the same area can have different perimeters, and rectangles with the same perimeter can have different areas. The generalization about squares is often over-generalized causing a common misconception about the relationship between area and perimeter in rectangles and in other polygons.

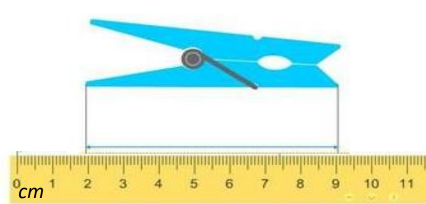
Area measures are often thought of as being flat. Students should understand that area tells about the space an object takes up on a flat surface. Students should learn that the area of a shape is preserved (i.e., the area of a shape does not change if it is cut up and rearranged to make a different shape).

A common misconception is for students to rely on numbers alone, without considering the units. As students explore area, reinforce the importance of naming the measurement unit each time a measurement is said because the units communicate how big the measurement is. Without the unit, there is no way of knowing what the numbers mean. It is also important that students learn that the units used to measure the area of an object or to compare the areas of two objects must be the same size. Comparison activities should be designed to help students discover that it is necessary to apply the same unit of measure when comparing two different areas.

A common error or misconception that many students have is the placement of the ruler when measuring an object. Teachers need to assist students with using rulers accurately. Some students fail to consider the gap between the end of the ruler and the zero mark. Many students begin to measure at other points on a ruler, other than the 0 cm mark when they first start to use a standard measurement such as a cm ruler. Some students ignore the 0 cm mark and begin at the 1 cm mark. This may indicate that students do not realize that the scale of the ruler begins at 0 cm, especially if it is not labelled at the beginning of the ruler. A strategy often used to help students with this misconception is to break up plastic rulers. Give a piece of broken ruler to each student and ask them to measure items in the classroom. Observe how they attempt to measure items, especially if there is not a familiar starting point for them such as 0 cm or 1 cm on their piece of ruler. It should be noted that some students may start at 1 on the ruler and still use the ruler accurately by taking this into consideration. It is important that students are encouraged to estimate measurements before verifying them using a measurement tool.



The clothes pin measures 7 cm.



The clothes pin measures 7 cm.

The student results of the **Nova Scotia Assessments between 2018-2019 and 2019-2020 in Grade 6** shows the following:

2018-2019 Results	***2019-2020 Results***
54% of the students did not know an estimate for the capacity of a spoon. *54% of the students made an error when calculating the area of a rectangle knowing its perimeter and one dimension.* *61% of the students were not able to calculate the perimeter of a rectangle knowing its length and its area.* *74% of the students have made an error when measuring the length of an object using a graduated ruler.* *81% of the students were not able to compare two rectangles having the same area but different perimeters.* *80% of the students had difficulty when determining the length and the width of a rectangle knowing its perimeter.*	*77% of students did not recognize that shapes with the same area can have different perimeters.* *59%of students were not able to estimate capacity using a referent.* *52% of the students were not able to determine the area of a regular rectangle knowing the perimeter and dimensions.*

It is essential to explain to students that to calculate

- the area of a rectangle, if they are given its perimeter and one dimension, they are able to calculate the other dimension. Once they have both dimensions, the area can be calculated using $A = L \times W$;
- the perimeter of a rectangle, if they are given its area and one dimension, they are able to calculate the other dimension. Once they have both dimensions, the perimeter can be calculated using $P = 2L + 2W$.

C. What are the next steps in instruction for the class and for individual students?

Since the focus of our curriculum is teaching through problem-solving, concepts of perimeter and area should be presented in a real-world problem-solving context. These concepts could be introduced by asking students to solve a problem such as the following: Zack has 20 pieces of interlocking fence, each one metre long. How many different rectangular-shaped enclosures can he make using all the pieces of fence? What shape should he choose if he uses the fenced space for his dog? What shape might he use if he uses the fenced space for a garden?

Students often do not make the distinction between area and perimeter and understand when to use each one in a problem. They may calculate the area instead of the perimeter or vice versa. It is important that students have many opportunities to construct rectangles of different areas and perimeters concretely and pictorially. Students should recognize that area and perimeter are independent of one another. Area and perimeter involve measuring length. Rules or formulas may be invented by students as they engage in learning opportunities, but formal instruction related to formulae occurs in Mathematics 6. When students are able to measure efficiently and effectively using standard units, their learning experiences can be directed to situations that encourage them to construct measurement formulas. When determining the area of a rectangle, students may realize as they count squares that it would be quicker to find the number of squares in one row and multiply this by the number of rows. When finding perimeters of rectangles, students may discover more efficient methods instead of adding all four sides to find the answer (e.g., add the length and width and double the sum).

It is important that students learn about area and perimeter together. Through explorations, students will discover that

- the perimeter and area are two independent concepts
- it is possible for rectangles of a certain area to have different perimeters
- it is possible for rectangles with the same perimeter to have different areas
- the closer the shape is to a square, the larger the area will be
- for any given perimeter, the rectangle with the smallest possible width will result in the least area

Geo-boards or grid paper can be used to create various rectangles all with the same perimeter of 20 cm. Students should be working toward the realization that rectangles of different dimensions can have the same perimeter. Students should also determine the area of each of these rectangles to understand that though each of these rectangles has a perimeter of 20 units, the area of each of the rectangles is different.

Show students the chart below. Encourage them to discuss observed patterns in small groups. The discussion should allow students to see that rectangles, of the same perimeter, can have different areas as well as to see that among these rectangles, the one that is a square has the largest area.

Perimeter = 2(L + W)	Half-Perimeter = L + W	Length = L	Width = W	Area = L × W
20 cm	10 cm	9 cm	1 cm	9 cm^2
20 cm	10 cm	8 cm	2 cm	16 cm^2
20 cm	10 cm	7 cm	3 cm	21 cm^2
20 cm	10 cm	6 cm	4 cm	24 cm^2
20 cm	10 cm	5 cm	5 cm	25 cm^2

D. What are the most appropriate methods and activities for assessing student learning?

Below are some sample questions related to measurement which will be used to represent some of the appropriate methods and activities for assessing student learning.

Encourage students to share their thinking, their strategies, and their solutions. Rich discussion fosters the development of alternative problem-solving strategies.

Examples:

1. Tom drew two rectangles, A and B. The dimensions of A are 3 cm by 4 cm.
The dimensions of B are 2 cm by 5 cm.
Which statement is **not true** about the two rectangles?

- ○ The area of A is larger than the area of B.
- ○ A and B have the same perimeter, but they do not have the same area.
- ○ The perimeter of B is the same as the perimeter of A.
- ○ A and B have the same area, but they do not have the same perimeter.

2. The perimeter of a rectangular garden is 120 m.
What are the length and the width of this garden?

- ○ 12 m and 10 m
- ○ 40 m and 20 m
- ○ 60 m and 60 m
- ○ 100 m and 20 m

3. The area of a rectangle is 24 cm^2.
What are the possible measurements of the dimensions of this rectangle?

- ○ Length = 12 cm and width = 12 cm
- ○ Length = 14 cm and width = 10 cm
- ○ Length = 16 cm and width = 8 cm
- ○ Length = 6 cm and width = 4 cm

4. Mr. MacDonald wants to construct an enclosed rectangular kennel for his dog.
The perimeter of the kennel is 20 m.
The kennel must have the biggest possible area.
What is the area of the kennel?

- ○ 20 m^2
- ○ 25 m^2
- ○ 100 m^2
- ○ 400 m^2

5. The area of a rectangular carpet is 18 m^2.
What is the largest perimeter of the rectangular carpet?

- ○ 18 m
- ○ 22 m
- ○ 38 m
- ○ 72 m

6. The perimeter of a rectangular wall is 12 m.
Its width is 2 m.
What is the area of the wall?

- ○ 6 m^2
- ○ 8 m^2
- ○ 10 m^2
- ○ 28 m^2

7. Mrs. Harris has four sunflower plants.
The table below shows heights of the four sunflower plants.

Plant	**Height**
A	1230 mm
B	1.2 m
C	127 cm
D	1389 mm

What is the order of the heights of the four plants from tallest to shortest?

- ○ A, B, C, D
- ○ D, C, B, A
- ○ D, A, B, C
- ○ D, C, A, B

8. Andrea uses the ruler to measure the length of her eraser.

What is the length of the eraser?

- ○ 15 cm
- ○ 8 cm
- ○ 6 cm
- ○ 2 cm

Mathematics in Grade 6 Lesson Learned 5
Geometry

Students had difficulty when asked to draw upon their previous knowledge of 2-D shapes to assist them in their identification and descriptions of prisms. Students require more experiences to identify and name common attributes of both rectangular-based and triangular-based prisms. Students need to sort a given set of right rectangular-based and triangular-based prisms, using the shape of the base. Sets of 3-D objects usually include a variety of prisms. Students need to be able to identify examples of rectangular-based prisms and triangular-based prisms in their environment. Students need more exposure and experience with the Cognitive Levels of Questioning (knowledge, application and analysis) in order to apply these higher order thinking skills when working with all geometry concepts.

A. What conclusions can be drawn from the Nova Scotia Assessment: Mathematics in Grade 6?

Students did very well when presented with questions related to 3-D objects. They were able to identify 3-D objects being described according to some attributes, such as the number of vertices. Identifying a net for a given 3-D object was also done well. When working with transformation questions, students were able to recognize which image was a reflection. These questions were knowledge and application questions.

In the 2019–2020 Item Description Report, the following descriptions describe the questions that students had difficulty with

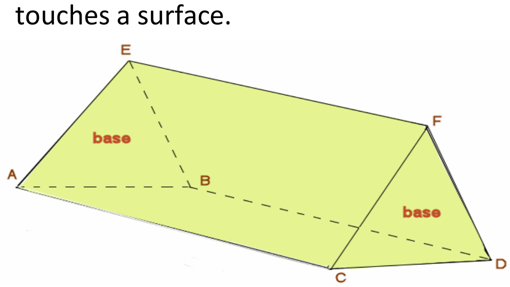
- identifying which 3-D object does not have parallel faces
- recognizing a rectangular-based prism in a context involving the number of faces, vertices, and edges
- identifying a given rotation from its picture
- identifying a sorting rule for two sets using a Venn diagram

These questions were knowledge, application, and analysis questions.

B. Do students have any misconceptions or errors in their thinking?

It is important to distinguish between misconceptions and errors. Misconceptions are, in effect, the misunderstandings about mathematical ideas which students entertain, and which usually lead to errors occurring. These are the most serious kind of errors which require urgent action on the part of the teacher. Errors may occur for a variety of underlying reasons, ranging from careless mistakes to errors resulting from misconceptions. - John SanGiovanni – (*Mine The Gap For Mathematical Understanding Grades 3-5*)

A common error or misconception that students have is understanding that the base of a rectangular-based and triangular-based prism is whatever face it is sitting on. The student thinks that the base is the face that touches a surface.



For this triangular prism, the two congruent triangles ABE and CDF are the two bases (also they are two faces) and the rectangles ABDC, ACFE and BDFE are the lateral faces.

All prisms have faces, two of which are customarily referred to as bases. These two bases may take the shape of any polygon. For clarification purposes, prisms can be thought of as having a two-part name. The first part refers to the shape of the bases, and the second part, which is "prism" (e.g., triangular-based prism, rectangular-based prism).
A common error or misconception that students encountered was identifying parallel faces of given 3-D objects. It is important that students become familiar with the vocabulary associated with describing the attributes of 2-D shapes and 3-D objects such as **parallel**, **intersecting**, **perpendicular**, **vertical**, and **horizontal**.

Common errors occurred when students were working with transformations. These transformations included students translating a shape on grid paper incorrectly. Many students had difficulty understanding how a translation in a straight line can be represented by movements left or right and up and down.

Other common errors many students have is they confuse clockwise with counterclockwise when working with rotation questions. Many students believe that a line of reflection can only be horizontal or vertical. When working with rotations, many students made the common error of not identifying the rotation around a turn centre.

The student results of the **Nova Scotia Assessments between 2018-2019 and 2019-2020 in Grade 6** shows the following:

2018-2019 Results

51% of the students did not recognize a rectangular-based prism in a context involving the number of faces, vertices and edges.

52% of the students had difficulty when determining a statement that correctly describes a given rotation.

54% of the students were not able to identify a 3-D object that does not have parallel faces.

57% of the students had difficulty identifying the sorting rule of two sets of quadrilaterals.

2019-2020 Results

52% of the students were not able to describe a given rotation from its picture.

54% of the students were not able to recognize a rectangular-based prism in a context involving the number of faces, vertices and edges.

54% of the students were not able to identify which 3 D-object does not have parallel faces.

58% of the students were not able to identify a sorting rule for two sets using a Venn diagram.

C. What are the next steps in instruction for the class and for individual students?

Students should draw upon their previous knowledge of 2-D shapes (polygons) to assist them in their general characteristics and will now develop more detailed ways to describe objects. Students will identify properties of shapes and objects and learn to use proper mathematical vocabulary to describe them.

Students should focus on comparing the number of sides as the key attribute for classifying polygons. Students should be able to name the specific polygons—triangle, quadrilateral, pentagon, hexagon, and octagon.

