Nova Scotia Assessment:
Mathematics in Grade 8

Lessons Learned

> Helping students to learn from their mathematical mistakes can give us insight into their misconceptions and, depending on our instructional reactions, can enable them to develop deeper understanding of the mathematics they are learning.
>
> (Linda M. Gojak, NCTM president)

12-2018

Contents

Purpose of this document 1

Overview of the Nova Scotia Assessment: Mathematics in Grade 8 2

- Performance Levels 4
- Assessment Results 4

Mathematics in Grade 8 Lessons Learned 5

Key Messages 7

Mathematics in Grade 8 Lesson Learned 1 – Problem Solving 9

Mathematics in Grade 8 Lesson Learned 2 – Number 16

Mathematics in Grade 8 Lesson Learned 3 – Patterns and Relations 23

Mathematics in Grade 8 Lesson Learned 4 – Measurement and Geometry 31

Mathematics in Grade 8 Lesson Learned 5 – Statistics and Probability 39

References 45

Appendix A: Cognitive Levels of Questioning 46

Appendix B: From Reading Strategies to Mathematics Strategies 48

Appendix C: Cognitive Levels of Sample Questions 52

Appendix D: Sample Questions Answers 53

Purpose of this document

This *Lessons Learned* document was developed based on an analysis of the Item Description Report for the 2017–2018 Nova Scotia Assessment (NSA): Mathematics in Grade 8 (M8). It is intended to support all Grade 6, 7, 8, and 9 classroom teachers (particularly Grades 7–8) and administrators at the school, region, and provincial levels, in using the information gained from this assessment to inform next steps for numeracy focus. The analysis of these items form the basis of this document, which was developed to support teachers as they further explore these areas through classroom-based instruction and assessment across a variety of mathematical concepts.

After the results for each mathematics assessment become available, an Item Description Report is developed to describe each item of the assessment in relation to the curriculum outcomes and cognitive processes involved with each mathematical strand. The percentage of students across the province who answered each item correctly is also connected to each item. Item Descriptor Reports for mathematics are made available to Regional Education Centers for distribution to schools, and they include provincial, region, and school data. Schools and regions should examine their own data in relation to the provincial data for continued discussions, explorations, and support for mathematics focus at the classroom, school, region, and provincial levels.

This document specifically addresses areas that students across the province found challenging based on provincial assessment evidence. It is essential that teachers consider assessment evidence from a variety of sources to inform the next steps most appropriate for their students. Effective classroom instruction and assessment strategies are responsive to the individual learners within a classroom.

The M8 assessment generates information that is useful in guiding classroom-based instruction and assessment in mathematics. This document provides an overview of the mathematics tasks included in the assessment, information about this year's mathematics assessment results, and a series of Lessons Learned for mathematics. Suggested next steps for classroom instruction and assessment are included for each Lesson Learned.

Overview of the Nova Scotia Assessment: Mathematics in Grade 8

Nova Scotia Assessments are large-scale assessments that provide reliable data about how well all students in the province are learning the mathematics curricula. It is different from many standardized tests in that all questions are written by Nova Scotia teachers to align with curriculum outcomes and the results reflect a snapshot of how well students are learning these outcomes. These results can be counted on to provide a good picture of how well students are learning curriculum outcomes within schools, boards and in the province. Since the assessments are based on the Nova Scotia curriculum, and are developed by Nova Scotia teachers, results can be used to determine whether the curriculum, approaches to teaching and allocation of resources are effective. Furthermore, because individual student results are available, these, in conjunction with other classroom assessment evidence, help classroom teachers understand what each student has under control and identify next steps to inform instruction.

The Nova Scotia Assessment: Mathematics in Grade 8 provides information about mathematics for each student and complements assessment data collected in the classroom. This assessment is administered at the end of Grade 8. It is designed to provide detailed information for every student in the province regarding his or her progress in achieving a selection of mathematics curriculum outcomes at the end of Grade 8. Information from this assessment can be used by teachers to inform their instruction and next steps in providing support for their students.

The design of the assessment includes the following:

- Mathematical tasks that reflect a selection of outcomes aligned with the mathematics curriculum to the end of Grade 8 from across all strands.
- All items are in selected response format.
- All items are designed to provide a broad range of challenge, thereby providing information about a range of individual student performance

Table 1: Specific Curriculum Outcomes Assessed in 2017-2018

Strand	**Specific curriculum outcome**
Number (N)	*6N04, 7N01, 7N02, 7N03, 7N05, 7N06, 7N07, 8N01, 8N02, 8N03, 8N04, 8N05, 8N06, 8N07
Patterns and Relations (PR)	7PR01, 7PR02, 7PR06, 8PR01, 8PR02
Measurement (M)	6M02, 6M03, 7M01, 7M02, 8M01, 8M03, 8M04
Geometry (G)	5G01, 6G03, 6G04, 7G01, 7G03
Statistics and Probability (SP)	7SP01, 7SP03, 7SP04, 8SP01, 8SP01, 8SP02

**6N: The 1st digit indicates the grade level (Grade 6), the letter N indicates the strand (Number) and the 3d digit indicates the curriculum SCO.*

Cognitive levels of questioning in mathematics:

- *Knowledge* questions require students to recall or recognize information, names, definitions, or steps in a procedure.
- *Application* questions require some degree of comprehension and students should apply their mathematical knowledge to answer correctly.
- *Analysis* questions require students to go beyond comprehension and application to higher order thinking skills, such as analysis, generalizations, and non-routine problem solving.

Table 2: Table of specification showing the percentages of cognitive levels

Table of Specifications: Cognitive Levels	
Cognitive Level	**Percentage**
Knowledge	20–25 %
Application	55–60 %
Analysis	20–25 %

These percentages are also recommended for classroom-based assessment.

Note: Please refer to Appendix A for further information about cognitive levels of questioning.

The Nova Scotia Assessment: Mathematics 8 includes 76 items distributed over two days for a duration of 90 minutes each day; 38 items on day one and 38 items on day two. The chart below shows the distribution, by mathematical strand and cognitive level, of items each day.

Table3: Number of items by strand and cognitive level

Number of items, Day 1				
	Knowledge	Application	Analysis	Total
Number	3	9	2	14
Patterns and Relations	2	3	1	6
Measurement	2	3	1	6
Geometry	1	3	1	5
Statistics and Probability	2	4	1	7
Number of items, Day 2				
	Knowledge	Application	Analysis	Total
Number	3	9	2	14
Patterns and Relations	2	3	1	6
Measurement	2	3	1	6
Geometry	1	3	1	5
Statistics and Probability	2	4	1	7

Performance Levels

The four performance levels for Nova Scotia Assessment: Mathematics in Grade 8 are described below. Level 3 is the expectation for this assessment.

Level 1: Students at Level 1 can generally solve problems when they are simple and clearly stated or where the method to solve the problem is suggested to them. They will have more success with problems dealing with mathematical concepts from earlier years. They can do some basic operations (+, -, x, ÷), but may not understand when each operation should be used. They can recognize some math terms and symbols, mainly from earlier grades.

Level 2: Students at Level 2 can generally solve problems similar to problems they have seen before. They depend on a few familiar methods to solve problems. They can usually do basic operations (+, -, x, ÷) and can usually understand where they should be used. They can understand and use some math terms and symbols, especially those from earlier grades.

Level 3: Students at Level 3 can generally solve problems that involve several steps and may solve problems they have not seen before. They can correctly apply number operations (+, -, x, ÷) using integers, decimal numbers and fractions and can judge whether an answer makes sense. They can understand and use many math terms and symbols, including those at grade level.

Level 4: Students at Level 4 can solve new and complex problems. They can apply number operations (+, -, x, ÷) using integers, decimal numbers and fractions with ease. They can think carefully about whether an answer makes sense. They find math terms and symbols easy to use and understand.

Assessment Results

The Nova Scotia Assessment: Mathematics in Grade 8 was implemented since the school year 2012–2013. It is important to mention that the evaluation of 2015–2016 was administered according to the new curriculum of 2017 in May – June, 2018. The following percentage of students performed at the expectation of Level 3 or above on the mathematics assessment: 54% (2012–2013), 57% (2013–2014), 55% (2014–2015), 62% (2015–2016) and 56% (2017–2018).

The following is a breakdown of the 2017–2018 M8 results for each performance level (7951 Grade 8 students participated in the M8 assessment):

- Performance Level 1: 10% of students in the province are below the expectations of this assessment
- Performance Level 2: 34% of students in the province are approaching the expectations of this assessment
- Performance Level 3: 47% of students in the province are at the expectations of this assessment
- Performance Level 4: 9% of students in the province are above the expectations of this assessment

Table 4: Percentage of students at each performance level

	2012 – 2013	2013 – 2014	2014 – 2015	2015 – 2016		2017 – 2018
Performance Level 1	14%	13%	13%	6%		10%
Performance Level 2	32%	30%	32%	32%		34%
Performance Level 3	48%	51%	48%	51%		47%
Performance Level 4	6%	6%	7%	11%		9%

Mathematics in Grade 8 Lessons Learned

The assessment information gathered from the 2017–2018 Nova Scotia Assessment: Mathematics in Grade 8 data has been organized into 5 Lessons Learned: Problem Solving, Number, Patterns and Relations, Measurement and Geometry, and Statistics and Probability.

Each Lesson Learned is divided into four sections that address the following questions:

A. What conclusions can be drawn from the Mathematics Assessment in Grade 8?
B. Do students have any misconceptions or errors in their thinking?
C. What are the next steps instruction for the class and for individual students?
D. What are the most appropriate methods and activities for assessing student learning?

Lessons Learned

1. **Problem Solving**: Learning through problem solving should be the focus of mathematics at all grade levels. Problem solving is one of the critical mathematical components that students must encounter in a mathematics program to achieve the goals of mathematics education and encourage lifelong learning in mathematics. When students encounter a new situation, and respond to questions such as “How would you… or How could you …?” the problem-solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.

2. **Number**: It is important to acknowledge that numbers are a fundamental strand in the study of many others domains. The work that students do to determine the general term of a given pattern is dependent on their ability to operate with numbers. Many algebra concepts that students learn in middle school and early high school are essentially generalized arithmetic. For example, consider the numbers and the operations involved in solving the equation $2x + (-4x) = -8$. Numbers must be meaningful for students to make sense of measurement and to interpret numerical data in statistics and probability.