In the diagram below, the shaded polygons are regular polygons, and all others are irregular polygons.

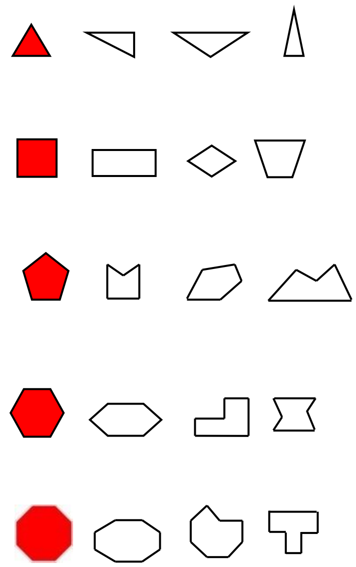
3 straight sides: triangles

4 straight sides: quadrilaterals

5 straight sides: pentagons

6 straight sides: hexagons

8 straight sides: octagons

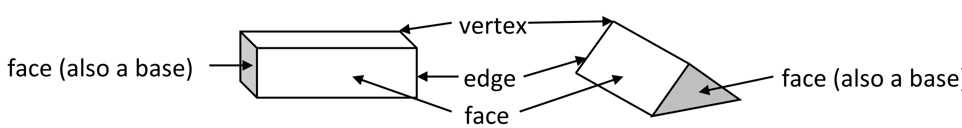


Regular polygons are shaded.

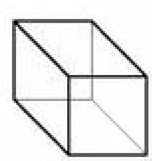
Provide students with various sizes of a particular polygon. Have students count the number of sides and identify the polygon. Having a variety of these experiences with different polygons, students should begin to realize that a polygon, regardless of its dimensions, remains the same shape.

There is a developmental sequence associated with how students think and reason geometrically. As levels of geometric thinking develop, students will notice more attributes of 3-D objects. These attributes are the components that go together to make up the form—**edges**, **vertices**, and **faces** (two of which are the **bases**). In the process of identifying and naming attributes of prisms, it may be necessary to review and encourage students to use appropriate vocabulary such as number of faces, number of edges, number of vertices, or shapes of the faces/bases.

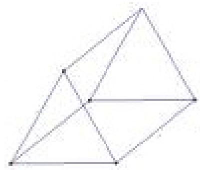
Students should be able to identify the faces, edges, and vertices as well as the shape of the faces of a given 3-D object. A strategy that may help with identifying the base of a geometric object is to remember that the base of a rectangular-based prism, can be any of the faces. The base of a triangular-based prism refers to its triangular face.



This rectangular-based prism has 6 rectangular faces, 12 edges, and 8 vertices.



This triangular-based prism has 5 faces (3 rectangular faces, 2 triangular faces), 9 edges, and 6 vertices.



To investigate the faces and edges of 3-D objects, hold up a cereal box (rectangular-based prism). Ask students to identify the faces and edges. Lead a discussion that will have students describe edges and faces in terms of parallel, intersecting, perpendicular, vertical, and horizontal. Then, have students work in pairs. One student chooses a geometric solid and describes it according to its attributes. The second student then tries to identify the solid. Once the solid is identified, students switch roles.

Another way to have students explore the edges and faces of 3-D objects is to have them work in small groups to stack pattern blocks to build prisms. Constructing prisms can take many forms. One way is to make concrete models. Pattern blocks are very good for this, but many teacher-made materials can be used. While the pattern block pieces themselves are prisms, they have been treated as 2-D shapes; however, stacking a number of triangles or squares from the pattern blocks would provide examples of different prisms. This stacking would help students conceptualize the uniform nature of prisms. Stacked pattern blocks will form triangular prisms, rectangular prisms, trapezoidal prisms, rhomboidal prisms, and hexagonal prisms. After students have constructed the prisms, discuss the following questions:

- Which solid has the most parallel faces?
- Which solid has the least number of edges?
- Which solid has only two parallel faces?
- Which solids have eight intersecting edges?
- Which solid has four sets of parallel faces?

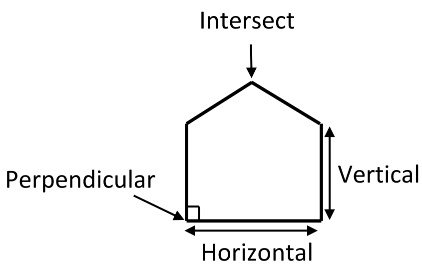
While the language of geometry is important, the teaching of mathematically correct geometric language should be done in the context of physical models rather than as definitions.

Another type of model is a skeleton. This is a model showing only the edges and vertices of a 3-D shape. Students can make skeletal models for prisms using rolled newspapers and tape, straws and string, toothpicks and miniature marshmallows, and small balls of modelling clay or straws with pieces of pipe cleaners. Display a variety of 3-D objects for students and invite students to build skeletons of those prisms. Some students may need to touch the edges and vertices in order to construct a skeleton. The process of making a skeleton helps students visualize the object and remember its properties.

Students should be given copies of nets of rectangular-based and triangular-based prisms to cut out and fold to construct the prisms. They should be encouraged to unfold them and examine the 2-D shapes that are connected to make each net. These shapes are the faces of the 3-D object and are one of the key attributes students should use to identify rectangular-based and triangular-based prisms. Other attributes students should consider when identifying 3-D objects are the number of edges and vertices and congruency. In addition to cutting out and assembling prepared nets, it is also expected that students will trace the various faces of the different prisms to make nets for rectangular-based and triangular-based prisms and explore other possible nets for these prisms. Have students visualize the folding and unfolding of the nets and then use materials to explore whether the net will successfully construct the prism.

Sorting rectangular-based prisms and triangular-based prisms, requires students to attend to specific attributes of objects. Give students a variety of 3-D prisms (real-life or commercially made models). Ask

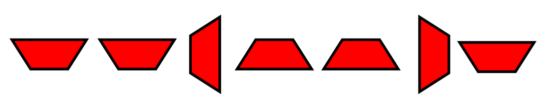
students to sort the prisms according to a given attribute, such as shape of the base. Place two hula hoops on the floor to represent a large-scale Venn diagram. Provide labels and have students sort the objects according to triangular-based prisms, rectangular-based prisms, and other prisms. As students place their object on the diagram, have them explain, to the class, why they placed objects in certain places.
There is a gradual progression from identifying and describing 2-D shapes and 3-D objects in students' own words to identifying and describing them in the formal language of geometry. It is important that students become familiar with the vocabulary associated with describing the attributes of 2-D shapes and 3-D objects such as **parallel**, **intersecting**, **perpendicular**, **vertical**, and **horizontal**. Lines in the same plane can be parallel or they can intersect. Parallel lines never intersect since they remain a constant distance apart. Intersecting lines meet at a single point. Perpendicular lines are intersecting lines that form right angles. Lines can also be vertical or horizontal. Vertical lines are up and down and are perpendicular to the horizon. Horizontal lines are parallel to the horizon or are from left to right.



Teachers may include pyramids to let students see that not all solids have parallel faces. This activity can include other prisms, for example, hexagonal or octagonal. Prisms, by definition, have two congruent parallel faces made of polygons called bases with line segments joining corresponding points on the two bases. These line segments are always parallel and are called edges.

Students may need to be reminded to always use a ruler when drawing straight lines. To draw a 2-D shape with parallel lines, students can use their rulers to measure equal distances between lines. For perpendicular lines, remind students they are drawing a right angle (i.e., a 90-degree angle). A simple index card can be used to draw perpendicular lines (right angles) and to draw straight lines.

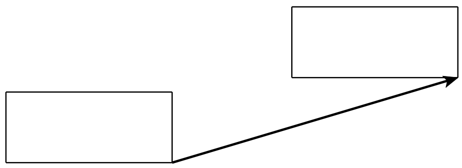
Mathematics 5 is the first-time students are introduced to transformations. Students should be encouraged to observe, describe, and create patterns using translations, reflections, and rotations. Pattern blocks are an ideal concrete material for students to manipulate when creating patterns.



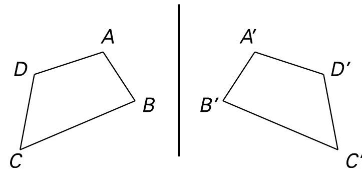
There are three types of transformations that will be explored in this grade: translation (slide), reflection (flip), and rotation (turn). Students need to be exposed to numerous examples of each of the transformations to recognize when one has been performed.

Students should be able to predict where an image will be before performing the transformation. This is a similar process to estimating the result of a calculation before performing the calculation. Teachers should encourage students to make such predictions as one way to develop spatial reasoning. Students should also be able to explain whether a transformation is or is not the result of a given translation.

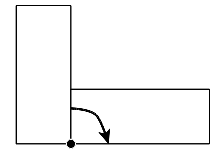
Example of a translation (slide)



Example of a reflection (flip)

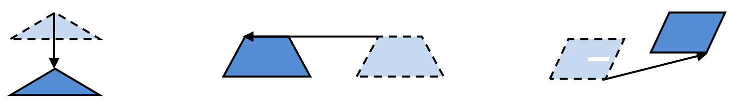


Example of a rotation (turn)



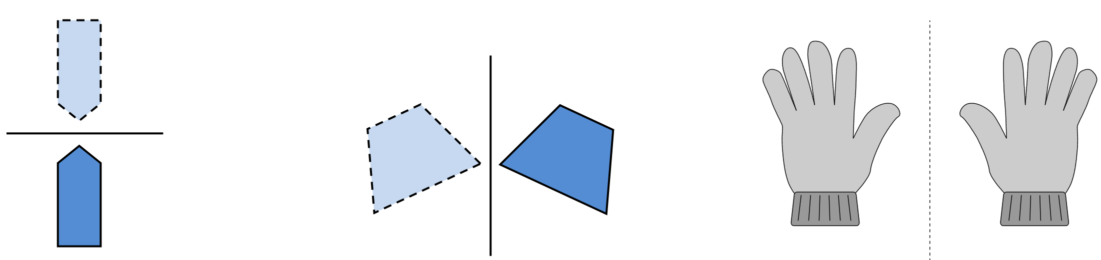
Introducing transformations to students, the teacher should begin with informal language (slide, flip, and turn) making the connection to mathematical terms (**translation**, **reflection**, and **rotation**). Encouraging students to think in terms of sliding, flipping, and turning shapes is a very useful strategy to help with their visualization in geometry since much of students' geometric development depends on understanding how shapes do and do not change when they are transformed in different ways. Students could begin by using concrete materials such as pattern blocks and geo-boards to demonstrate translations, reflections, and rotations. Once they have concretely demonstrated understanding of performing the transformations, they can use square dot paper or grid paper to draw the transformations.

Translations (slide) move a shape left, right, up, down, or diagonally without changing its orientation in any way. A real-life example of a translation would be a piece moving on a chessboard.



A **reflection** (flip) does change the orientation of a shape; the effect is one of reversing a shape (i.e., right becomes left or up becomes down). To describe reflections (flips), students should be able to use language such as, "reflected up" or "reflected to the left." Students should draw or trace shapes, and using a Mira, draw mirror lines, and the reflected images. Students should then compare the shapes and their reflected images using tracing paper or by folding over and looking through the paper at a light source. They should conclude that the two shapes are congruent. Students should label the original shape with *A*, *B*, *C*, *D*, ...; the corresponding vertices of the reflected image with prime notation, *A′, B′, C′, D′*. They should name both shapes clockwise starting at *A* and *A′*. Students should conclude that the original shape and its image are of opposite orientation.

Reflections can be thought of as the result of picking up a shape and flipping it over. The reflection image is the mirror image of the original shape. A real-life example of a reflection could be a pair of gloves beside each other.

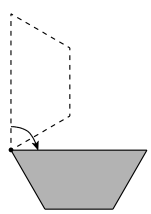


The line of reflection creates symmetry between the pre-image and image, whereas a line of symmetry typically refers to symmetry within a given object.

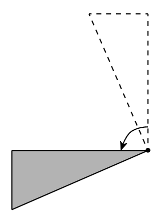
Students have no prior experiences with rotations. As well, students will only be expected to rotate the shape about a vertex at this grade level.

A **rotation** moves shapes in a circular motion. Rotations are commonly the most challenging of the transformations. Students need many first-hand experiences making rotations and examining the results before they will be able to identify such rotations given to them. At this grade level, the emphasis should be on drawing rotation images and identifying a rotation image with centres on one of the vertices and angles that are $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ turns.

When students first begin working with turns, they identify and describe them in terms of fractions of a circle $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ turn. In addition to describing the amount of turn, students also need to identify the turn direction (clockwise or counter-clockwise). Clockwise and counter-clockwise are abbreviated as "cw" and "ccw."



Quarter turn clockwise



Quarter turn counter-clockwise

Students need to make the connection between their prior knowledge of symmetry and the line of reflection. The line of reflection creates symmetry between the pre-image and image, whereas a line of symmetry typically refers to symmetry within a given object.

The general properties students should use to identify translations are

- the 2-D shape and its image are congruent
- the 2-D shape and its image have the same orientation (That is, if we go around the object *ABCD* in a clockwise direction, we should be able to also go around its image *A'B'C'D'* in a clockwise direction.)

Model, how to describe given translations, using mathematical language such as two units right and three units down.

Students could look for the use of transformations by searching the Internet. They could examine the transformations used by artists of various cultures including First Nations and African Nova Scotian, wallpaper designers, and quilt makers.

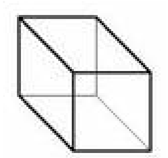
D. What are the most appropriate methods and activities for assessing student learning?

Below are some sample questions related to geometry which will be used to represent some of the appropriate methods and activities for assessing student learning.

Encourage students to share their thinking, their strategies, and their solutions. Rich discussion fosters the development of alternative problem-solving strategies.

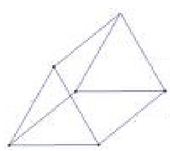
Examples:

1. Which statement is true about the following prism?



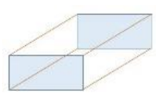
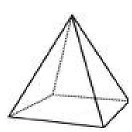
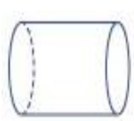
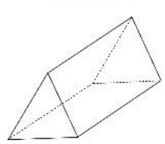
- ○ It has 6 faces, 6 vertices, and 8 edges.
- ○ It has 6 faces, 8 vertices, and 8 edges.
- ○ It has 6 faces, 8 vertices, and 9 edges.
- ○ It has 6 faces, 8 vertices, and 12 edges.

2. Which statement is true about the following prism?



- ○ It is a rectangular-based prism.
- ○ Its base is a rectangle.
- ○ It has 3 rectangular faces, 6 vertices, and 2 triangular bases.
- ○ It has 5 faces, 6 vertices, and 6 edges.

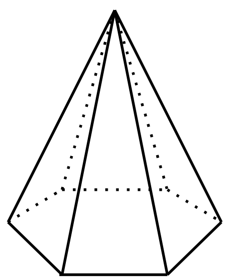
3. Which of these objects **does not** have parallel faces?

- ○ a rectangular-based prism 
- ○ a square-based pyramid 
- ○ a cylinder 
- ○ a triangular-based prism 

4. I have 5 faces.
I have 6 vertices.
I have 9 edges.
Which 3-D object am I?

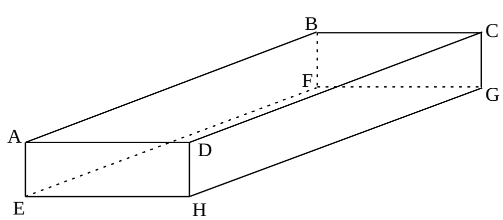
- ○ hexagonal-based prism
- ○ octagonal-based prism
- ○ rectangular-based prism
- ○ triangular-based prism

5. How many edges does this pyramid have?



- ○ 6
- ○ 7
- ○ 10
- ○ 12

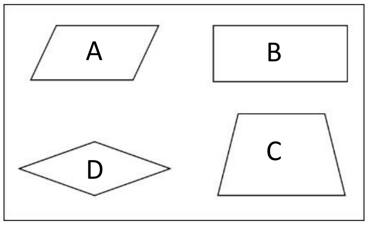
6. Look at the following rectangular-based prism.



Which statement about this prism is **true**?

- ○ The two faces ADHE and ABCD are perpendicular.
- ○ The two faces ADHE and EFGH are parallel.
- ○ The two edges AB and AD are parallel.
- ○ The two edges HG and EF are perpendicular.

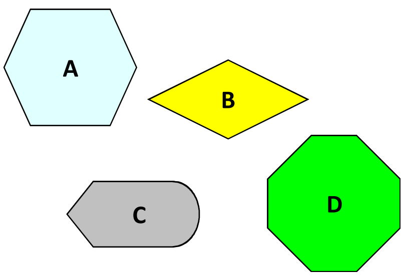
7. Paul drew the following quadrilaterals.



Which quadrilateral has two sides that are perpendicular?

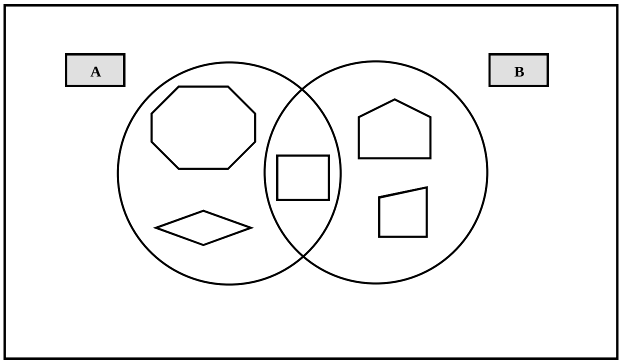
- ○ A
- ○ B
- ○ C
- ○ D

8. Which figure is **not** a polygon?



- ○ A
- ○ B
- ○ C
- ○ D

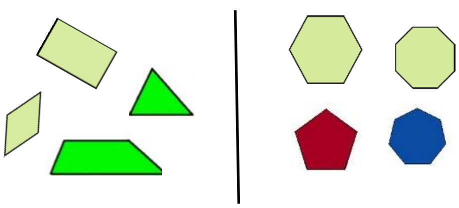
9. Andrew sorted polygons in the Venn diagram below:



What sorting rule did Andrew use?

- ○ A – all angles are right angles
  B – one pair of sides are not parallel
- ○ A – one axis of symmetry
  B – the four sides are equal.
- ○ A – at least two axes of symmetry
  B – at least one pair of sides are perpendicular
- ○ A – at least three axes of symmetry
  B – all angles are right angles

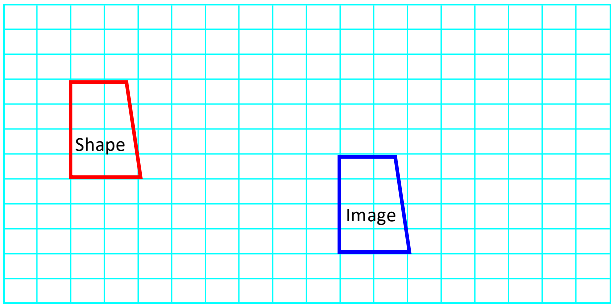
10. Stéphanie has sorted the following group of polygons.



What sorting rule did Stéphanie use?

- ○ Polygons with 4 sides and polygons with 5 sides
- ○ Polygons with 4 sides and polygons with less than 8 sides
- ○ Polygons with a maximum of 4 sides and polygons with more than 4 sides
- ○ Polygons with 3 sides and polygons with more than 5 sides

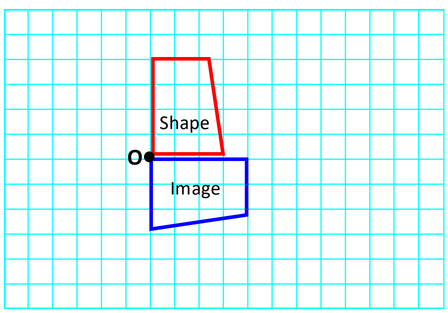
11. The picture below shows a translation that moves the shape to the image.



Which statement would describe this translation?

- ○ 6 squares right and 1 square down
- ○ 8 squares left and 3 squares up
- ○ 6 squares right and 3 squares up
- ○ 8 squares right and 3 squares down

12. The picture below shows a shape that rotates about a point O to obtain its image.



Which statement would describe this rotation?

- ○ $\frac{1}{2}$ turn counterclockwise about point O
- ○ $\frac{1}{4}$ turn counterclockwise about point O
- ○ $\frac{3}{4}$ turn clockwise about point O
- ○ $\frac{1}{4}$ turn clockwise about point O

Mathematics in Grade 6 Lesson Learned 6
Statistics and Probability

Students have improved when asked to construct and interpret double bar graphs. But they still require experiences when constructing and interpreting double bar graphs. Students need to develop the skill of interpreting and drawing conclusions from double bar graphs when asked questions about the data displayed. Both literal comprehension questions and inferential comprehension questions need to be asked. They need more opportunities to identify outcomes from a given probability experiment that are less likely, equally likely, or more likely to occur than other outcomes.

A. What conclusions can be drawn from the Nova Scotia Assessment: Mathematics in Grade 6?

Students did very well when asked to interpret a table showing the results of a survey using tally marks. They understood the concept of the tally table and what it represented, and could interpret what they were being asked to answer for the data displayed. This was a knowledge question.

We noticed that students did very well when interpreting the data from a double bar graph when asked to answer a straightforward literal comprehension question about the data. They were not asked to interpret and draw any conclusions about the graph. Again, this was a knowledge question.

Students did well when asked to determine what scale was most appropriate for the set of data displayed in a pictograph. For example, if students want to display a graph to show their marble collection and they have 36 blue, 24 red and 42 clear marbles, they may decide to draw a pictograph where each symbol represents 2 marbles or one where each symbol represents 6 marbles. This type of question was an analysis question.

Double bar graphs caused difficulty for students, whether horizontal or vertical, in that students interpreted data from the wrong bars on the graph. Therefore, when asked to answer questions concerning the double bar graph data, the conclusions drawn were not correct. This was an application question.

Students had difficulty when asked to interpret a double bar graph and to draw conclusions. They understood the concept of the double bar graph and what it represented, but they did not realize that in order to draw a conclusion to answer the question, they had to perform an operation of addition with all of the information given in each bar being displayed. This was an application question.

Students had difficulty solving word problems involving a probability of four outcomes. They should have experience using spinners, number cubes, and other concrete materials to gather data that may be used to predict the probability of an outcome. They also found comparing the likelihood of two possible outcomes occurring using words such as **less likely, equally likely,** or **more likely** difficult. These types of questions were analysis questions.

B. Do students have any misconceptions or errors in their thinking?

It is important to distinguish between misconceptions and errors. Misconceptions are, in effect, the misunderstandings about mathematical ideas which students entertain, and which usually lead to errors occurring. These are the most serious kind of errors which require urgent action on the part of the teacher. Errors may occur for a variety of underlying reasons, ranging from careless mistakes to errors resulting from misconceptions. - John SanGiovanni – (*Mine The Gap For Mathematical Understanding Grades 3-5*)

A misconception or error that students have when interpreting a double bar graph to draw conclusions about the given data is that they did not understand that double bar graphs show two sets of data at the same time. Double bar graphs are used to compare two sets of data. Many students have the misconception that they only need to read the bar graph to draw a conclusion and do not realize that there are times when they must go a step further and perform an operation on the data in order to draw a conclusion to answer a given question concerning the data.
A misconception that many students have when solving a word problem involving probability of different outcomes, is that many students believe that the results of a random spin, the rolling of number cubes or the use of other concrete materials may be influenced by their thoughts or wishes.

Another error or misconception students had was that they were confused when their results did not match their prediction. Because students base their prediction on likelihood (probability), this only tells how likely it is an event may happen. It does not tell exactly how often the event will happen or whether an event will happen on the next roll or spin.

The student results of the **Nova Scotia Assessments between 2018-2019 and 2019-2020 in Grade 6** shows the following:

- that student achievement in statistics and probability is very satisfactory

C. What are the next steps in instruction for the class and for individual students?

Prior to Mathematics 5, students created and labelled bar graphs using appropriate scales and appropriate attributes. Students should be aware that sometimes when two pieces of data are collected about a certain population, it is desirable to display both sets of data side by side, using the same scale. This is usually done using a **double bar graph**. A **legend** is used to help the reader interpret a double bar graph.

When the teacher introduces a new type of graph, it is essential to do so as part of a contextual activity that touches the interests of the students, for which this graph would be an ideal way to organize or display the resulting data. It should be kept in mind that the construction of a graph offers opportunities to integrate other mathematical concepts belonging to other mathematical stands, such as number and measurement, and other concepts related to other disciplines, such as the humanities and sciences.

A double-bar graph shows how two different sets of data are alike or different. Using a legend helps the reader interpret a double-bar graph. Using sport statistics, a connection can be made between some students' out-of-school interests and the area of mathematics. Hockey, soccer, baseball, and football statistics lend themselves to the construction of a double-bar graph.

Remind students that in a double-bar graph, each set of data must use the same scale, have a title, scale, and legend and the order of data must remain the same throughout. Provide students with examples of various graphs displaying and/or describing the above attributes.

Invite students to discuss the following questions to draw a conclusion:

- What have I learned from this graph?
- What conclusions can you gather from this data?
- What message is conveyed in this double-bar graph?
- Who did the collecting?
- Who was the data collected for?
- What message is the data telling us?

Model the construction of a double-bar graph before students work independently to construct their own. Teachers may use chart paper grid pads in constructing the double-bar graphs. At the beginning, students could use grid paper to construct bar graphs to ensure that the squares are all of equal size. Students should have regular opportunities to examine graphs to interpret the information displayed, draw conclusions about the data, look for patterns, make predictions, pose questions, and solve problems. Students should also have opportunities to read and interpret graphs found in other sources. Many newspapers use a variety of graphs in their articles and presentations. These can be sources of graphs for discussion and to show how graphs are used in the world around us. Census at School Canada, a project of the Statistical Society of Canada, also has a wide range of data displays appropriate for students.