3. **Patterns and Relations**: Patterns and relations are part of all strands of the mathematics curriculum from primary to grade 9. The study of patterns and relations aims to develop algebraic reasoning which requires the intersection of several abstract and procedural skills such as generalization, problem-solving, communication, use of models and representation of real situations using symbols as well as analysis of change. We have to be aware that mathematics is not an arbitrary constructions. It arises from the evolution of our brain in a world which has interesting patterns, number patterns, space patterns, time patterns, etc. The human brain tends to internalize these patterns with the aim of strengthening its capacities of analysis and synthesis to reconcile with natural, societal and environmental phenomena.

4. **Measurement and Geometry**: “The general aim of the study of measurement and geometry is stated as making an individual acquire the mathematical knowledge needed in daily basis, teaching how to solve problems and acquiring reasoning methods. Learning geometry is not just learning the definitions or the attributes of geometrical concepts but also to have the ability of analyzing the properties of two (2D) and three dimensional (3D) geometric shapes and develop mathematical arguments about geometric relationships, to specify locations and spatial relationship, to apply transformations, visualization, spatial reasoning and geometric modeling to solve problems.” (NCTM, *Principles and Standards 2000 for School Mathematics*)

5. **Statistics and probability:** Data analysis is not only about calculating statistics and constructing graphs and diagrams. It is a process of inspecting, transforming, and modeling data with the goal of discovering useful information, suggesting conclusions, and supporting decision-making. It includes collecting, organizing, displaying, and analyzing data. Probability is a branch of mathematics that deals with calculating the likelihood of a given events occurrence. Probability theory had its start in the 17th century, when two French mathematicians, Blaise Pascal and Pierre de Fermat carried on a correspondence discussing mathematical problems dealing with games of chance. Contemporary applications of probability theory run the gamut of human inquiry, and include aspects of computer programming, astrophysics, music, weather prediction, and medicine.
(Margaret Rouse, http://whatis.techtarget.com/definition/probability)

Key Messages

The following key messages should be considered when using this document to inform classroom instruction and assessment.

The Nova Scotia mathematics curriculum is based upon several key assumptions or beliefs about mathematics learning that have grown out of research and practice. These beliefs include the following:

- Mathematics learning is an active and constructive process.
- Learning is most effective when standards of expectation are made clear with ongoing assessment and feedback.
- Learners are individuals who bring a wide range of prior knowledge and experiences and who learn via various styles and at different rates.
- Learning is most likely to occur when placed in meaningful contexts and in an environment that supports exploration, risk taking, and critical thinking and that nurtures positive attitudes and sustained effort.
- Students are curious, active learners with individual interests, abilities, and needs. They come to classrooms with varying knowledge, life experiences, and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.
- Students learn by attaching meaning to what they do and need to construct their own meaning of mathematics. This meaning is best constructed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of models and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students and enhance the formation of sound, transferable, mathematical concepts. At all levels, students benefit from working with and translating through a variety of materials, tools, and contexts when constructing meaning about new mathematical ideas. Meaningful discussions can provide essential links among concrete, pictorial, contextual, and symbolic representations of mathematics. (EECD, 2013a, p. 23)
- Research has shown that assessment for learning (formative assessment) practices produce significant and often substantial learning gains, close achievement gaps, and build students' ability to learn new skills (Black, P. & William 1996, OECD, 2006). Student involvement in assessment promotes learning. Timely and effective teacher feedback and student self-assessment allow students to reflect on and articulate their understanding of mathematical concepts and ideas.
- Provincial assessment results form part of the larger picture of assessment for each student and complements assessment data collected in the classroom. Ongoing assessment for learning (formative assessment) is essential to effective teaching and learning. Assessment for learning can and should happen every day as part of classroom instruction. Assessment of learning (summative assessment) should also occur regularly and at the end of a cycle of learning. Both forms of assessment are necessary to guide teaching, stimulate learning, and produce achievement gains.
- It is important to construct assessment activities that require students to complete tasks across the cognitive levels. While it is important for students to be able to answer factual and procedural type questions, it is also important to embed activities that require strategic reasoning and problem-solving.
- Ongoing assessment for learning involves the teacher focusing on how learning is progressing during the lesson and the unit, determining where improvements can be made, and identifying the next steps. "Such assessment becomes 'formative assessment' when the evidence is actually used to adapt the teaching to meet learning needs," (Black, Harrison, Lee, Marshall & William, 2003, p.2). Effective strategies of assessment for learning during a lesson include: strategic questioning, observing, conversing (conferring with students to "hear their thinking"), analyzing student's work (product), engaging students in reviewing their progress, as well as, providing opportunities for peer and self-assessment.

- Assessment of learning involves the process of collecting and interpreting evidence for the purpose of summarizing learning at a given point in time and making judgments about the quality of student learning on the basis of established criteria. The information gathered may be used to communicate the student's achievement to students, parents, and others. It occurs at or near the end of a learning cycle.
- All forms of assessment should be planned with the end in mind. Think about the following questions:
  - What do I want students to learn? (identify clear learning targets)
  - What does the learning look like? (identify clear criteria for success)
  - How will I know they are learning?
  - How will I design the learning so that all will learn?
- Before planning for instruction using the suggestions for instruction and assessment, it is important that teachers review individual student results in conjunction with current mathematics assessment information. A variety of current classroom assessments should be analyzed to determine specific strengths and areas for continued instructional focus or support.

Balanced Assessment in Mathematics

The following ways are effective to gather information about a student's mathematical understanding:

- Conversations/Conferences/Interviews: individual, group, teacher-initiated, student-initiated
- Products/Work Samples: Mathematics journals, portfolios, drawings, charts, tables, graphs, individual and classroom assessment, pencil-and-paper tests, surveys, self-assessment
- Observations: Planned (formal), unplanned (informal), read-aloud (literature with mathematics focus), shared and guided mathematics activities, performance tasks, individual conferences, anecdotal records, checklists, interactive activities

(EECD, 2013a, p. 4)

Mathematics in Grade 8 Lesson Learned 1
Problem Solving

Learning through problem solving should be the focus of mathematics at all grade levels. Problem solving is one of the critical mathematical components that students must encounter in a mathematics program to achieve the goals of mathematics education and encourage lifelong learning in mathematics. When students encounter a new situation, and respond to questions such as "How would you… or How could you …?" the problem-solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.

"The single most important principle for improving the teaching of mathematics is to allow the subject of mathematics to be problematic for students (Hiebert et al., 1996). That is, students solve problems not to apply mathematics but to learn new mathematics. When students engage in well-chosen problem-based tasks and focus on the solution methods, what results is new understanding of the mathematics embedded in the task. When students are actively looking for relationships, analyzing patterns, finding out which methods work and which don't, justifying results, or evaluating and challenging the thoughts of others, they are necessarily and optimally engaging in reflective thought about the ideas involved. The appropriate thoughts in their cognitive structure are acting to give meaning to new ideas. Most, if not all, important mathematics concepts and procedures can best be taught through problem solving."
(*Teaching Student-Centered mathematics: 5–8*, Van de Walle and Lovin)

For an activity to be problem-solving based, it must challenge students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. A true problem-solving activity requires students to use prior learning in new ways and contexts. Problem-solving requires and builds depth of conceptual understanding and student engagement, perseverance, and collaboration. Problem-solving is also a powerful teaching tool that fosters multiple, creative, and innovative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident mathematical risk takers.

Students need to be able to explore a wide variety of methods for solving and verifying problems in all areas of mathematics. They must be challenged to find multiple solutions for problems and be given opportunities to create and solve their own problems.

A. What conclusions can be drawn from the Nova Scotia Assessment: Mathematics in Grade 8?

The analysis of the results of *2017–2018 Nova Scotia Assessment: Mathematics in Grade 8*, reveals that students have a good understanding of basic facts and procedures, when explicitly given all the information needed to do knowledge and application questions. But when given analysis questions, they are not able to apply their higher order thinking skills. For example, students were not sure how to calculate the mark of a student on a test knowing the marks of other students and the mean of the class. Overall, our students are experiencing challenges with problem solving across all mathematical strands. This appeared to be a theme throughout the assessment data when students were asked to "solve a word problem" or "solve a multi-step problem". The data also showed that, across all mathematics strands, students struggle with items that require translating among representations in problem-solving contexts.

B. Do students have any misconceptions or errors in their thinking?

It is important to distinguish between misconceptions and errors. Misconceptions are, in effect, the misunderstandings about mathematical ideas which students entertain and which usually lead to errors occurring. These are the most serious kind of errors which require urgent action on the part of the teacher. Errors may occur for a variety of underlying reasons, ranging from careless mistakes to errors resulting from misconceptions.

It is important to mention that when students encounter a problem-solving situation, they become confused by mathematical expressions which are not familiar to them or which are difficult to understand. In addition, when students consider that a problem is a mathematical problem, they believe, wrongly, that they should associate it simply to routine calculations with few concerns to the meaning of the context and to the credibility of the answers.

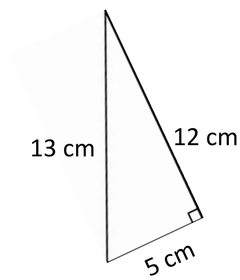
Example 1

Find the area of this right-angle triangle.

$$\text{Area} = \frac{\text{Base} \times \text{Height}}{2}$$

$$\text{Area} = \frac{5 \times 13}{2}$$

$$\text{Area} = 32.5\ \text{cm}^2$$



In example 1, the students incorrectly apply the formula for the area of a triangle, by considering the upwards side as the height and the bottom side as the base. This misconception is the source of the error.

Example 2

3.65 + 2.38 = 603

In example 2, the students do not pay attention to the decimals and fail to place a decimal in their answers.

Many students, when given a problem-solving task, especially a contextual problem, have the belief that it is always too hard for them to attempt. Putting words around the numbers obstructs their ability to think about the question. Another belief students have is that there is only one way to solve a word problem. When asked to solve a problem in a different context, they struggle to identify a possible strategy and often fail to even attempt to solve the problem. Along with not having problem-solving strategies to help them attempt the problem, students also tend to forget any knowledge of translating between and among representations when problem solving.

For example, if students are asked to solve a word problem using map scale (application question) to calculate a distance, they do not seem to realize that they can use proportions when asked to solve such a word problem.

The student results of the 2017–2018 Nova Scotia Assessment: Mathematics in Grade 8 shows the following data:

- 60% of the students made errors when solving a multistep contextual problem involving the mean of a set of data.
- 61% of the students made errors when solving a contextual problem involving the division of fractions.
- 67% of the students did not understand how to apply proportion the solve a multistep contextual problem involving a road map scale.

- 70% of the students had difficulty when solving a multistep contextual problem involving fractions to determine a rate.
- 70% of the students made errors when solving a word problem when asked to determine the length of a fence around a composite geometric shape.
- 70% of the students had difficulty to solve a word problem involving mixed and whole numbers.
- 77% of the students were not able to solve a contextual problem involving the surface area of a cylinder.

C. What are the next steps in instruction for the class and for individual students?

Teaching mathematics through problem solving is a familiar approach to some teachers and new to others. "To teach through problem solving, the teacher provides a context or reason for the learning by beginning the lesson with a problem to be solved, so he allows to establish a context which favors and justifies the learning. This approach contrasts with the more traditional approach, for example, presenting a new procedure and then adding a couple of word problems at the end for students to solve. Instead, the teacher gives students the opportunity to think about the problem and work through the solution in a variety of ways, and only then draws the procedure out of their work''. (Marian Small, (2010). *Prime: Number and Operations, background and Strategies,* Toronto, On: Nelson Education Ltd)

A Problem-Solving Approach

A problem-solving approach is embedded and expected throughout our curriculum, grades Primary to 12, in all strands.

"Why teach through problem solving? There are many reasons for teaching through problem solving:

- The math makes more sense.
- A problem-solving approach provides the teacher with better insight into a student's mathematical thinking.
- Problems are motivating when they are more of a challenge.
- Problem solving builds perseverance.
- Problem solving builds confidence, maximizes understanding, and allows for differences in style and approach.
- Problems can provide practice, both with concepts and with skills.
- A problem-solving approach provides students with better insight into what mathematics is all about.
- Students need to practice problem solving."

Marian Small, (2010). *Prime: Number and Operations, background and Strategies,* Toronto, On: Nelson Education Ltd

Van de Walle and Lovin (2006), in their resource, *Teaching Student-Centered Mathematics Grades 5–8*, suggest a three-part lesson format for teaching through problem-solving: **before**, **during**, and **after**.

Before

Before students begin:

- Get students mentally prepared to accomplish the task.
- Be sure the task is understood.
- Establish and explain the expectations.

During

As students work through the problem:

- Let students approach the problem in a way that makes sense to them.
- Listen to the conversations to observe thinking.
- Provide hints or suggestions if students are on the wrong path.
- Encourage students to test their ideas.
- Ask questions to stimulate ideas.
- Assess student understanding of her/his solution.

After

After students have solved their problem:

- Provide ample time for all students to discuss the solution.
- Gather for a group meeting to reflect and share.
- Highlight the variety of answers and methods.
- Encourage students to justify their solutions.
- Encourage students to comment positively or ask questions regarding their peer's solutions.

For more details on using a problem-solving approach to teach mathematics, see Van de Walle and Lovin (2006), *Teaching Student-Centered Mathematics Grades 5–8,* from which these ideas are drawn (pp. 15–26).

Note: There are 10 problem-solving strategies in *Math Makes Sense 8,* textbook page 368.

- Make a table.
- Use a model.
- Draw a diagram.
- Solve a simpler problem.
- Work backward.
- Guess and test.
- Make an organized list.
- Use a pattern.
- Draw a graph.
- Use logical reasoning.

From Reading Strategies to Mathematics Strategies

With a problem-solving approach embedded and expected throughout our curriculum grades Primary to 12 in all strands, there are definite implications for the teaching of reading strategies in mathematics. During the middle school years, students are making connections between every day, familiar language, and formal mathematical language. The process of making connections, in turn, facilitates learning. Concepts and skills should also be connected to everyday situations and other curricular areas. Students can also make connections to make mathematics come alive through contextual and multidisciplinary problems.

There are classroom strategies with suggestions of how to use the strategy in a mathematical context that support teachers as they develop students' mathematical vocabulary, initiate effective ways to maneuver informational text, and encourage students to reflect on what they have learned.

When teachers use these strategies in the instructional process or embedded in assessment tasks, the expectations for students must be made explicit. The students' understanding of the mathematics involved and maintaining the integrity of the curriculum are still the foremost concerns.

Please refer to Appendix B for some strategies with illustrative examples.

D. What are the most appropriate methods and activities for assessing student learning?

The following questions from different mathematics strands will be used to represent some of the appropriate methods and activities for assessing student learning.

Encourage students to share their thinking, their strategies, and their solutions. Rich discussion fosters the development of alternative problem-solving strategies.

Examples:

1. George, Alex and Pauline have $96 altogether.
If George gives five dollars to Alex and four dollars to Pauline, each person then has the same number of dollars.

How many dollars did George start with?

- ○ 9
- ○ 32
- ○ 41
- ○ 87

2. A fruit basket contains apples, oranges and pears. The ratio of apples to oranges is 1:3 and the ratio of oranges to pears is 4:3.

What is the ratio of apples to pears?

- ○ 3: 4
- ○ 1: 4
- ○ 4: 9
- ○ 1:12

3. Pierre buys a blueberry pie and eats one-half of it the first day.
On the second day, he eats one-third of the remaining part.
On the third day, he eats one-half of the remaining part.

What fraction of the original pie is still uneaten?

- ○ $\frac{1}{6}$
- ○ $\frac{1}{4}$
- ○ $\frac{1}{3}$
- ○ $\frac{1}{2}$

4. Kim travelled six kilometres in the following manner:
- She ran the first two kilometres at 10 km/hour.
- She biked the next two kilometres at 12 km/hour.
- She drove the final two kilometres at 60 km/hour.

How many minutes did it take Kim to travel the six kilometres?

- ○ 2 minutes
- ○ 10 minutes
- ○ 12 minutes
- ○ 24 minutes

5. 12 of the students of Mrs. LeBlanc's class like to play soccer, 8 students like to play tennis and 5 like to both play soccer and tennis.

How many students are there in Mrs. LeBlanc's class?

- ○ 13
- ○ 15
- ○ 20
- ○ 25

6. Robert has 128 marbles in a jar. Each time he puts his hand into the jar, he removes half of the marbles that are in the jar.

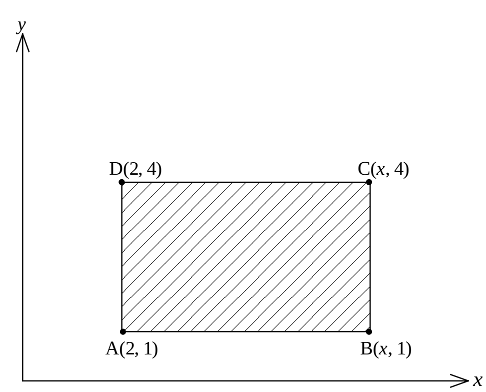
How many times must he put his hand in the jar and remove marbles from it so that exactly 1 marble remains in the jar?

- ○ 4
- ○ 5
- ○ 6
- ○ 7

7. David draws a rectangle ABCD in a Cartesian plane where the cm is the unit of measurement. The area of the rectangle is 30 cm^2. The coordinates of the vertices A, B, C and D are given in the diagram at the right.

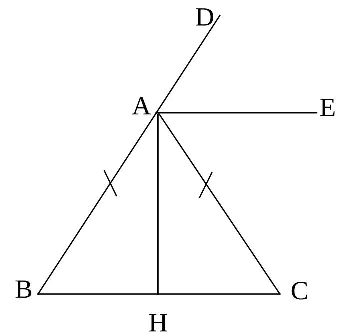
What is the value of x?

- ○ 8
- ○ 10
- ○ 12
- ○ 15



8. The isosceles triangle ABC has: AB = AC and $\angle ABC = 40^o$.

We extend BA till D. AH is the angular bisector of $\angle BAC$.
AE is the angular bisector of $\angle CAD$. (The diagram is not to scale.)



Which of the following statements is **not** true?

- ○ AH is the perpendicular bisector of BC.
- ○ AE is parallel to BC.
- ○ AE is perpendicular to AH.
- ○ $\angle BAC = 60^o$.

9. The students of the School des Voyageurs have sold 180 lottery tickets. Nancy bought a certain number of tickets. A winning ticket will be drawn randomly. The probability of drawing one of Nancy's tickets is $\frac{1}{20}$.

How many tickets did Nancy buy?

- ○ 1 ticket
- ○ 9 tickets
- ○ 20 tickets
- ○ 36 tickets

10. Suppose you roll two identical number cubes labelled 1 to 6, and you toss a coin.

How many possible outcomes are there?

- ○ 14
- ○ 24
- ○ 38
- ○ 72

Mathematics in Grade 8 – Lesson Learned 2
Number

It is important to acknowledge that numbers are a fundamental strand in the study of many other domains. The work that students do to determine the general term of a given pattern is dependent on their ability to operate with numbers. Many algebra concepts that students learn in middle school and early high school are essentially generalized arithmetic. For example, consider the numbers and the operations involved in solving the equation $2x + (-4x) = -8$. Numbers must be meaningful for students to make sense of measurement and to interpret numerical data in statistics and probability.

Fractions, decimal numbers, percentages, ratios, and proportions play an essential role in the learning and the use of mathematics in daily life. Students are constantly bombarded by numbers, at the shopping mall, in the supermarket, on television, on the radio, in newspapers and magazines, in computers games, etc. For that purpose, they need a certain competency to effectively manipulate numbers to be able to estimate quantities, evaluate announcements and advertisements, make an informed decision when making a purchase, etc.

Students must be exposed to a variety of representations of fractions, decimal numbers and percentages to build a permanent mental image of these numbers, to be capable to establish links between these representations and strengthen their understanding. The use of models to organize, record, and communicate mathematical ideas facilitates the representations. Using concrete materials, diagrams, and symbols, the models show the mathematical ideas.