Once students have mastered the concept of **likelihood** (probability) of a single outcome occurring, they can then begin to compare the likelihood of two outcomes occurring, using the comparative language **less likely**, **equally likely**, and **more likely**.

Students will design and conduct probability experiments for the likelihood of single outcomes occurring, as well as a comparison of two outcomes. They will be expected to record the outcomes and explain the results. Students should have experience using spinners, number cubes, and other concrete materials to gather data that may be used to predict the probability of an outcome.

The following activities provide next steps for instruction for students for probability:
Using overhead spinners, ask students (class discussion)

- Which spinner is most likely to spin a 2?
- Which spinner is less likely to spin a 2?
- Which spinner is equally likely to spin a 2 or 3?

Using a variety of coloured multi-link cubes, ask students to identify the number of coloured cubes needed to produce an outcome that is less likely to occur. For example, they may place 15 red, 10 blue, and 5 green in a bag. They would then state a colour that is less likely to be drawn.

Provide students with blank spinners and have them design an experiment with an event that is less likely to occur. For example, they may design a spinner with eight sections, four of which are yellow, two of which are red, one of which is blue, and one of which is green. Possible events would be, the spinner is less likely to land on blue than red, or green than yellow, etc.

Using a variety of coloured multi-link cubes, ask students to identify the number of coloured cubes needed to produce an outcome that is equally likely to occur. For example, place 10 red, 10 blue, and 10 green in a bag. Ask students which colour is likely to be drawn and explain their thinking.

Using a variety of coloured multi-link cubes, ask students to identify the number of coloured cubes needed to produce an outcome that is more likely to occur. For example, place 15 red, 10 blue, and 5 green in a bag. Ask students which colour is most likely to be drawn.

Games offer many opportunities to place probability in a context. Students should be encouraged to create games and share their ideas with the class.

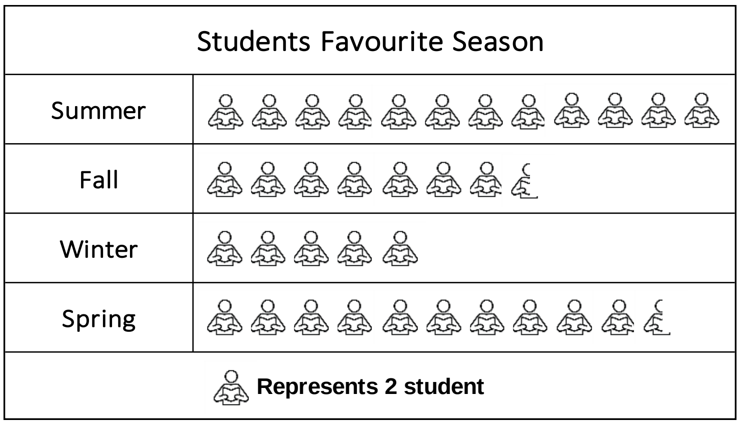
D. What are the most appropriate methods and activities for assessing student learning?

Below are some sample questions related to statistics and probability which will be used to represent some of the appropriate methods and activities for assessing student learning.

Encourage students to share their thinking, their strategies, and their solutions. Rich discussion fosters the development of alternative problem-solving strategies.

Examples:

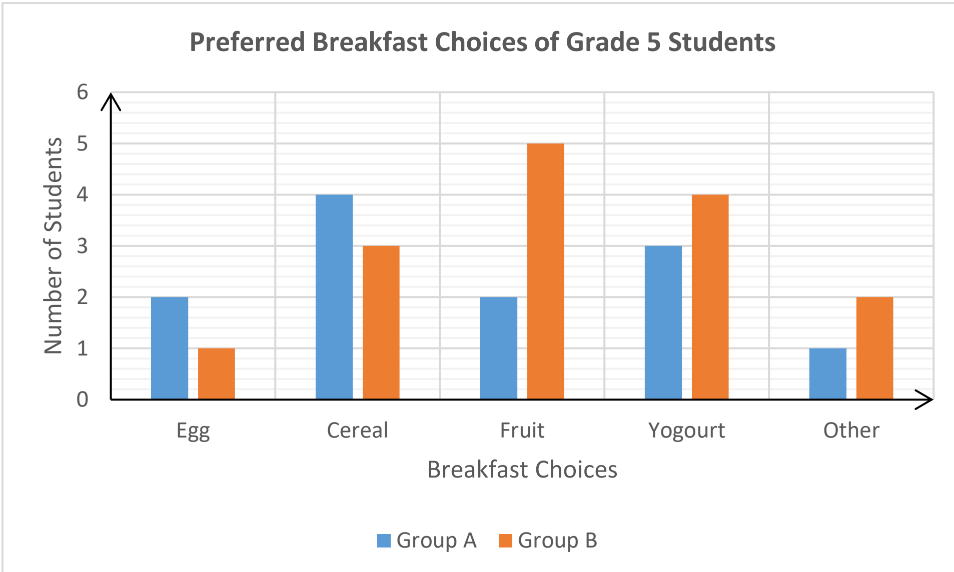
1. Tony surveyed the grade 5 students about their favourite season.
The following pictograph shows the results of Tony's survey.



How many students like spring?

- ○ 10.5
- ○ 11
- ○ 21
- ○ 34

2. Mrs. Smith surveyed her Grade 5 students about what they eat for breakfast. Here is a double bar graph showing the students responses.
Examine the following double bar graph and respond to questions A, B and C.



A. How many students in Group B prefer fruit?

- ○ 5
- ○ 7
- ○ 15
- ○ 27

B. How many more students in Group A than students in Group B prefer cereal?

- ○ 7
- ○ 4
- ○ 3
- ○ 1

C. Which statement is **true** about this bar graph?

- ○ More students in Group A than students in Group B prefer fruit than any other breakfast choice.
- ○ Less students in Group B than students in Group A prefer yogurt than any other breakfast choice.
- ○ More students in Group B than students in Group A participated in the survey.
- ○ More students in Group B than students in Group A prefer cereal than any other breakfast choice.

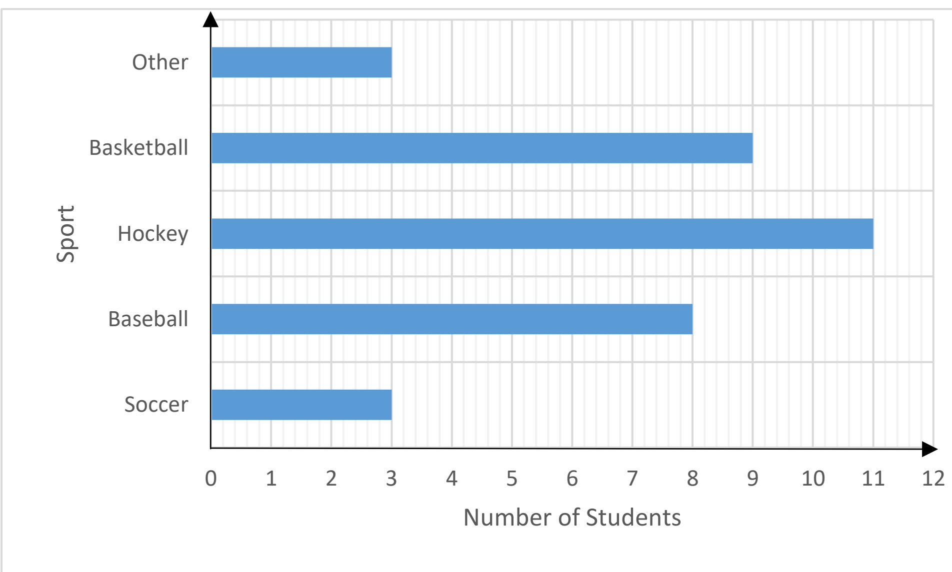
3A. Melina surveyed her grade 6 classmates about their preferred choice of favourite sports. The table below shows the results of Melina's survey.

Preferred Sports of Grade 6 Students	
Sport	**Tally**
Soccer	|||
Baseball	𝍸 |||
Hockey	𝍸 𝍸 |
Basketball	𝍸 ||||
Other	|||

How many more students preferred hockey to soccer?

- ○ 3
- ○ 8
- ○ 11
- ○ 14

3B. Melina constructed the following bar graph below to present the data of the previous table. Melina made an error when constructing the bar graph.

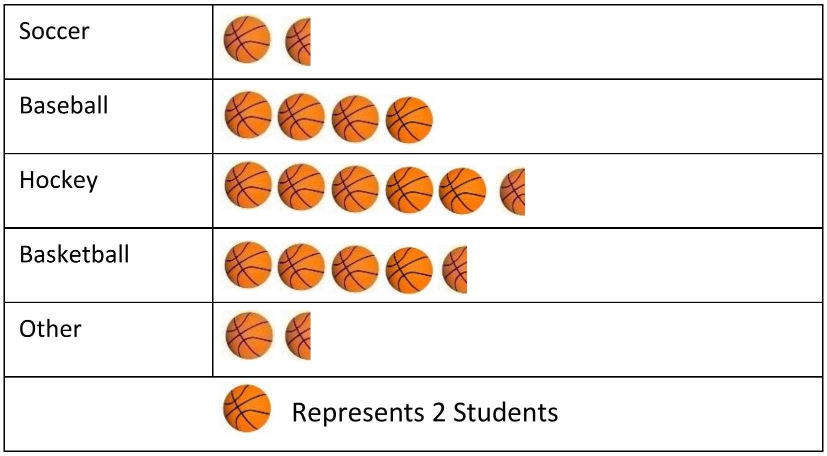


What error did Melina make?

- ○ The bars do not have the same width.
- ○ The axis labels of the graph are not indicated.
- ○ The title of the graph is not indicated.
- ○ The distance between the bars is not equal.

3C. Then Melina constructed the pictogram below to represent the same data of question 3a (Preferred Sports of Grade 6 Students).

Preferred Sports of Grade 6 Students



Which statement is true about this pictogram?

- ○ There are 20 students who prefer baseball and hockey.
- ○ There are 23 students who prefer hockey and baseball.
- ○ There are 35 students in Melina's class.
- ○ There are 17 students who prefer baseball and basketball.

4. Bernie has 3 blue marbles, 2 red marbles, 5 orange marbles and 2 yellow marbles in a bag. Without looking, Bernie picks a marble from the bag, then he puts this marble back into the bag. Which statement below is **true**?

- ○ Bernie is more likely to choose a red marble than a blue marble.
- ○ Bernie is less likely to choose an orange marble than a yellow marble.
- ○ Bernie is more likely to choose a blue marble than an orange marble.
- ○ Bernie is equally likely to choose a red marble than a yellow marble.

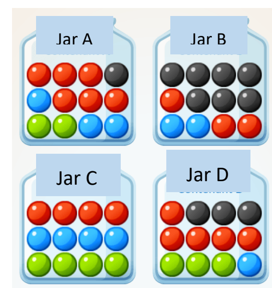
5. Gisela performs the following probability experiment:
She 4 red, 3 white, 1 blue and 2 yellow tiles in a bag.
She chooses a tile from the bag without looking.
Then Gisela put the tile she chose back into the bag.

A. Which event is impossible to occur?

- ○ Gisela chose a red tile.
- ○ Gisela chose a green tile.
- ○ Gisela chose a yellow tile.
- ○ Gisela chose a white tile.

B. Which event is most likely to occur?

- ○ Gisela chose a blue tile.
- ○ Gisela chose a red tile.
- ○ Gisela chose a yellow tile.
- ○ Gisela chose a white tile.



6. John has 4 jars A, B, C and D. Each jar contains 12 balls of different colours.

Jar A contains 1 black ball, 2 green balls, 3 blue balls and 6 red balls.
Jar B contains 2 blue balls, 3 red balls and 7 black balls.
Jar C contains 4 red balls, 4 blue balls and 4 green balls.
Jar D contains 1 blue ball, 3 green balls, 3 black balls and 5 red balls.

A. In which jar, would John have the least chance to draw a green ball without looking?

- ○ D
- ○ C
- ○ B
- ○ A

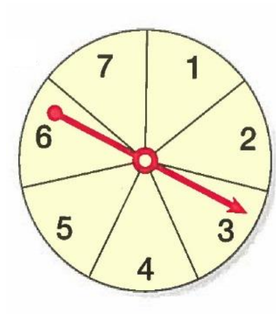
B. In which jar, it is equally likely that John could draw a red ball or a blue ball or a green ball without looking?

- ○ D
- ○ C
- ○ B
- ○ A

7. Jen performs the following probability experiment: Jen puts 5 blue buttons, 7 yellow buttons, some pink buttons, 2 orange buttons and 3 purple buttons into a bag. Jen chooses a button from the bag without looking.
Which statement below helped Jen to determine the number of buttons in the bag?

- ○ It is less likely to choose a purple button than a blue button.
- ○ It is equally likely to choose a blue button or a pink button.
- ○ It is more likely to choose a yellow button than a blue button.
- ○ It is less likely to choose a purple button than a yellow button.

8. The spinner below is divided into 7 equal sections. The sections are numbered from 1–7.
Gabriella spins the arrow on the spinner.
What is the chance that the arrow stops on the section number 6?