A. What conclusions can be drawn from the Nova Scotia Assessment: Mathematics in Grade 8?

A meticulous examination of the assessment reveals that students encountered difficulty understanding and applying proportional reasoning with numerical reasoning. The pillar of proportional reasoning is multiplicative reasoning which exists between quantities that appear in the form of fractions, decimal numbers, percentages, ratios, and proportions. On the other hand, numerical reasoning is based on additive reasoning which we find among quantities that appear in the form of whole numbers and integers. The examples below give an idea of the difference between these two types of reasoning in the strand of number.

Example 1

The mass of a kitten A is 300 g and the mass of a kitten B, of the same age, is 500 g. At the end of two weeks, the mass of A increases to 600 g and the mass of B increases to 800 g. Which kitten grew the most?

The students' answers are divided into two categories. Some students say that it is kitten A and others say that it is kitten B. The students who say that it is kitten B, think about the numbers in an absolute frame. They compare the number 800 g with the number 500 g, in spite of the same number of grams, 200 g, which was added to the mass of each kitten. These students used additive reasoning. Why do other students say that it is kitten A rather than B that grew the most? These students consider the numbers within the framework of multiplicative or proportional reasoning. They determine that the mass of kitten A increased by approximately 67%, whereas that of B increased by 33.3%.

Example 2
The perimeter of a square is 32 cm. If the length of its sides is doubled, what is the perimeter?

Students who use additive reasoning solve this question as follows: They calculate the length of the side, 8 cm. They double this length, 16 cm. Then, they calculate the new perimeter, 64 cm.
Students who use multiplicative reasoning see that the perimeter is proportional to the length of the side. Thus, if this length doubles, the perimeter also doubles and it becomes equal to 32 cm x 2 = 64 cm.

From a young age, students begin to apply the strategies of additive reasoning. As early as grade 3, they learn that multiplication is repeated addition, subtraction is the opposite operation of addition and division is repeated subtraction or the inverse operation of multiplication. Up until the end of grade 8, students study mathematical concepts related to numbers such as fractions, decimal numbers, percentages, ratios, rates, proportions and linear relationships which are the cornerstones of multiplicative reasoning. Nevertheless, they face gaps in the acquisition of proportional reasoning, which gives them the capacity to analyze several ideas or quantities at the same time. It is essential to help students move from additive reasoning to multiplicative reasoning, from an early age.

Overall, most students are having issues when translating between and among representations of fractions, decimal numbers, and percentages. However, many students show good understanding of basic facts and procedures, but they struggle with computation of percentage increase and percentage decrease. They also struggle with translating from fractional percentages to decimal numbers, particularly when the percentage is between 0 and 1.

B. Do students have any misconceptions or errors in their thinking?

Teaching becomes more effective when misconceptions are identified, challenged and corrected. To deal with misconceptions, get students to explain how they came to their answers or rules. If there is a misconception, challenge it and contrast it with correct mathematical conception, solid math methodology, and good teaching.

Examples of some misconceptions:

- When multiplying fractions the numerators are multiplied as are the denominators. Students misapply this concept to the addition of fractions. In the following example, the student transfers the algorithm for multiplying two fractions to adding two fractions: $\frac{2}{7}+\frac{3}{5}=\frac{5}{12}$ instead of $\frac{31}{35}$.
  In this example, the error made is due to a misconception that is a result of inappropriately applying a different rule the student has been taught about fractions.

- Another common misconception is noticed when ordering fractions. Some students have this misconception that the larger the denominator the smaller the fraction. They do not consider the role of the numerator when comparing fractions.
  This misconception leads to the following error: $\frac{6}{7} < \frac{1}{2}$ instead of $\frac{6}{7} > \frac{1}{2}$.

- The most common misconception is perhaps the order of operations. Students often misuse the distributive rule. Here is an example $a - 3(b - c) = a - 3b - 3c$. The number –3 is not correctly distributed.

The student results of the 2017–2018 Nova Scotia Assessment Mathematics in Grade 8 shows the following data:

- 56% of the students made errors when adding fractions.
- 57% of students were not able to compare and correctly order whole numbers with square roots when they are in the same set.
- 57% of the students did not know how to express a given mixed number in percent form.
- 59% of the students had difficulty using combined percentages in problem-solving situations.
- 61% of students made errors when dividing two fractions.
- 62% of the students were not able to correctly use the distributive property to determine the value of a given numerical expression.
- 67% of students did not understand how to use the concept of proportion to determine the actual distance between two cities using the scale of a road map.
- 70% of students encountered difficulty when they were asked to solve problems involving fractions, whole numbers and operations.

C. What are the next steps in instruction for the class and for individual students?

Teachers should be aware that students construct meaning internally by accommodating new concepts within their existing mental framework and some of them are known to misapply algorithms and rules in domains where they are inapplicable.

Next steps in instruction should provide opportunities for students to translate among fractions, decimal numbers and percentages to solve real world problems, and to communicate mathematical ideas. Our goal is to help our students develop the ability to work flexibly and easily with numbers, not the ability to repeatedly perform the same procedure. Students should be given opportunities to develop the ability to transition from the different representations of numbers with ease in order to build up a computational fluency with fractions, decimal numbers, and percentages.

Flexibility with a variety of computational strategies is an important tool for successful daily living. It is time to broaden our perspective of what it means to compute. There are strategies that provide opportunities to support students' use of numbers in context, such as:

- engage students in discussion about all possible connections among fractions, decimals, and percentages
- ask students challenging questions
- ask students to show more than one way of translating among fractions, decimal numbers and percentages
- provide students many learning opportunities to develop their proportional reasoning
- encourage students to communicate their ideas orally and in writing

D. What are the most appropriate methods and activities for assessing student learning?

The following questions using numbers and operations will be used to represent some of the appropriate methods and activities for assessing student learning.

Encourage students to share their thinking, their strategies, and their solutions. Rich discussion fosters the development of alternative problem-solving strategies.

Examples:

1. I am an odd number, situated between 12^2 and 13^2.
I contain the digit 5.
I am divisible by 3.

Who I am?

- ○ 145
- ○ 152
- ○ 157
- ○ 159

2. Evaluate: $\frac{[9-(-3)](-2)}{(-4)(-3)}$

- ○ 1
- ○ 2
- ○ −1
- ○ −2

3. What whole number is equal to $(1\times 2)(\frac{1}{1}-\frac{1}{2})+(2\times 3)(\frac{1}{2}-\frac{1}{3})+(3\times 4)(\frac{1}{3}-\frac{1}{4})$?

- ○ 1
- ○ 2
- ○ 3
- ○ 4

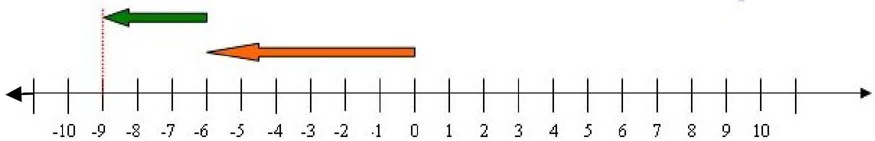
4. Which of the following numbers is divisible both by 3 and by 4?

- ○ 726
- ○ 462
- ○ 216
- ○ 126

5. Which decimal is equal to 0.65%?

- ○ 6.5
- ○ 0.65
- ○ 0.065
- ○ 0.0065

6. Which operation does the following number line represent?



- ○ (–6) + (– 9)
- ○ (–6) – (– 9)
- ○ (–6) – (– 3)
- ○ (–6) + (– 3)

7. Which percent is equal to $2\frac{3}{4}$?

- ○ 2.34%
- ○ 2.75%
- ○ 0.275%
- ○ 275%

8. Find the value of the following expression: $4\frac{1}{3} - 2\frac{1}{2} \times 1\frac{1}{3}$.

- ○ 1
- ○ $1\frac{5}{6}$
- ○ $2\frac{4}{9}$
- ○ $10\frac{2}{3}$

9. At a local bank, the interest rate on a savings account is 1.25% per year.
John deposits $340.00 in the bank, on January 1, 2016.

What is the interest earned at the end of December 2016?

- ○ $425.00
- ○ $374.85
- ○ $34.85
- ○ $4.25

10. In July 2016, the regular price of a computer was $449.00.
In September 2016, it was $399.00.

What was the percentage decrease in the price of the computer?

- O 50%
- O 12.5%
- O 11.1%
- O 1.25%

11. A musician has $8\frac{3}{4}$ m of piano wire. She cuts the wire into $\frac{2}{3}$ m pieces.

How much wire does the musician have left?

- O $\frac{1}{12}$ m
- O $\frac{2}{3}$ m
- O $\frac{3}{4}$ m
- O $\frac{5}{6}$ m

12. The price of a bike, in a clearance store, went through two reductions.
The regular price was $200. The bike was first reduced by 20%.
One week later, the bike was reduced by a further 10%.

What is the total percentage decrease in price of this bike?

- O 26%
- O 28%
- O 30%
- O 39%

13. In 1999, the population of a city was 1500.
In 2004, its population became 1800.

What was the percentage increase in population of this city from 1999 to 2004?

- O 12%
- O 15%
- O 20%
- O 30%

14. Paul needs 6 hours to paint his room.
To do the same job alone, his brother Lee needs 4 hours.
To do the same job alone, his sister Mona needs 3 hours.

If Paul, Lee and Mona together paint the room, how many minutes do they need?

- ○ 13 minutes
- ○ 45 minutes
- ○ 80 minutes
- ○ 540 minutes

15. The scale on a road map is 1:5 000 000.
This means that 1 cm on the map represents 5 000 000 cm of actual distance.
The actual straight line distance between two cities is 250 km.

What is the map distance in centimeters between these two cities?

- ○ 0.5 cm
- ○ 5 cm
- ○ 20 cm
- ○ 200 cm

Mathematics in Grade 8 – Lesson Learned 3
Patterns and Relations

Patterns and relations are part of all strands of the mathematics curriculum from primary to grade 9. The study of patterns and relations aims to develop algebraic reasoning which requires the intersection of several abstract and procedural skills such as generalization, problem-solving, communication, use of models and representation of real situations using symbols as well as analysis of change. We must be aware that mathematics is not an arbitrary construction. It arises from the evolution of our brain in a world which has interesting patterns, number patterns, space patterns, time patterns, etc. The human brain tends to internalize these patterns with the aim of strengthening its capacities of analysis and synthesis to reconcile with natural, societal and environmental phenomena.

Grade 8 students should demonstrate that they are able of representing mentally a concrete situation and that they are capable of generalizing by drawing conclusions from the observation and interpretation of simple examples. They should demonstrate that they are capable of working with symbols which represent unknowns to translate a real situation in the form of an algebraic equation which allows them to argue, to infer, and to produce new information from existing data. Students should develop fluency in moving from one representation – concrete, table of values, graph, algebraic equation… – to another. Students must learn to recognize, extend, create, and use mathematical patterns. Patterns allow students to make predictions and justify their reasoning when solving problems. Learning to work with patterns in the middle school grades helps develop students' algebraic reasoning, which is foundational for working with more abstract mathematics in secondary grades.

A. What conclusions can be drawn from the Nova Scotia Assessment: Mathematics in Grade 8?

We noticed that students could successfully solve simple equations. Students also performed well on patterns and relations questions (knowledge and application) that required them to read graphs and tables, to match a table of values or a graph to a linear equation, and to model an equation using algebra tiles. However, our assessment information also shows that many students experienced a challenge in completing tasks that require problem solving (analysis questions). They were challenged when asked to apply their knowledge to find the expression of the n^{th} term of a pattern. They also struggled with translating between a pattern described contextually, pictorially, or verbally. Students need to progress from recursive thinking to functional thinking to be able to determine the n^{th} term.

When students are working with patterns and attempting to determine the general term for an algebraic expression, it is very important for students to understand how to logically use the given information to determine this expression and not just rely on the use of trial and error.

B. Do students have any misconceptions or errors in their thinking?

Student misconceptions in conceptual knowledge of patterns and relations lead to incorrectly solving problems involving the determination of the n^{th} term of a given pattern. Without adequate knowledge of the problem features, the student is unable to distinguish between the situations in which the strategy will work and the ones it is not applicable.

Transitioning from numbers and operations to patterns and relations (from arithmetic reasoning to algebraic reasoning) is often one of the key stumbling blocks preventing students from learning patterns and relations deeply in middle school. Teachers should take into consideration that students may attempt to incorporate new information into their current knowledge base without sufficient understanding to successfully bridge the ideas.

Misapplying the step by step rule, to analyze the table of values or the consecutive terms of a given pattern when students try to determine the algebraic equation of a given pattern, is the most serious misconception for grade 8 students. Is this because students do not have the preliminary knowledge necessary to understand this concept or because the problem situation is too complex to execute a nevertheless familiar task? Is this misconception a cause of misunderstanding of patterns and relations or a consequence?

Examples of some misconceptions:

- The table of values below shows an electrician's earnings, which are made up of a fixed rate and a variable rate.

Time *t* (h)	**Earnings *e* ($)**
1	65
2	90
3	115
4	140

  The table shows that the variation of the earnings is constant, $25/h, because when the time increases by 1 h, the earnings increase by $25. Then the relation between *e* and *t* is a linear relationship. Its equation is $e = mt + b$.
  Most students misapply the recursive thinking to calculate m ($25/h) and b ($40).
  The equation that describes this relation is $e = 25t + 40$.

- Students misapply dividing all terms. They divide only the coefficients of *x* by 2.
  Example: $2x – 4 = 6x$
  $x – 4 = 3x$
  $x = –2$
  Instead of $x – 2 = 3x$
  $x = –1$

- How to properly use the inverse operations to solve equations is another misconception for some students. One common strategy students have for solving an equation is that they want to remove a term from the equation, they subtract it from both sides of the equation. This works fine in an equation like $3x + 5 = 11$
  $3x + 5 – 5 = 11 – 5$
  $3x = 6$
  $x = 2$
- However, when they encounter an equation like $3x – 5 = 11$, those students still try to subtract 5 from both sides instead of adding 5 to solve the equation for *x*.
  $3x – 5 – 5 = 11 – 5$
  Instead of $3x – 5 + 5 = 11 + 5$

- One explanation for this conceptual mistake is that those students may be deficient in their understanding of the preservation of equality.

The student results of the 2017–2018 Nova Scotia Assessment: Mathematics in Grade 8 shows the following data:

- 63% of the students were not able to use the distributive property to expand an algebraic expression correctly.
- 68% of the students did not know how to determine the algebraic expression of the n^{th} term of a given pattern.

C. What are the next steps in instruction for the class and for individual students?

Patterns are powerful ideas and their mathematical applications have helped to find solutions for many real-world problems. When students correctly translate among the different representations of patterns (tables, charts, diagrams, graphs, expressions, equations), we say that they have concept attainment. To succeed in this mathematical domain, students can use these various representations to mathematize real situations. To reach there, teachers should provide students learning opportunities which allow them to think algebraically and to organize their recursive thought.

Even though students have had many opportunities to work with patterns and relations, many have not moved from numerical reasoning to proportional reasoning and then to algebraic reasoning. Students need numerous experiences to develop their mathematical symbolism and their algebraic reasoning. Symbolism, using variables, equations, and linear relationships, is a powerful strategy that students can use to conceptualize ideas, to solve problems, and to support their thinking. Supports and extensions can be beneficial for all students. For any given activity, there will always be some students who require more or less support, or for whom extensions will increase interest and deepen understanding.

Students were challenged when asked to determine the expression of the n^{th} term of a pattern to enable them to find the numerical value of any term. They need to continue to work on analysis questions of pattern representations using tables, pictures, graphs and equations.

D. What are the most appropriate methods and activities for assessing student learning?

The following questions using patterns and relations will be used to represent some of the appropriate methods and activities for assessing student learning.

Encourage students to share their thinking, their strategies, and their solutions. Rich discussion fosters the development of alternative problem-solving strategies.

Examples:

1. Examine the following pattern:

What is the number of stars in the 80th term?

- ○ 240
- ○ 241
- ○ 244
- ○ 320

2. Abraham has a set of square tiles. The dimensions of the first tile is 1 cm by 1 cm.
The dimensions of the 2nd tile are 2 cm by 2 cm.
The dimensions of the 3rd tile are 3 cm by 3 cm, and so on.

Which table of values shows how the area, in cm^2, of tiles changes as the side length, in cm, increases?

○

Side (cm)	Area (cm^2)
1	2
2	4
3	6
4	8

○

Side (cm)	Area (cm^2)
1	1
2	4
3	9
4	12

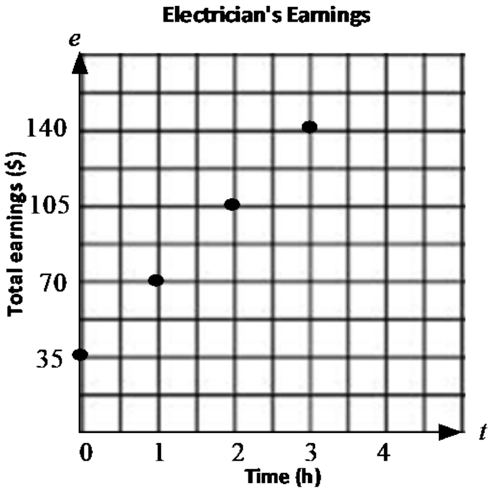
○

Side (cm)	Area (cm^2)
1	1
2	4
3	10
4	16

○

Side (cm)	Area (cm^2)
1	1
2	4
3	9
4	16

3. The graph below shows the earnings of an electrician, where t is the time worked, in hours, and e represents his earnings, in dollars.



Which linear relation represents the data shown in the graph?

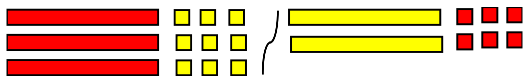
- ○ $e = 35t$
- ○ $e = 35t - 35$
- ○ $e = -35t + 35$
- ○ $e = 35t + 35$

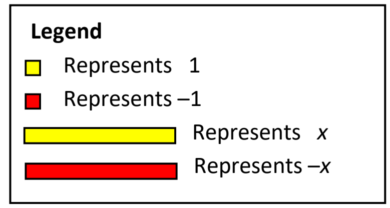
4. Robin has a rectangular flower bed.
The length of this flower bed is 50 cm more than its width x.

Which equation represents the perimeter, P, of the flower bed?

- ○ $P = 6x + 100$
- ○ $P = 3x + 50$
- ○ $P = 2x + 100$
- ○ $P = 4x + 100$

5. Nicole uses algebra tiles to model an equation as shown below.

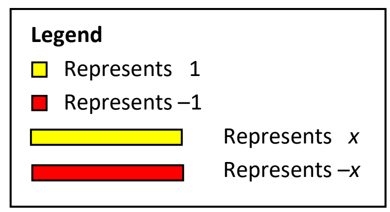


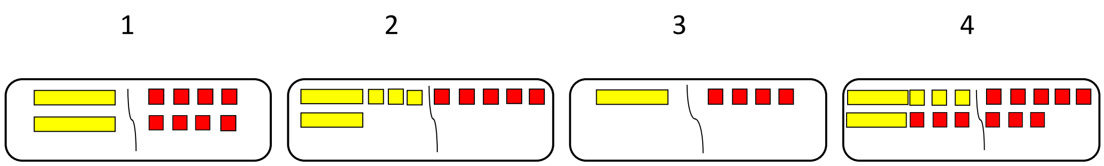


Which of the following equations describes this representation?

- ○ $3x + 9 = -2x - 6$
- ○ $-3x - 9 = 2x - 6$
- ○ $3x - 9 = 2x + 6$
- ○ $-3x + 9 = 2x - 6$

6. Nabil solved the equation $2x + 3 = -5$ correctly using algebra tiles.





Which of the following shows the order of steps Nabil used to solve the equation?