- ○ 1 chance out of 6
- ○ 1 chance out of 7
- ○ 6 chances out of 6
- ○ 6 chances out of 7

Mathematics in Grade 6 Lesson Learned 7
Problem Solving

Students had difficulty with application and analysis items when applying these higher order thinking skills when problem solving. Students need to continue to work on translating between and among representations when problem solving.

Learning through problem solving should be the focus of mathematics at all grade levels. Problem solving is one of the critical mathematical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics. When students encounter new situations, and respond to questions such as, How would you…? or How could you …? the problem-solving approach is being modelled. Students develop problem-solving strategies by being open to listening, discussing, and trying different strategies.

For an activity to be problem-solving based, it must challenge students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. A true problem-solving activity requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement, perseverance, and collaboration. Problem solving is also a powerful teaching tool that fosters multiple, creative, and innovative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident mathematical risk takers.

Students need to be able to explore a wide variety of methods for solving and verifying problems in all areas of mathematics. They must be challenged to find multiple solutions for problems and be given opportunities to create and solve their own problems.

A. What conclusions can be drawn from the Nova Scotia Assessment: Mathematics in Grade 6?

We found that students have a good understanding of basic facts and procedures, when explicitly given all the information needed to do a knowledge question. But when given application and analysis problem solving items, they are not able to apply higher order thinking skills. For example, students were not sure whether they should add, subtract, multiply, or divide when questions were presented in the context of a story problem. Overall, our students are experiencing challenges with problem solving across all mathematical strands. This appeared to be a theme throughout the assessment data when students were asked to “solve a word problem” or “solve a multi-step problem”. The data also showed that, across all mathematics strands, students struggle with items that require translating among representations in problem-solving contexts.

B. Do students have any misconceptions or errors in their thinking?

It is important to distinguish between misconceptions and errors. Misconceptions are, in effect, the misunderstandings about mathematical ideas which students entertain, and which usually lead to errors occurring. These are the most serious kind of errors which require urgent action on the part of the teacher. Errors may occur for a variety of underlying reasons, ranging from careless mistakes to errors resulting from misconceptions. - John SanGiovanni – (*Mine The Gap For Mathematical Understanding Grades 3-5*)

It is important to mention that when students encounter a problem-solving situation, they become confused by mathematical expressions which are not familiar to them or which are difficult to understand. In addition, when students consider that a problem is a mathematical problem, they believe, wrongly, that they should associate it simply to routine calculations with few concerns to the meaning of the context and to the credibility of the answers.

Many students, when given a problem-solving task, especially a "word problem" (solving a problem in a context) have the misconception that is it always too hard for them to attempt. Putting words around the numbers obstructs their ability to think about the question. Another misconception students have is that there is only one way to solve a word problem. When asked to solve a problem in a context, they struggle to identify a possible strategy and often fail to even attempt to solve the problem.

Along with not having problem-solving strategies to help them attempt the problem, students also tend to forget any knowledge of translating between and among representations when problem solving. If asked to solve a word problem using words, symbols and/or pictures, most students only provide symbols. They do not seem to realize that they can use all three representations when asked to solve a word problem. These other representations may support their thinking and problem solving.

For example, when given an analysis problem in the context of comparing and ordering fractions, students struggle with answering "what strategy could I use to solve this problem"? There are many strategies and ways to solve the problem. Some students find it difficult to find an entry point to begin to solve the problem.

Problem: Kelly has a quilt that has 24 squares. One-fourth of the squares are yellow, $\frac{3}{6}$ are green, and the rest are red. What colour is the greatest number of squares?

Solution 1

- What do I know? The quilt has 24 squares; one-fourth are yellow; $\frac{3}{6}$ are green, and the rest are red. I think I should see if I can change the two fractions into equivalent fractions with a denominator of 24.
- $\frac{1}{4}$ and $\frac{3}{6}$, I could change them into 24ths because I know 4 and 6 will divide evenly into 24 and my quilt has 24 squares.
- $\frac{1}{4} = \frac{6}{24}$ and $\frac{3}{6} = \frac{12}{24}$, so I can add the two fractions together, so I now have $\frac{18}{24}$. If I have 24 squares in the quilt altogether, then I need 6 more squares which are red, because in my question I did not know how many red squares were in my quilt.
- So, the green squares have the greatest number of squares which is 12, because there are 6 yellow squares and 6 red squares.

Solution 2

- The students could have also drawn a picture using the information given as another entry point. I know that the quilt has 24 squares.

- I know that $\frac{1}{4}$ of the squares are yellow.
  So, I will colour $\frac{1}{4}$ of the squares yellow.

Y			
Y			
Y			
Y			
Y			
Y			

- I know $\frac{3}{6}$ is the same as $\frac{1}{2}$.
  So, I can colour $\frac{1}{2}$ of the squares green.

Y		G	G
Y		G	G
Y		G	G
Y		G	G
Y		G	G
Y		G	G

- There are 6 squares left. They are red.
  Now, I can see that the greatest number of squares are green.

Y	R	G	G
Y	R	G	G
Y	R	G	G
Y	R	G	G
Y	R	G	G
Y	R	G	G

Students could have used concrete materials /manipulatives such as coloured tiles, two-sided counters, cube-a-link blocks, and other personal strategies to solve this question.

The student results of the **Nova Scotia Assessments between 2018-2019 and 2019-2020 in Grade 6** shows the following:

- a slight variation in student competence in this area

2018-2019 Results

64% of students had difficulty to solve a multi-step word problem involving computation and interpreting the remainder.

57% of the students were not able to solve a word problem containing an expression like “2 times more and 3 times more” correctly.

58% of the students had difficulty to solve a word problem when required to read, and to show their work using pictures, words or symbols.

59% of the students were not able to solve a multi-step word problem in which hundredths are used.

80% of the students made errors when solving a word problem about perimeter and area of a rectangle.

2019-2020 Results

58% of students had difficulty to solve an everyday context word problem in which hundredths are used in a multi-step problem.

63% of students had difficulty to solve a multi-step story problem involving whole number multiplication and division.

59% of students made errors when solving a word problem involving area and perimeter

C. What are the next steps in instruction for the class and for individual students?

A significant part of learning to solve problems is to learn about the problem-solving process. It is generally accepted that the problem-solving process consists of four steps – understanding the problem, devising a plan, carrying out the plan, and looking back to determine the reasonableness of an answer. Teachers need to teach their lessons through a problem-solving approach. Students learn mathematics as a result of solving problems. It is important to point out that not all lessons students encounter must be taught through problem solving. If the purpose of the lesson being taught is to develop a certain skill for conceptual understanding, then some practice is required.

The teacher provides a context or reason for the learning by beginning the lesson with a problem to be solved. This approach contrasts with the more traditional approach of, for example, presenting a new procedure and then adding a couple of word problems at the end for students to solve. Instead, the teacher gives students the opportunity to think about the problem and work through the solution in a variety of ways, and only then draws the procedures out of their work. (Small, 2005, p.154)

A Problem-Solving Approach

A problem-solving approach is embedded and expected throughout our curriculum, grades Primary to 12, in all strands.

As noted in Pearson (2009), problem-solving is a key strategy:

> Problem-solving is a key instructional strategy that enables students to take risks, secure in the knowledge that their thoughts, queries, and ideas are valued. As students share their solutions and findings, the teacher has the opportunity to provide direct instruction on problem-solving strategies. After students share their solutions and justifications, teachers can elaborate on their methods and encourage students to comment or ask questions of their peers. Using student findings and solution methods as a means to guide instruction also allows students to see the value in their work, and encourages peers to share their strategies. While some strategies may be more efficient than others, several strategies may work and often a combination of strategies is required to solve a problem. Students must use strategies that are meaningful to them and make sense to them. (p.13)

Problem-Solving Strategies

Students are already drawing on personal strategies for problem solving, in many of the activities they undertake. *Strategies Toolkit* lessons found in the *Pearson Math Makes Sense Series,* allows teachers to expand their students' personal repertoires through explicit instruction on a specific strategy. "When students develop a name for the strategy, they develop a stronger self-awareness of the personal strategies they are starting to use on their own" (Pearson, 2009, p.13).

The *Strategies Toolkit* lessons highlight these problem-solving strategies (Pearson, 2009, p.13):

- Make a chart or table
- Draw a picture
- Work backward
- Make an organized list
- Use a model
- Solve a simpler problem
- Guess and test
- Use a pattern

Van de Walle and Lovin (2006), in their resource, *Teaching Student-Centered Mathematics Grades 3–5*, suggest a three-part lesson format for teaching through problem-solving. This same approach is used in our core resource, *Math Makes Sense* (Pearson, 2009, pp. 13–14):

Before

Before students begin:

- Prepare meaningful problem scenarios for students. These should be sufficiently challenging, but easy to solve.
- Ensure the problem is understood by all.
- Begin with a simpler version of the task.
- Brainstorm possible strategies to use.
- Estimate what the answer might be.
- Explain the expectations for the process and the product.

During

As students work through the problem:

- Let students approach the problem in a way that makes sense to them.
- Listen to the conversations to observe thinking.
- Provide hints or suggestions if students are on the wrong path. (Could also come from another student, "Look what _______ is trying to do. Might that be helpful for you?")
- Encourage students to test their ideas.
- Ask questions to stimulate ideas.
- Assess student understanding of her/his solution.

After

After students have solved their problem:

- Gather for a group meeting to reflect and share.
- Provide ample time for all students to discuss the solution.
- Highlight the variety of answers and methods.
- Encourage students to justify their solutions.
- Encourage students to comment positively or ask questions regarding their peer's solutions.
- Encourage students to compare strategies, to find similarities and differences.

For more details on using a problem-solving approach to teach mathematics, see Van de Walle and Lovin (2006), *Teaching Student-Centered Mathematics Grades 3–5,* from which these ideas are drawn (pp. 11–28).

For further information on using a rubric to score problem-solving sample questions in either Mathematics in Grade 4 (M4) or Mathematics in Grade 6 (M6), please go to the Program of Learning Assessment for Nova Scotia (PLANS) website and on the Documents tab of each assessment page, you will find a Problem-Solving and Communication document (EN/FI Rubric and Exemplars) which includes the provincial rubric and sample questions (plans.ednet.ns.ca/grade6/documents).

From Reading Strategies to Mathematics Strategies
With a problem-solving approach embedded and expected throughout our mathematics curriculum grades Primary to 12 in all strands, there are definite implications for the teaching of reading strategies in mathematics. During the elementary years, students are making connections between every day, familiar language, and formal mathematical language. The process of making connections, in turn, facilitates learning. Concepts and skills should also be connected to everyday situations and other curricular areas. For example, when developing literacy skills students learn to make text-to-world, text-to-text, and text-to-self connections. Students can also make connections to make mathematics come alive through math-to-world, math-to-math, and math-to-self connections.
There are classroom strategies with suggestions of how to use the strategy in a mathematical context that support teachers as they develop students' mathematical vocabulary, initiate effective ways to maneuver informational text, and encourage students to reflect on what they have learned.

When teachers use these strategies in the instructional process or embedded in assessment tasks, the expectations for students must be made explicit. The students' understanding of the mathematics involved and maintaining the integrity of the curriculum are still the foremost concerns.

Please refer to **Appendix E** (page 92) found at the end of this document for further strategies with illustrative examples.

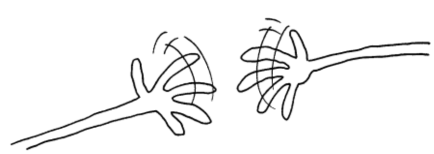
D. What are the most appropriate methods and activities for assessing student learning?

Below are some sample questions related to problem solving which will be used to represent some of the appropriate methods and activities for assessing student learning.

Encourage students to share their thinking, their strategies, and their solutions. Rich discussion fosters the development of alternative problem-solving strategies.

Examples:

1. Seven children exchange handshakes, each one at once.
How many handshakes are there?



Answer: 21 handshakes
Mathematics: Number, Pattern and Relations
Cognitive Level: Analysis

Suggested Strategies: Draw a picture, Make a Chart or table, Use a Pattern, Act out the problem

For more information related to the sample question 1, please refer to **Appendix E (page 92).**

2. Sophie builds a tower with green and blue blocks. There are 9 levels in the tower. Each level has 6 green blocks and 7 blue blocks.
How many green blocks are in Sophie's tower?
How many blue blocks are in Sophie's tower?
How many blocks are in Sophie's tower altogether?

Answer: 54 green blocks + 63 blue blocks = 117 blocks altogether
Mathematics: Number
Cognitive Level: Analysis

Suggested Strategies: Draw a picture, Make a Chart or table, Use a Pattern

3. Sally checked a book out of the library, and it is now 12 days overdue. If a book is 1 day overdue, the fine is 10¢, 2 days overdue, 20¢, 3 days overdue, 30¢, and so on.
How much is her fine?

Answer: 120 cents or $1.20
Mathematics: Number
Cognitive Level: Application

Suggested Strategies: Draw a picture, Make a Chart or table, Use a Pattern

4. Lee bought 32 batteries. He put 8 batteries in his remote-control model car. His 3 sisters divided the rest equally.
How many batteries did each sister receive?