- ○ 1, 2, 3, 4
- ○ 4, 2, 3, 1
- ○ 2, 3, 1, 4
- ○ 2, 4, 1, 3

7. Evaluate the expression $-3x + 4y - 5z$, for $x = -1$, $y = -3$ and $z = -2$.

- ○ –19
- ○ 1
- ○ 4
- ○ 25

8. What is the value of *x* that satisfies the equation $3 - \frac{x}{4} = -2$?

- ○ −20
- ○ 5
- ○ 14
- ○ 20

9. Which of the following tables of values corresponds to a linear relation?

○

x	*y*
2	−2
3	0
4	4
5	6

○

x	*y*
1	1
2	4
3	9
4	16

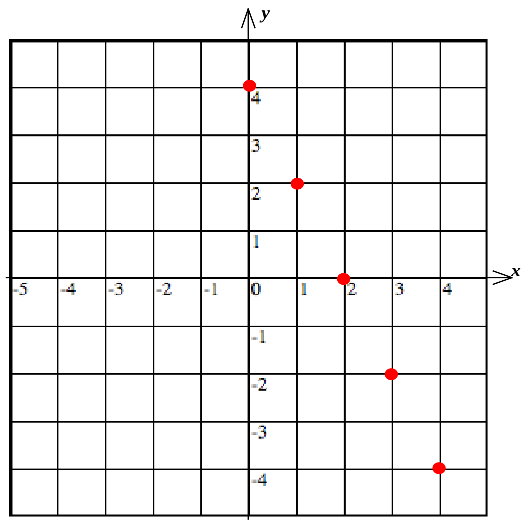
○

x	*y*
0	2
1	3
2	5
3	8

○

x	*y*
1	3
2	6
3	9
4	12

10. Examine the following graph:



Which of the following equation represents the data shown in the graph?

- ○ $y = 2x - 4$
- ○ $y = -2x - 4$
- ○ $y = -2x + 4$
- ○ $y = 2x + 4$

Mathematics in Grade 8 – Lesson Learned 4
Measurement and Geometry

“The general aim of the study of measurement and geometry is stated as making an individual acquire the mathematical knowledge needed in daily basis, teaching how to solve problems and acquiring reasoning methods. Learning geometry is not just learning the definitions or the attributes of geometrical concepts but also to have the ability of analyzing the properties of two (2D) and three dimensional (3D) geometric shapes and develop mathematical arguments about geometric relationships, to specify locations and spatial relationship, to apply transformations, visualization, spatial reasoning and geometric modeling to solve problems.” (NCTM, Principles and Standards 2000)

A true sense of measurement goes well beyond the skills of simply using a measurement tool. Mathematics is used to describe and explain relationships. As part of the study of measurement and geometry, students look for relationships among measurement strategies, geometric properties and proportional reasoning. Measurement sense can be developed by providing rich mathematical tasks that allow students to make connections between measurement and geometry. The relationship between these two mathematical strands is evident in the development of formulas. By correctly applying the formulas, students can more easily determine indirect measurements.

“As an object of study, geometry is well suited to push student’s thinking in many directions. While working with spatial objects, for example, students can reason concretely and abstractly, discuss relationships, and make logical connections.” (*NCTM*, *Mathematics teaching in the Middle School*, May 2016, p. 543)

A. What conclusions can be drawn from the Nova Scotia Assessment: Mathematics in Grade 8?

We noticed that students performed well on knowledge questions that required them to use a simple formula or to recognize a simple geometrical transformation. For example, most students were able to correctly calculate the volume of a rectangular prism knowing the values of the three dimensions. However, when students were asked to solve application and analysis questions, they were quite challenged, especially if these questions evoked the algebraic manipulation of geometrical formula. Some students had difficulty using the Pythagorean relationship to solve problems involving composite figures. Most students were challenged when asked to apply their knowledge of the properties of the angles of a triangle combined with the definition of the perpendicular bisector or the angle bisector.

The assessment analysis also showed that grade 8 students did not understand the connections between algebraic reasoning and measurement thinking and procedures (e. g. using scale to solve map problems). At times, students were not confident when they had to use proportional reasoning in measurement contexts.

B. Do students have any misconceptions or errors in their thinking?

Student misconceptions in conceptual knowledge of measurement and geometry are related to some misunderstandings and misapplying of a set of misconceptions in the domains of numbers, operations and solving equations. Often in mathematics, a misconception is a valid knowledge in a certain domain, but it is inadequate knowledge, in other domains, which prevents the implementation or the understanding of another knowledge which is adequate. For example, the area of a circle, as being a function of its radius, is valid knowledge in measurement. This knowledge becomes inadequate when it comes to applying the proportional reasoning and prevents from understanding how the area of the circle becomes quadruple or nine times as great when the length of its radius becomes respectively double or triple. It is essential to deal with misconceptions by getting students to explain how they came to their answers. If there is a misconception, challenge it or contrast it with the correct conception.

Some examples of misconceptions

- *The use of proportional reasoning and the square of a number misconception*
  The area of a square is 25 cm^2. The length of side is doubled. The new area is
  - ○ 25 cm^2
  - ○ 50 cm^2
  - ○ 75 cm^2
  - ○ 100 cm^2

  The answer of most students is 50 cm^2. May be the error is due to lack of understanding of using the proportional reasoning to solve the problem or to lack of awareness of the quadratic nature of area.
  Suggestions:
  - More exercise on this topic to help students develop the proportional reasoning
  - More exercise on area to understand its quadratic nature
  - More exercise on volume to understand its cubic nature
  - Frequent use of geometry software to implement more interactive teaching

- *Scale factor misconception*
  This misconception arises frequently when students are engaged to determine the real distance between two points using a road map. This misconception is due to misapplying the concept of proportion.
  Suggestions:
  - More warm-up before teaching about scales and proportions
  - More emphasis on properties of proportions
  - Variety of activities can be used in the classroom

- *Pythagorean relationship misconception*
  It has been noted that many students tend to have common misconceptions in their applying of Pythagorean relationship when one leg of a right-angle triangle is the unknown.
  Suggestions:
  - More variety of questions on different illustrations involving right angle triangle
  - More emphasis on solving equations related to measurement and geometry formula

- *Solving equations involving measurement misconception*
  Some students misapply the geometric formula of the surface area or the volume to determine the height of a cylinder. It seems that this misconception is due to a misunderstanding of how to solve algebraic equations.

The student results of the 2017–2018 Nova Scotia Assessment: Mathematics in Grade 8 shows the following data:

Measurement

- 53% of students made errors when calculating the volume of a triangular prism.
- 55% of the students were not able to distinguish between the surface area and the volume of a rectangular prism.
- 63% of the students were not able to correctly calculate the area of a circle inscribed in a square whose side length is given. The students did not know that the diameter of the circle is equal to the side of the circumscribed square.
- 69% of students showed a misunderstanding of the concept of the perimeter when they were asked to calculate the length of the fence of a yard.
- 77% of students encountered difficulty when calculating the surface area of a cylinder using a given formula.

Geometry

- 56% of students were not able to identify parallel or perpendicular edges of a right rectangular prism.
- 62% of the students mistakenly identified the quadrant of a Cartesian plane where the final image of a given point should be situated after undergoing two consecutive transformations.
- 66% of the students had difficulty determining the coordinates of the final image of a point that undergoes two consecutive transformations.
- 67% of the students had difficulty determining the coordinates of the vertices of the final image of a shape that undergoes a single transformation.

C. What are the next steps in instruction for the class and for individual students?

Teachers must be aware of the importance of the context in the study and the conceptual understanding of measurement, 2D shapes and 3D objects, and transformations.

First, teachers should make sure that students understand that multiplicative relationships are the core of proportional reasoning. The next step in instruction is the appropriate use of manipulative and concrete material to model the concepts of area, volume and capacity. The final step is making sure that students are exposed to and can correctly solve geometric formulas as it helps to determine the right measurement. Grade 8 students need to know that every indirect measurement can be determined by using a formula. To calculate the perimeter or the area of 2D geometric shapes, they should correctly use the appropriate formula. Also, to calculate the surface area or the volume of 3D objects, there is a given formula for each one.

Technology enables both students and teachers to access wide range of tools to use in mathematics. *The Geometer's Sketchpad, Autograph and GeoGebra* are dynamic geometry software. They can be used for instructing basic geometric figures to maximize student learning in measurement and geometry. They also enable students to examine mathematical ideas on their own, allowing them not to be just consumers of technology but producers of knowledge through technology. The use of technology depends on the teacher, and the teacher's role in making curriculum decisions is very important. Recognizing that today's students are using technology outside of the classroom and the study of mathematics should keep pace, teachers need to take advantage of technology for teaching and learning.

D. What are the most appropriate methods and activities for assessing student learning?

The following questions using measurement and geometry will be used to represent some of the appropriate methods and activities for assessing student learning.

Encourage students to share their thinking, their strategies, and their solutions. Rich discussion fosters the development of alternative problem-solving strategies.

Examples

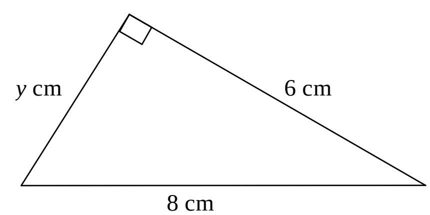
1. Which of the following sets of numbers below are **not** a Pythagorean triple?

- ○ 3, 4, 5
- ○ 5, 12, 13
- ○ 12, 16, 20
- ○ 9, 30, 35

2. The diagram at the right shows a right-angle triangle.

Which of the following equations helps to determine the length *y*?

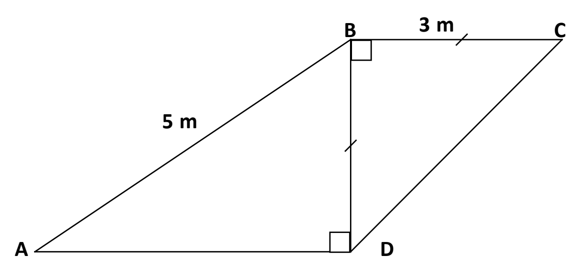
- ○ $y = 36 + 64$
- ○ $y = \sqrt{6+8}$
- ○ $y^2 = 6+8$
- ○ $y^2 = 64-36$



3. Louis plans to build a deck for his brother as shown.
BCD is a right isosceles triangle.

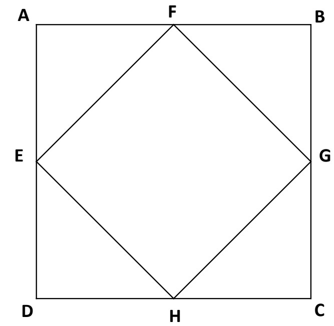
What is the perimeter of the deck?