Answer: 8 batteries
Mathematics: Number
Cognitive Level: Application

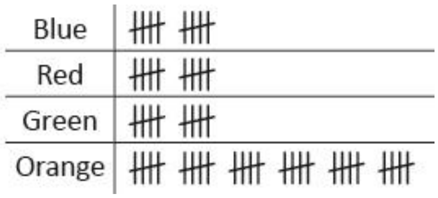
Suggested Strategies: Draw a picture, Use a model, Make a Chart or table, Use a Pattern

5. Melinda wants to build a rectangular pen for her chickens. She bought 6 large squares of plywood for the floor of her chicken pen. Each sheet of plywood has a length that measures 2.5 m. She arranges the squares so that all sides are touching. Find all possible dimensions for the floor of the pen.
How much fencing would she need for each pen?

Answer: Rectangle 1: 15 m x2.5 m; **Fence:** 35 m; **Rectangle 2:** 7.5 m x 5 m; **Fence:** 25 m
Mathematics: Measurement
Cognitive Level: Analysis

Suggested Strategies: Draw a picture, Use a model, Make a Chart or table,

6. Patrick did an experiment. He used a spinner with blue, red, green and orange parts. His results are recorded in a tally chart.
What might Patrick's spinner look like?



Answer: Orange section = one-half; Blue, Red and Green sections are one-sixth each
Mathematics: Statistics and Probability
Cognitive Level: Analysis

Suggested Strategies: Draw a picture, Work backwards

7. Norbert and Jean-Guy have 430 hockey cards altogether. Norbert has 20 more cards than Jean-Guy.

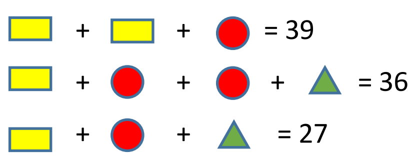
How many hockey cards do they have?

Answer: Norbert has 225 cards and Jean-Guy has 205 cards.
Mathematics: Number
Cognitive Level: Application

Suggested Strategies: Draw a picture, Make a table or an Organized list

8. Each shape represents a different number.

Find a number that each shape could represent.



Answer: Rectangle = 15 Circle = 9 Triangle = 3
Mathematics: Number, Patterns and Relations
Cognitive Level: Analysis

Suggested Strategies: Guess and test, Predict and verify

[Appendix E: Problem-Solving Scenarios](#)

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Appendix A: Cognitive Levels of Questioning

Cognitive Levels of Questioning

Knowledge
Knowledge questions may require students to recall or recognize information, names, definitions, or steps in a procedure.

Key verbs: *identify, calculate, recall, recognize, find, evaluate, use and measure*

Knowledge questions, items, and/or tasks:
- rely heavily on recall and recognition of facts, terms, concepts, or properties
- recognize an equivalent representation within the same form, for example, from symbolic to symbolic
- perform a specified procedure; for example, calculate a sum, difference, product, or quotient
- evaluate an expression in an equation or formula for a given variable
- draw or measure simple geometric figures
- read information from a graph, table, or figure

Application
Application questions may require students to make connections, represent a situation in more than one way (translating between representations), or solve contextual problems.

Key verbs: *sort, organize, estimate, interpret, compare and explain*

Application questions, items, and/or tasks:
- select and use different representations, depending on situation and purpose
- involve more flexibility of thinking
- solve a word problem
- use reasoning and problem-solving strategies
- may bring together skills and knowledge from various concepts or strands
- make connections between facts, terms, properties, or operations
- represent a situation mathematically in more than one way
- compare figures or statements
- explain and provide justification for steps in a solution process
- translate between representations
- extend a pattern
- use information from a graph, table, or figure to solve a problem
- create a routine problem, given data, and conditions
- interpret a simple argument

Analysis
Analysis questions may require students to go beyond comprehension and application to higher order thinking skills, such as generalizations and non-routine problem-solving.

Key verbs: *analyze, investigate, formulate, explain, describe and prove*

Analysis questions, items, and/or tasks:

- require problem solving, reasoning, planning, analysis, judgment, and creative thought
- thinking in abstract and sophisticated ways
- explain relationships among facts, terms, properties, or operations
- describe how different representations can be used for different purposes
- analyze similarities and differences between procedures and concepts
- generalize a pattern
- solve a novel problem, a multi-step, and/or multiple decision point problem
- solve a problem in more than one way
- justify a solution to a problem and/or assumption made in a mathematical model
- describe, compare, and contrast solution methods
- formulate a mathematical model for a complex situation, such as probability experiments
- provide a mathematical justification and/or analyze or produce a deductive argument

Below are the percentages of knowledge, application, and analysis questions in the Nova Scotia provincial assessments for Mathematics in Grade 6:

- Knowledge 20–30%
- Application 50–60%
- Analysis 10–20%

These percentages are also recommended for classroom-based assessments.

Appendix B: From Reading Strategies to Mathematics Strategies

The following table illustrates when strategies are to be used, and during what part of the three-part lesson format (as referenced in Lesson Learned 1).

Name of Strategy	Before	During	After	Assessment
1. Concept Circles	X	X	X	X
2. Frayer Model	X	X	X	X
3. Concept Definition Map	X	X	X	X
4. Word Wall	X	X	X	
5. Three-Read		X	X	
6. Graphic Organizer	X	X	X	X
7. K-W-L	X		X	X
8. Think-Pair-Share	X	X	X	X
9. Think-Aloud	X	X		X
10. Academic Journal-Mathematics	X	X	X	X
11. Exit Cards			X	X

1. **Concept Circle**

A concept circle is a way for students to conceptually relate words, terms, expressions, etc. As a "before" activity, it allows students to predict or discover relationships. As a "during" or "after" activity, students can determine the missing concept or attribute or identify an attribute that does not belong.

The following steps illustrate how the organizer can be used:

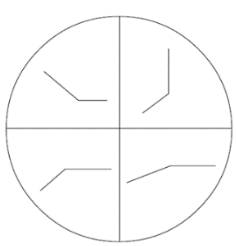
- Draw a circle with the number of sections needed.
- Choose the common attributes and place them in the sections of the circle.
- Have students identify the common concepts to the attributes.

This activity can be approached in other ways.

- Supply the concept and some of the attributes and have students apply the missing attributes.
- Insert an attribute that is not an example of the concept and have students find the one that does not belong and justify their reasoning.

Example of a Concept Circle

Concept: Can you find any right angles in the concept circle?

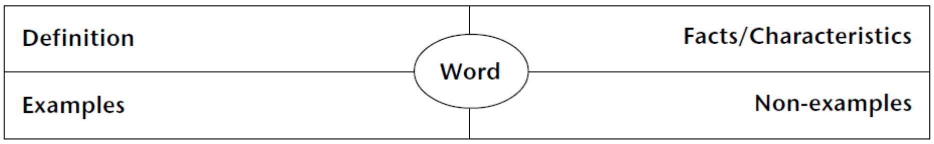


2. Frayer Model

The Frayer Model is a graphic organizer used to categorize a word and build vocabulary. It prompts students to think about and describe the meaning of a word by

- giving a definition
- describing main characteristics
- providing examples and non-examples of the word or concept

It is especially helpful to use with a concept that might be confusing because of its close connections to another concept.

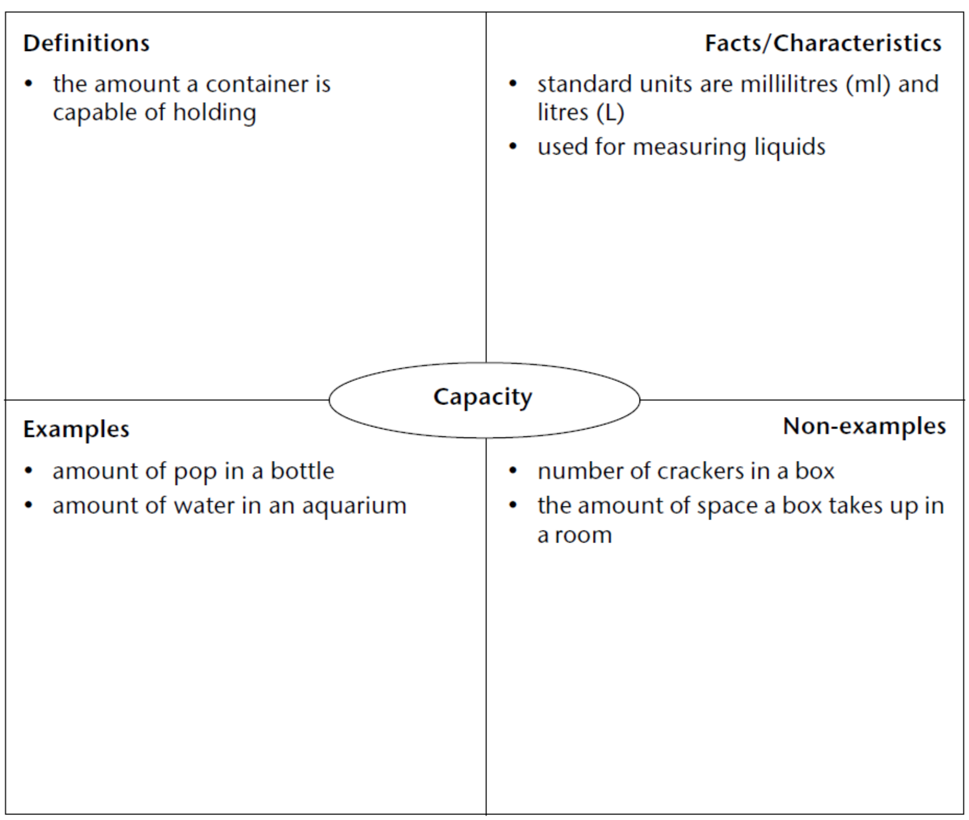


The following steps illustrate how the organizer can be used.

1. Display the template for the Frayer model and discuss the various headings and what is being sought.
2. Model how to use this example by using a common word or concept. Give students explicit instructions on the quality of work that is expected.
3. Establish the groupings (e.g. pairs) to be used and assign the concept(s) or word(s).
4. Have students share their work with the entire class.

This is an excellent activity to do in poster form to display in class. Each group might do the same word or concept, or different words or concepts could be assigned.

Example of a Frayer Model



3. **Concept Definition Map**

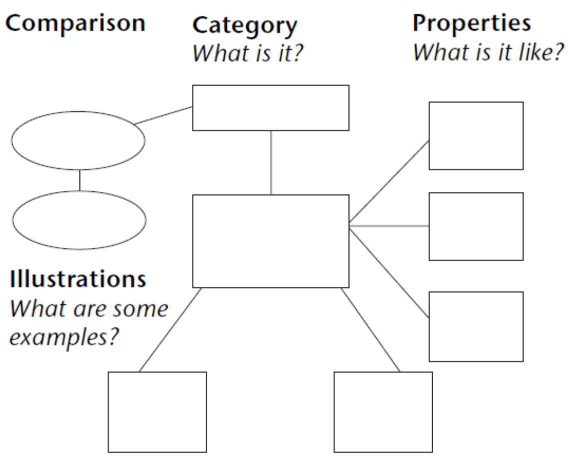
The purpose of a concept definition map is to prompt students to identify the main components of a concept, show the interrelatedness, and build vocabulary. Information is placed into logical categories, allowing students to identify properties, characteristics, and examples of the concept.

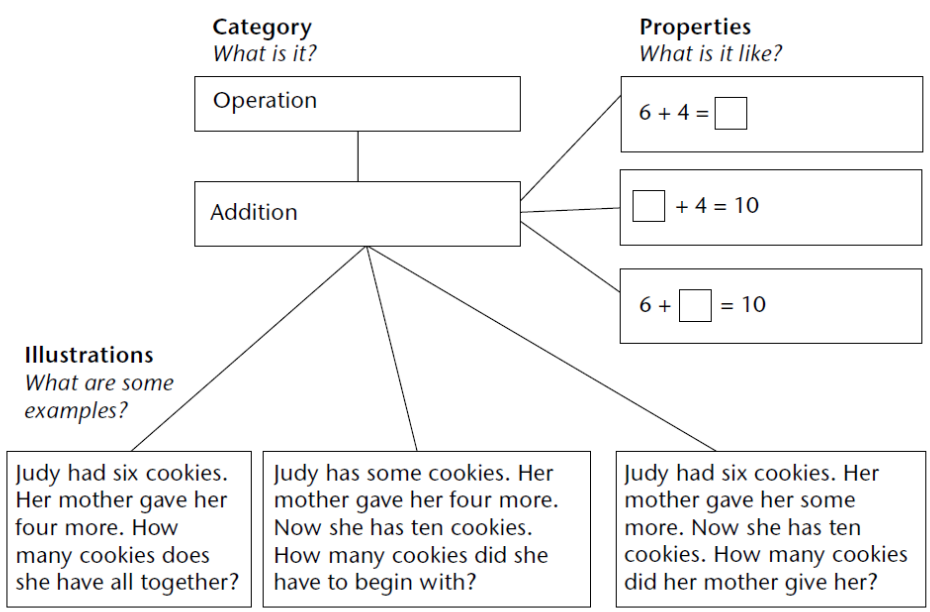
The following steps illustrate how this organizer can be used.

1. Display the template for the concept definition map.
2. Discuss the different headings, what is being sought, and the quality of work that is expected.
3. Model how to use this map by using a common concept.
4. Establish the concept(s) to be developed.
5. Establish the groupings (e.g. pairs) and materials to be used to complete the task.
6. Complete the activity by having the students write a complete definition of the concept.

Encourage students to refine their map as more information becomes available.

Example of a Concept Definition Map





4. **Word Wall**
A mathematics word wall is based upon the same principle as a reading word wall, found in many classrooms. It is an organized collection of words that is prominently displayed in the classroom and helps students learn the language of mathematics. A word wall can be dedicated to a concept, big idea, or unit in the mathematics curriculum. Words are printed on cards and then posted on the wall or bulletin board.

Illustrations placed next to a word on the word wall can add to the students' understanding. Students may also elaborate on the word in their journals by illustrating, showing an example, and using the word in a meaningful sentence or short paragraph. Students can be assigned a word and its illustration to display on the word wall. Room should be left to add more words and diagrams as the unit or term progresses.

As a new mathematical term is introduced to the class, students can define and categorize the word in their mathematics journal under an appropriate unit of study. Then the word can be added to the mathematics word wall so students may refer to it as needed. Students will be surprised at how many words fall under each category and how many new words they learn to use in mathematics.
Note: The word wall is developed one word at a time as new terminology is encountered.

Use the following steps to set up a word wall.
1. Determine the key words that students need to know or will encounter in the topic or unit.
2. Print each word and add the appropriate illustrations.
3. Display cards when appropriate.
4. Regularly review the words as a warm-up or refresher activity.

5. **Three-Read Strategy**
Using this strategy, found in *Toward a Coherent Mathematics Program: A Study Document for Educators* (Nova Scotia Department of Education 2002), the teacher encourages students to read a problem three times before they attempt to solve it. There are specific purposes for each reading.

First Read
The students try to visualize the problem in order to get an impression of its overall context. They do not need specific details at this stage, only a general idea so they can describe the problem in broad terms.

Second Read
The students begin to gather facts about the problem to make a more complete mental image of it. As they listen for more detail, they focus on the information to determine and clarify the question.

Third Read
The students check each fact and detail in the problem to verify the accuracy of their mental image and to complete their understanding of the question.

During the Three-Read strategy, the students discuss the problem, including any information needed to solve it. Reading becomes an active process that involves oral communication among students and teachers; it also involves written communication as teachers encourage students to record information and details from their reading and to represent what they read in other ways with pictures, symbols, or charts. The teacher facilitates the process by posing questions that ask students to justify their reasoning, support their thinking, and clarify their solutions.

To teach the Three-Read Strategy, teachers should exaggerate each step as they model it. When students practice the strategy, teachers should ask questions that stimulate the kinds of questions that students should be asking themselves in their internal conversations. In every classroom, an ongoing discussion of this Three-Read Strategy must be conducted, and many students will need to be reminded to use this strategy.

6. **Graphic Organizer**

A graphic organizer can be of many forms: web, chart, diagram, Venn diagram, etc. Graphic organizers use visual representations as effective tools to do such things as

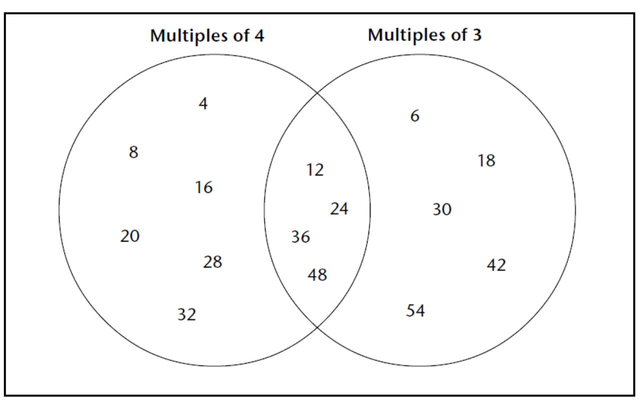
- activate prior knowledge
- analyze
- compare
- make connections
- organize
- summarize

The following steps illustrate how the organizer can be used.

1. Present a template of the organizer and explain its features.
2. Model how to use the organizer, being explicit about the quality of work that is expected.
3. Present various opportunities for students to use graphic organizers in the classroom.

Students should be encouraged to use graphic organizers on their own as ways of organizing their ideas and work. If the graphic organizer being used is a Venn diagram, it is important to draw a rectangle around Venn diagrams to represent the entire group that is being sorted. This will show the items that do not fit the attributes of the circle(s) outside of them, but within the rectangle. Therefore, elements of the set that do belong to Attribute A or Attribute B are shown within the rectangle but not within the circles in the rectangles.

Example of a Venn diagram



7. **K-W-L (Know/Want to Know/Learned)**
K-W-L is an instructional strategy that guides students through a text or mathematics word problem and uses a three-column organizer to consolidate the important ideas. Students brainstorm what they know about the topic and record it in the K column. They then record what they want to know in the W column. During and after the reading, students' record what they have learned in the L column.
The K-W-L strategy has several purposes:
   - to illustrate a student's prior knowledge of a topic
   - to give a purpose to the reading
   - to help a student monitor his or her comprehension

Example: Double Bar Graph

Know	Want to Know	Learned
A bar graph may have vertical bars or horizontal bars. *A bar graph has a title and labels for the axes.*	*Are there other types of bar graphs?* *How to read a double bar graph?* *The characteristics of a double bar graph.* *How do I build a double bar graph with a computer?*	*There are horizontal and vertical double bar graphs.* *A double bar graph must have a legend.* *I can read a double bar graph.* *The title, axis labels and legend are important for reading a bar graph.* *The bars must be the same width.* *We didn't use a computer to build a bar graph.*

The following steps illustrate how the K-W-L can be used.

1. Present a template of the organizer to students, explain its features, and be explicit about the quality of work that is expected.
2. Ask them to fill out the first two sections (what they know and what they want to know before proceeding.
3. Check the first section for any misconceptions in thinking or weakness in vocabulary.
4. Have the students read the text, and taking notes as they look for answers to the questions they posed.
5. Have students complete the last column to include the answers to their questions and other pertinent information.
6. Discuss this new information with the class, and address any questions that were not answered.

8. **Think-Pair-Share**

   Think-Pair-Share is a learning strategy designed to encourage students to participate in class and keep them on task. It focusses students' thinking on specific topics and provides them with an opportunity to collaborate and have meaningful discussion about mathematics.

   - First, teachers ask students to think individually about a newly introduced topic, concept, or problem. This provides essential time for each student to collect his or her thoughts and focus on his or her thinking.
   - Second, each student pairs with another student, and together the partners discuss each other's ideas and points of view. Students are more willing to participate because they do not feel the peer pressure that is involved when responding in front of the class. Teachers ensure that sufficient time is allowed for each student to voice his or her views and opinions. Students use this time to talk about personal strategies, compare solutions, or test ideas with their partners. This helps students to make sense of the problem in terms of their prior knowledge.
   - Third, each pair of students shares with the other pairs of students in large-group discussion. In this way, each student can listen to all of the ideas and concerns discussed by the other pairs of students. Teachers point out similarities, overlapping ideas, or discrepancies among the pairs of students and facilitate an open discussion to expand upon any key points or arguments they wish to pursue.

9. **Think-Aloud**

   Think-aloud is a self-analysis strategy that allows students to gain insight into the thinking process of a skilled reader as he or she works through a piece of text. Thoughts are verbalized, and meaning is constructed around vocabulary and comprehension. It is a useful tool for such things as brainstorming, exploring text features, and constructing meaning when solving problems. When used in mathematics, it can reveal to teachers the strategies that are part of a student's experience and those that are not. This is helpful in identifying where a student's understanding may break down and may need additional support.

   The think-aloud process will encourage students to use the following strategies as they approach a piece of text.

   - Connect new information to prior knowledge.
   - Develop a mental image.
   - Make predictions and analogies.
   - Self-question.
   - Revise and fix up as comprehension increases.

   The following steps illustrate how to use the think-aloud strategy.

   1. Explain that reading in mathematics is important and requires students to be thinking and trying to make sense of what they are reading.
   2. Identify a comprehension problem or piece of text that may be challenging to students; then read it aloud and have students read it quietly.
   3. While reading, model the process verbalizing what you are thinking, what questions you have, and how you would approach a problem.
   4. Then model this process a second time, but have a student read the problem and do the verbalizing.
   5. Once students are comfortable with this process, a student should take a leadership role.

10. **Academic Mathematics Journal**

An academic journal in mathematics is an excellent way for students to keep personal work and other materials that they have identified as being important for their personal achievement in mathematics. The types of materials that students would put in their journals would include:

- strategic lessons – lessons that they would identify as being pivotal as they attempt to understand mathematics
- examples of problem-solving strategies
- important vocabulary

Teachers are encouraged to allow students to use these journals as a form of assessment. This will emphasize to the student that the material that is to be placed in his or her journal has a purpose. Provide feedback for these journals only based on how students are using them and whether or not they have appropriate entries.

The goal of writing in mathematics is to provide students with opportunities to explain their thinking about mathematical ideas and then to re-examine their thoughts by reviewing their writing. Writing will enhance students' understanding of math as they learn to articulate their thought processes in solving math problems and learning mathematics concepts.

11. **Exit Card**

Exit cards are quick tools for teachers to become better aware of a students' mathematics understanding. They are written student responses to questions that have been posed in class or solutions to problem-solving situations. They can be used at the end of a day, week, lesson, or unit. An index card is given to each student (with a question that promotes understanding on it), and the student must complete the assignment before he or she can "exit" the classroom. The time limit should not exceed 5 to 10 minutes, and the student drops the card into some sort of container on the way out. The teacher now has a quick assessment of a concept that will help in planning instruction.

Appendix C: Cognitive Levels of Sample Questions

Lesson Learned 1: Number		**Lesson Learned 2: Estimation**		**Lesson Learned 3: Patterns and Relations**	
Question	**What type of question?**	**Question**	**What type of question?**	**Question**	**What type of question?**
1	Application	1	Application	1	Knowledge
2	Application	2	Application	2	Application
3	Application	3	Application	3	Analysis
4	Application	4	Application	4	Application
5	Application	5	Analysis	5	Application
6	Application	6	Application	6	Analysis
7	Application	7	Analysis	7	Analysis
8	Application			8	Application
9	Application			9	Application
10	Application			10	Analysis
11	Application			11	Analysis
12	Application			12	Analysis
13	Knowledge			13	Analysis
14	Application			14	Application
15	Knowledge			15	Analysis
16	Application				
17	Application				
18	Application				
19	Analysis				
20	Application				
21	Application				
22	Analysis				
23	Application				

Lesson Learned 4: Measurement		Lesson Learned 5: Geometry		Lesson Learned 6: Statistics and Probability		Lesson Learned 7: Problem Solving	
Question	**What type of question?**	**Question**	**What type of question?**	**Question**	**What type of question?**	**Question**	**What type of question?**
1	Knowledge	1	Knowledge	1	Application	1	Analysis
2	Application	2	Knowledge	2 A	Application	2	Analysis
3	Application	3	Knowledge	2 B	Application	3	Application
4	Analysis	4	Application	2 C	Application	4	Application
5	Analysis	5	Knowledge	3 a	Knowledge	5	Analysis
6	Analysis	6	Knowledge	3 b	Application	6	Analysis
7	Analysis	7	Knowledge	3 c	Knowledge	7	Application
8	Application	8	Knowledge	4	Application	8	Analysis
		9	Analysis	5A	Knowledge		
		10	Analysis	5B	Knowledge		
		11	Application	6A	Application		
		12	Application	6B	Application		
				7	Analysis		
				8	Application		

Appendix D: Answers to the Sample Questions

Lesson Learned 1: Number		Lesson Learned 2: Estimation		Lesson Learned 3: Patterns and Relations	
Question	**Answer**	**Question**	**Answer**	**Question**	**Answer**
1	D	1	D	1	A
2	B	2	B	2	D
3	B	3	B	3	D
4	C	4	D	4	D
5	B	5	B	5	C
6	C	6	D	6	A
7	D	7	C	7	B
8	B			8	D
9	B			9	B
10	D			10	D
11	C			11	D
12	B			12	C
13	D			13	D
14	A			14	D
15	D			15	C
16	D				
17	B				
18	B				
19	D				
20	A				
21	C				
22	D				
23	D				

Lesson Learned 4: Measurement		Lesson Learned 5: Geometry		Lesson Learned 6: Statistics and Probability		Lesson Learned 7: Problem Solving	
Question	Answer	Question	Answer	Question	Response	Question	Answer
1	D	1	D	1	C	1	21 handshakes
2	B	2	C	2A	A	2	54 green blocks 63 blue blocks Total: 117 blocks
3	D	3	B	2B	D	3	120 cents
4	B	4	D	2C	C	4	8 batteries each
5	C	5	D	3A	B	5	**Rectangle 1:** 15 m x 2.5 m, Fence 35 m **Rectangle 2:** 7.5 m x 5 m, Fence 25 m
6	B	6	A	3B	C	6	Orange section = one half Blue, Red and Green section is one sixth each
7	D	7	B	3C	D	7	225 cards and Jean-Guy has 205 cards
8	C	8	C	4	D	8	Rectangle = 15 Circle = 9 Triangle = 3
		9	C	5A	B		
		10	C	5B	B		
		11	C	6	C (Jar B)		
		12	D	7	B (Jar C)		
				8	B		
				8	B		

Appendix E: Problem-Solving Sample Question 1: Page 73

1. Seven children exchange handshakes, each one at once.
   How many handshakes are there?

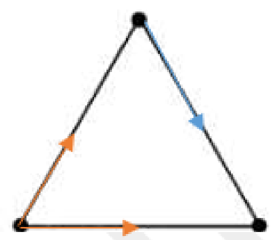
Problem number 1 (handshakes) can be solved by choosing from the strategies below.