- ○ 15 m
- ○ 16.2 m
- ○ 19.2 m
- ○ 30 m



4. EFGH is a square inside the square ABCD.
E, F, G and H are the midpoints of DA, AB, BC and CD respectively.

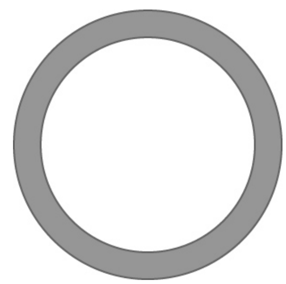
What is the area of square EFGH, if AB = 12 cm?



- ○ 8.2 cm^2
- ○ 24 cm^2
- ○ 72 cm^2
- ○ 144 cm^2

5. In the diagram shown, two circles have the same center.
The radius of the outer circle is 10 cm and the radius of the inner circle is 8 cm.

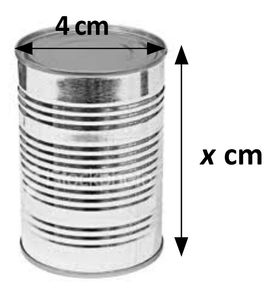
What is the area of the shaded region, to the nearest cm^2?
($\pi = 3.14$)



- ○ 20 cm^2
- ○ 13 cm^2
- ○ 36 cm^2
- ○ 113 cm^2

6. The surface area of this cylindrical can is 125.6 cm^2.
The diameter of its base is 4 cm and its height is *x* cm.

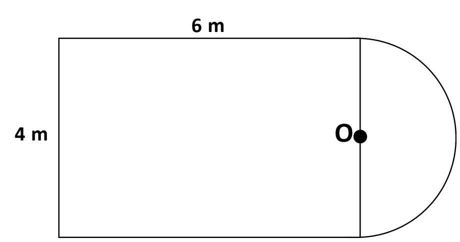
What is the value of *x*? ($\pi = 3.14$)



- ○ *x* = 5
- ○ *x* = 8
- ○ *x* = 10
- ○ *x* = 31.4

7. Talia bought a carpet in the shape shown below.
O is the center of the semi-circle.

What is the area of the carpet to the nearest tenth of m^2? ($\pi = 3.14$)



- ○ 26.3 m^2
- ○ 30.3 m^2
- ○ 36.6 m^2
- ○ 48.8 m^2

8. Nicole uses a road map that has a scale of 1:200 000, where 1 cm on the map represents an actual distance of 200 000 cm.

What is the actual distance in kilometres between two cities that are 5 cm apart on the map?

- O 10 km
- O 100 km
- O 200 km
- O 1000 km

9. A cylindrical can has a radius of 3 cm and a height of 10 cm.

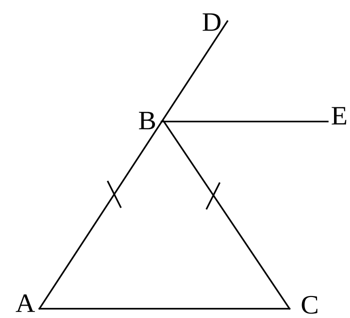
If the radius is doubled and the height is halved, what will happen to the volume of the can?

- O The volume will be doubled.
- O The volume will be four times as great.
- O The volume will be halved.
- O The volume does not change

10. The isosceles triangle ABC has $\angle BAC = \angle BCA = 40^\circ$ and BE is the angle bisector of $\angle DBC$. (The diagram is not to scale)

What is the measurement of $\angle DBE$?

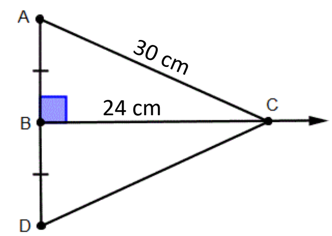
- O 40°
- O 80°
- O 100°
- O 180°



11. Which statement is the best definition of the perpendicular bisector of a line segment?

- O It divides the line segment into two parts.
- O It is exactly at the right angles to the line segment.
- O It is perpendicular to the line segment in one of its two extremities.
- O It is perpendicular to the line segment and passes through its midpoint.

12. In the diagram at the right, BC is the perpendicular bisector of the line segment AD.

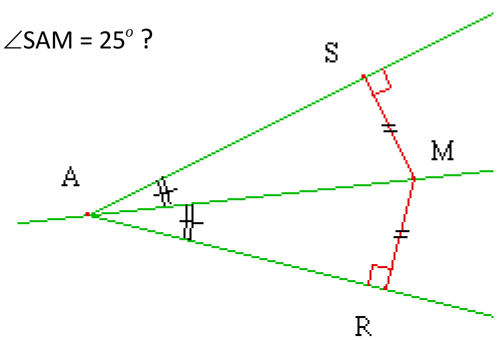


What is the length of the line segment AD?

- ○ 12 cm
- ○ 18 cm
- ○ 36 cm
- ○ 54 cm

13. In the diagram below, the line AM is the angle bisector of $\angle SAR$, and MS = MR.

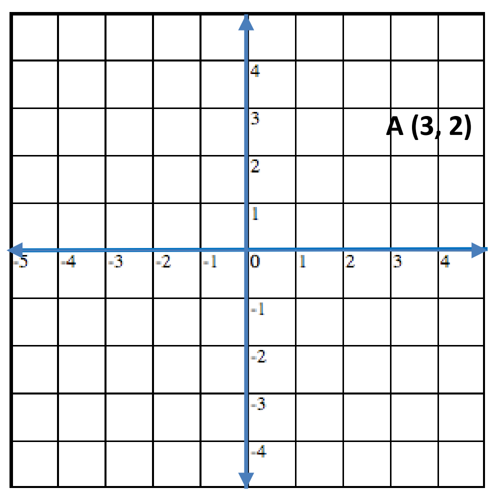
What is the measurement of $\angle SMR$ if $\angle SAM = 25^o$?



- ○ 50°
- ○ 65°
- ○ 115°
- ○ 130°

14. Theo plots the point A (3, 2) on the Cartesian plane below.
Sherene rotates point A 90° clockwise about the origin O.
Let B be the image of A.

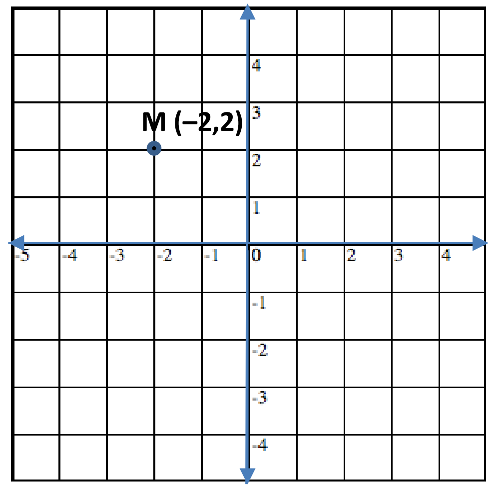
What are the coordinates of B?



- ○ B (–3, 2)
- ○ B (3, –2)
- ○ B (–3, –2)
- ○ B (2, –3)

15. Let M (–2, 2) be a point on a Cartesian plane.
N is the image of M obtained by reflection in the x-axis.
N undergoes a rotation of 90° clockwise about the point P (0, –2).
Q is the image of N after this rotation.

What are the coordinates of Q?



- ○ Q (–2, –2)
- ○ Q (–2, 0)
- ○ Q (0, 2)
- ○ Q (0, 0)

Mathematics in Grade 8 – Lesson Learned 5
Statistics and Probability

Data analysis is not only about calculating statistics and constructing graphs and diagrams. It is a process of inspecting, transforming, and modeling data with the goal of discovering useful information, suggesting conclusions, and supporting decision-making. It includes collecting, organizing, displaying, and analyzing data. Probability is a branch of mathematics that deals with calculating the likelihood of a given events occurrence. Probability theory had its start in the 17th century, when two French mathematicians, Blaise Pascal and Pierre de Fermat carried on a correspondence discussing mathematical problems dealing with games of chance. Contemporary applications of probability theory run the gamut of human inquiry, and include aspects of computer programming, astrophysics, music, weather prediction, and medicine.
(Margaret Rouse, http://whatis.techtarget.com/definition/probability)

In grade 8, students are becoming more aware of and interested in the real-world life. By studying data analysis and probability in contexts provided by other subject areas, such as social studies, science and physical education, they can connect with the real-world.

Probability is prevalent in our real-world life. It is important for Grade 8 students to explore probability situations through a variety of authentic experiments in a variety of contexts using a variety of methods of analysis. Grade 8 students should know that theoretical probability is based on the logical analysis of the situation whereas experimental probability is generated through experiments and data collection.

A. What conclusions can be drawn from the Nova Scotia Assessment: Mathematics in Grade 8?

We noticed that students performed well on application questions that required them to use a scatterplot with the line of the best fit, to ask questions for collecting data, to calculate the theoretical probability of an event by determining correctly the number of favorable outcomes and the number of possible outcomes. For example, most students could correctly calculate the theoretical probability of randomly selecting a marble from a bag.

However, students were quite challenged when they were asked to solve some application and analysis questions about interpreting circle graphs, investigating the effect of data variation on the mean, and calculating the experimental probability of complementary events. 64% of students had difficulty in linking the sector angles of a circle graph to percentages. Most students struggled with determining the mean when the number of data changes (analysis question). The assessment analysis also showed that Grade 8 students did not understand how to determine the probability of complementary events, and how to apply the concept of experimental probability.

B. Do students have any misconceptions or errors in their thinking?

What is a misconception? It is not simply a mistake. Mistakes can be made for various reasons, for example, of student's carelessness. A misconception is a student's erroneous concept that produces a systematic pattern of errors. A misconception identified in data management and probability, if not addressed, create persistent difficulties for student mathematics learning.

A major misconception in probability arises when two identical number cubes are used, students often do not understand that a 3 on one cube and a 5 on the other cube is a different roll than a 5 on the first cube and a 3 on the second. The analysis of the assessment results shows that many students misunderstand how to compute the probability of independent events and the impacts of the words **AND** and **OR** on the determination of probability.