<u>Draw a picture</u>

2 children

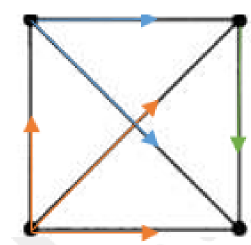
1 handshake

3 children



3 handshakes

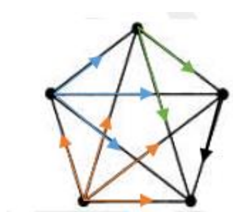
4 children



6 handshakes

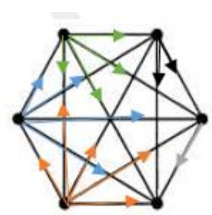
---

5 children



10 handshakes

6 children



15 handshakes

7 children

21 handshakes

The 1st child gives 6 handshakes, the 2nd child gives 5 handshakes, the 3rd child gives 4 handshakes, the 4th child gives 3 handshakes, the 5th child gives 2 handshakes, and the 6th child gives 1 handshake. The mathematical phrase is:

6 + 5 + 4 + 3 + 2 + 1 = 21, so it has 21 handshakes

Make a table

Children	2	3	4	5	6	7
Handshakes	1	3	6	10	15	21

For 2 children, the number of handshakes is 1.
For 3 children, this number is 1 + 2 = 3
For 4 children, this number is 1 + 2 + 3 = 6
For 5 children, this number is 1 + 2 + 3 + 4 = 10
For 6 children, this number is 1 + 2 + 3 + 4 + 5 = 15
For 7 children, this number is 1 + 2 + 3 + 4 + 5 + 6 = 21

Look for a pattern

For 2 children, the number of handshakes is 1
For 3 children, the number of handshakes is 1 + 2 = 3
For 4 children, the number of handshakes is 1 + 2 + 3 = 6
For 5 children, the number of handshakes is 1 + 2 + 3 + 4 = 10
The rule of the pattern observed is: From 1 hand, add 2, then 3, then 4, then 5, then 6, …
The terms of this pattern are:

+2 +3 +4 +5 +6
1 3 6 10 15 21

There are 21 handshakes with 7 children.

Have 7 children act out the number of handshakes

Have the 7 children stand in a straight line.
The 1st child is asked to give each of the 6 other children a handshake, note the number of handshakes they have given, and step out of line.
We do the same thing with the second child, the third child, the fourth child, the fifth child and the sixth child.
The first child gives 6 handshakes, the second child gives 5 handshakes, the third gives 4 handshakes, the fourth gives 3 handshakes, the fifth child gives 2 handshakes, and the sixth child gives 1 handshake.
The mathematical phrase is 1 + 2 + 3 + 4 + 5 + 6 = 21.
There are 21 handshakes.

Appendix F: Problem-Solving Scenarios

In mathematics, problem solving is seen as a complex process of mathematical modelling. A scripted contextual problem is a research exercise that is a challenge to the individual, one that engages their faculties and comprehension skills. The idea is to present students with real problems, authentic problem scenarios, which they will be motivated to solve. This approach enables them to acquire a methodical approach to exploration aimed at finding a solution that clearly describes their strategy using appropriate terminology.

Example 1: Mark and the Food Bank

Mark works as a volunteer for one of the busiest food banks in the city. This food bank distributes 372 food boxes each month from donations, fundraising, recoveries from supermarket surpluses. These boxes are delivered on a regular and timely basis to many individuals and families.

1. Mark wants to collect 2900 empty milk containers to raise money for this food bank. Mark collected 1825 containers.

   How many containers must Mark still collect to have the 2900 containers?

- ○ 1075
- ○ 1125
- ○ 1175
- ○ 4725

Work Space

2. The box, in which Mark arranges the food, has the shape of a rectangular prism.

   What statement is true about this prism?

- ○ the prism has 6 vertices
- ○ the prism has 6 faces
- ○ the prism has 8 edges
- ○ the prism has 12 faces

3. In each food box, Mark puts four cans of tuna 180 g each, a jar of peanut butter 454 g, two bottles of jam 500 g each.

Calculate the mass of tuna, peanut butter and jam that the food bank distributes in one food box.

- ○ 1134 g
- ○ 1174 g
- ○ 2174 g
- ○ 12 740 g

Solve this problem. Show all your work. You may use words, pictures, or symbols.

Example 1: Mark and the Food Bank – Answers and Justifications

Example 1: Mark and the Food Bank
Mark works as a volunteer for one of the busiest food banks in the city. This food bank distributes 372 food boxes each month from donations, fundraising, recoveries from supermarket surpluses. These boxes are delivered on a regular and timely basis to many individuals and families.

1. Mark wants to collect 2900 empty milk containers to raise money for this food bank. Mark collected 1825 containers.

   How many containers must Mark still collect to have the 2900 containers?

   - ● 1075
   - ○ 1125
   - ○ 1175
   - ○ 4725

Work Space

SCO: 4N03
Item Description: Subtract two four-digit numbers in context.

Big Idea: Operations on whole numbers help explain everyday life situations.

Cognitive Level: Application
Difficulty Level: Medium

Attention: Some students made errors caused by the misconception of regrouping and trading incorrectly when performing the operation of subtraction.

Selected Answers/Next Steps

The student chose A (1075) *A is the correct answer.* The student understands the concept of subtracting a four-digit number from a four-digit number with regrouping and trading. ***The Next Step*** It is important to probe the student's response to ensure their response can indicate their understanding of subtracting a four-digit number from a four-digit number with regrouping and trading. To do this, present the student with similar word problems when subtracting. When introducing addition and subtraction of four-digit numerals, it is important to use base-ten blocks to model the operations correctly.	***The student chose B (1125)*** This student subtracted a four-digit number from a four-digit number without regrouping or trading. ***The Next Step*** The student's response shows that they do not understand the concept of subtracting a four-digit number from a four-digit number with regrouping and trading.
The student chose C (1175) The student subtracted these two numbers with some consideration of regrouping of numbers and trading. ***The Next Step*** The student's response shows that they do not understand the concept of subtracting a four-digit number from a four-digit number with regrouping and trading.	***The student chose D (4725)*** The student took the two numbers 2900 and 1825 and added them instead of subtracting them. It appears that the student is not able to distinguish between addition and subtraction in a real context. ***The Next Step*** The student's response shows that they do not understand the concept of subtracting a four-digit number from a four-digit number with regrouping and trading.

Schedule a meeting with the students who got the wrong answer. Provide them with relevant learning activities that enable them to understand the concept of subtracting a four-digit number from a four-digit number with regrouping and trading. To do this, present the students with similar word problems for subtracting or adding a four-digit number from a four-digit number with regrouping and trading.

When introducing addition and subtraction of four-digit numerals, it is important to use base-ten blocks to model the operations correctly. If the students are having problems aligning the digits according to their place value positions, you could provide lined grid paper to the students.

2. The box, in which Mark arranges the food, has the shape of a rectangular prism.

What statement is true about this prism?

- ○ the prism has 6 vertices
- ● the prism has 6 faces
- ○ the prism has 8 edges
- ○ the prism has 12 faces

SCO: 3G01
Item Description: Recognize an attribute of a rectangular prism.

Big Idea: 3-D objects are found everywhere in the environment.

Cognitive Level: Knowledge
Difficulty Level: Easy

Attention: The common mistake made by students is they do not recognize the attributes of 3-D objects.

Selected Answers/Next Steps

The student chose A (**the prism has 6 vertices)** The student does not distinguish between the vertices and faces of a rectangular prism. ***The Next Step*** The student's response shows that they have not acquired the sense of space, the attributes or characteristics that define a three-dimensional object.	***The student chose B (*****the prism has 6 faces)** B is the right answer. The student's work shows that he can easily recognize the vertices, faces and edges of a rectangular prism. ***The Next Step*** It is important to probe the student's response to ensure their response is capable of indicating and naming the attributes of a prism. To do this, offer the student an object in the form of a rectangular prism (for example: a box of Kleenex, the mathematics manual, etc.) and ask them to indicate and name the vertices, the faces and edges and specify their numbers.
The student chose C (**the prism has 8 edges)** The student counts the vertices instead of the edges. ***The Next Step*** The student's response shows that they have not acquired the sense of space and the attributes or characteristics that define a three-dimensional object.	***The student chose D (*****the prism has 12 faces)** The student counts the edges instead of the faces. ***The Next Step*** The student's response shows that they have not acquired the sense of space and the attributes or characteristics that define a three-dimensional object.

Schedule a meeting with the students who got the wrong answer. Provide them with relevant learning activities that enable them to visualize a three-dimensional object in space and develop spatial skills. The use of handling equipment and concrete equipment is essential to familiarize students with the geometric differences between the concrete object and its pictorial representation.

It is obvious that a student will only be able to work on the drawing of an object located in three-dimensional space if it has a good mental image of that object. It is important to mention that to acquire and retain the mental image of a three-dimensional object (prisms, pyramids, cylinders, etc.), the student needs to use prior geometric knowledge such as a point, a line, a rectangle, a triangle, a circle…

3. In each food box, Mark puts four cans of tuna 180 g each, a jar of peanut butter 454 g, two bottles of jam 500 g each.

Calculate the mass of tuna, peanut butter and jam in kilograms that the food bank distributes
in one food box?

- ○ 1134 g
- ○ 1174 g
- ● 2174 g
- ○ 12 740 g

Solve this problem.
Show all your work.
You may use words, pictures, or symbols.

Solution:

The student uses addition.

The mass of the 4 cans of tuna:
180 g + 180 g + 180 g + 180 g = 720 g
The mass of 2 jars of jam:
500 g + 500 g = 1000 g
The total mass of the 3 food products:
720 g + 454 g + 1000 g = 2174 g

In a box, Mark put 2174 grams of tuna, peanut butter and jam.

or

The student uses multiplication and addition.

The mass of the 4 cans of tuna:
4 x 180 g = 720 g
The mass of 2 jars of jam:
2 x 500 g = 1000 g
The total mass of the 3 food products:
720 g + 454 g + 1000 g = 2174 g

In a box, Mark put 2174 grams of tuna, peanut butter and jam.

SCO: 4N06
Item Description: Solve a contextual problem using addition and multiplication.

Big Idea: The operations on natural numbers make it possible to explain everyday situations.

Cognitive Level: Application
Difficulty Level: Medium

Attention: The errors made by the students are mainly because of the misalignment of the numerals according to their place value or lack of regrouping.

Selected Answers/Next Steps

The student chose A (*1134 g)** The student does not multiply (repeated addition) or use the double strategy to help them solve this contextual multi-step problem. ***The Next Step The student's response shows that they do not understand the concept of multiplication (addition) or the double strategy when solving a multi-step contextual problem.	***The student chose B (1174 g)*** The student again does not multiply (repeated addition) but does use the double strategy to help them solve this contextual multi-step problem. ***The Next Step*** The student's response shows that they do not understand the concept of multiplication, and addition when solving a multi-step contextual problem.
The student chose C (*2174 g)** *C is the correct answer.* The student understands the concept of solving a contextual multi-step problem using the operations of addition and multiplication with regrouping and trading. They also understand the double strategy. ***The Next Step It is important to probe the student's response to ensure their understanding when solving a contextual multi-step problem using the operations on whole numbers with regrouping and trading. It will also indicate their understanding of addition, multiplication and the double strategy. To do this, present the student with similar contextual problems using multiple operations and other strategies.	***The student chose D (*12 740 g)** The student did not understand the place value of the digits in the question. The alignment of the digits was also in correct. ***The Next Step*** The student's response shows that they do not understand the concept of multiplication, addition, place value and alignment of numbers when solving a multi-step contextual problem.

Schedule a meeting with the students who got the wrong answer. Provide them with relevant learning activities that enable them to understand the concept of multiplication, repeated addition, place value and proper alignment of numbers.

If the students are having problems aligning the digits according to their place value positions, you could provide lined grid paper to the students.

To do this, present the students with similar word multistep problems for multiplication and addition using regrouping and trading.