When solving data analysis problems, many students performed much better when the question was simple and clear. It has been noted that students tend to have a common misconception in their understanding of the circle graph. This misconception is due to misunderstanding the relationship between a central angle in a sector and the percentage of the circle that sector occupied. Misapplying the formula of the mean is another misconception in data management. It is rooted in a misconception of numbers, operations, and equations.

The student results of the 2017–2018 Nova Scotia Assessment: Mathematics in Grade 8 shows the following data:

- 53% of the students made errors when determining the median of a given set of data. They had difficulty distinguishing between median, mode and mean.
- 58% of the students encountered difficulty when they were asked to determine the probability of independent events involving "or".
- 60% of the students were not able to identify the best circle graph that represents given data.
- 60% of the students did not know how to manipulate the formula of the mean in problem-solving situations.
- 61% of the students were not able to determine the range of a given set of data. They had difficulty distinguishing between range, mode and median.
- 62% of the students encountered difficulty when they were asked to determine the probability of independent events involving "and".

C. What are the next steps in instruction for the class and for individual students?

Students study the circle and the circle graph in Grade 7. In Grade 8, they should review this graph to remember the following points:

- In a circle graph, data are shown as parts of a whole.
- Each sector of a circle graph represents a percent of the whole circle.
- The whole circle represents 100%. The measurement of its central angle is 360^o.
- Each sector is labelled with a category and a percent.
- A circle graph compares the number in each category to the total number.
- A circle graph has a title and sometimes a legend that shows what category each sector represents.
-

A suggested strategy for dealing with the creation of a circle graph is to follow a systematic approach to creating circle graphs:

- Add the numbers of each category to form the total or whole.
- Divide each of the numbers in the categories by the whole by using a calculator (decimals between 0 and 1) and round to hundredths, to become percentage.
- Multiply each decimal value by 360 (The whole or 100% corresponds to 360^o) to find the size of the sector central angle.

Note: a hundredths disk helps students easily make a circle graph rather than getting bogged down with protractors and degrees.

A suggested strategy for dealing with the misconception of experimental probability is to begin probability explorations with experiments rather than theoretical analysis. The experiments provide the opportunity for authentic assessment for learning of students' understanding. During the experiments, you can ask students questions such as:

- What does it mean that an event has a probability of 89 percent or a probability of $\frac{2}{3}$?
- How many times do you think you should conduct the experiment before you feel confident?
- What should you do if your results don't make sense to you?
- If you know the probability of an event, can you predict the result of the next trial?
- What does "confidence in your results" mean?

(*Teaching Student-Centered Mathematics 5–8*, Van De Walle and Lovin 2006)

Area models and tree diagrams are useful and efficient tools to solve probability problems.

D. What are the most appropriate methods and activities for assessing student learning?

The following questions using statistics and probability will be used to represent some of the appropriate methods and activities for assessing student learning.

Encourage students to share their thinking, their strategies, and their solutions. Rich discussion fosters the development of alternative problem-solving strategies.

Examples

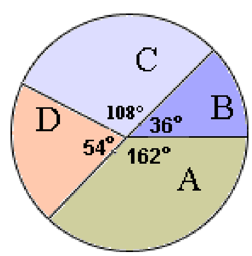
1. Noah was training for a 200–m race. His times, in seconds, for the first seven races were: 60, 59, 59, 69, 62, 58, 61.

Find the median time and the range.

- ○ Median = 59, Range = 69
- ○ Median = 69, Range = 11
- ○ Median = 60, Range = 1
- ○ Median = 60, Range = 11

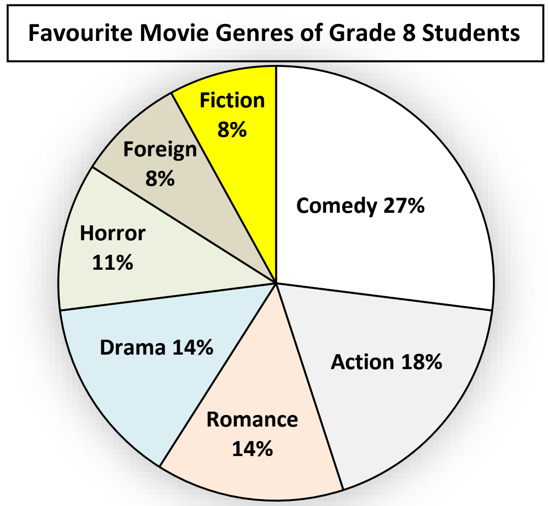
2. What percentage of the circle is covered by sectors B and D?

- ○ 25%
- ○ 30%
- ○ 35%
- ○ 40%



3. The circle graph at the right shows the favourite movie genres of Grade 8 students.

What is the size of the sector angle representing comedy movies to the nearest degree?



- ○ 90°
- ○ 85°
- ○ 91°
- ○ 97°

4. Hanna has the following marks:
Science – 74%, French – 76%, English – 86%, Art – 80%.

What mark will Hanna need in math if she wants her mean mark in these 5 subjects to be 75%?

- ○ 59
- ○ 63.2
- ○ 72
- ○ 75

5. Some bar graphs display data in a misleading way.

Which of the following statements is a characteristic of a misleading graph?

- ○ The two axes start at 0.
- ○ The vertical axis starts at 0.
- ○ The horizontal axis starts at 0.
- ○ The vertical axis does not start at 0.

6. Emma rolls a number cube, numbered from 1 to 6.

What is the theoretical probability of rolling a 2 or a 4?

- ○ $\frac{1}{6}$
- ○ $\frac{1}{3}$
- ○ $\frac{1}{2}$
- ○ $\frac{2}{3}$

7. If Emma rolls two 6-sided number cubes, numbered from 1 to 6, 72 times, how many times should she expect to roll a 2 and a 4?

- ○ 2
- ○ 18
- ○ 36
- ○ 48

8. In a pet store, there are 5 kittens, 7 puppies, 3 rabbits and 10 birds.

If a pet is chosen at random, what is the probability of choosing a kitten or a bird?

- ○ $\frac{2}{25}$
- ○ $\frac{5}{25}$
- ○ $\frac{3}{10}$
- ○ $\frac{3}{5}$

9. Suppose that you roll two 6 sided-number cubes, numbered from 1 to 6, and you toss a coin.

What is the number of possible outcomes?

- ○ 14
- ○ 24
- ○ 38
- ○ 72

10. On a day in September, there is a 75% probability of rain in Sydney, a 50% probability of rain in Halifax, and a 40% probability of rain in Yarmouth.

What is the probability that it will rain in all 3 cities on that day?

- ○ 15%
- ○ 16.5%
- ○ 84%
- ○ 165%

11. A bag contains 8 white marbles, 5 red marbles, and 7 blue marbles.
Andrea removes 1 marble from the bag without looking, records its color, then replaces it in the bag.
Andrea does this action 3 more times.

What is the probability that the first two marbles are red and the third marble is white?

- ○ $\frac{16}{684}$
- ○ $\frac{2}{80}$
- ○ $\frac{1}{10}$
- ○ $\frac{9}{10}$

12. Albert rolls a regular 6 sided-number cube labelled 1 to 6 and tosses a coin.

What is the probability of rolling a 4 and tossing a head?

- d $\frac{1}{12}$
- d $\frac{1}{8}$
- d $\frac{1}{6}$
- d $\frac{1}{2}$

References

Baron & Heideima, (2002). *Teaching Reading in the Content Area*. (Supplement), Mcrel.

Black, Harrison, Lee, Marshall & William, (2003). *Assessment for learning: Putting it into practice*. Buckingham, UK: Open University Press.

Black, P. & Wiliam, D. (2006). *Assessment and Classroom Learning*, OECD.

Davies, A. (2009). *Assessment for Learning.* Courtenay, BC: Connections Publishing.

Department of Education and Early Childhood Development (EECD), Province of Nova Scotia (2014a). *Mathematics 4 curriculum guide, Implementation draft*. Halifax, NS: Author.

Department of Education and Early Childhood Development (EECD), Province of Nova Scotia (2014b). *Mathematics 5 curriculum guide, Implementation draft*. Halifax, NS: Author.

Department of Education and Early Childhood Development (EECD), Province of Nova Scotia (2014). *Mathematics 6 Curriculum Guide, Implementation Draft*. Halifax, NS.

Department of Education and Early Childhood Development (EECD), Province of Nova Scotia (2015). *Mathematics 7 Curriculum Guide, Implementation Draft*. Halifax, NS.

Department of Education and Early Childhood Development (EECD), Province of Nova Scotia (2015). *Mathematics 8 Curriculum Guide, Implementation Draft*. Halifax, NS.

Hiebert and Collab, (1996), *Problem Solving as a Basis Reform in Curriculum and Instruction,* Vol 25, Issue 4, 1996.

(Margaret Rouse, http://whatis.techtarget.com/definition/probability)

Marian Small, (2010). *More Good Questions: Great Ways to differentiate Mathematics Instruction,* Colombia University.

Marian Small, (2010). *Prime: Number and Operations, background and Strategies,* Toronto, On: Nelson Education Ltd.

Marian Small, (2009). *Making mathematics meaningful to Canadian Students, K–8*. Toronto, On: Nelson Education Ltd.

National Council of Teachers of Mathematics (NCTM) (2000). *Principles and Standards for School Mathematics*. Reston, VA: NCTM.

National Council of Teachers of Mathematics (NCTM) (2016). *Mathematics Teaching in the MIDDLE SCHOOL* Vol. 21, No. 9, May 2016.

National Council of Teachers of Mathematics (NCTM). (2010, August). “Star students make connections,” *Teaching Children Mathematics*. Rexton, VA.

Van De Walle and Lovin (2006). *Teaching Student Centered mathematics: Grade 5–8,* New York, NY: Addison Wesley Longman.

Appendix A: Cognitive Levels of Questioning

Knowledge questions require students to recall or recognize information, names, definitions, or steps in a procedure.

Knowledge verbs: identify, compute, calculate, name, find, evaluate, use, and measure.

Knowledge questions, items, and/or tasks:

- rely heavily on recall and recognition of facts, terms, concepts, or properties
- recognize an equivalent representation within the same form, for example, from symbolic to symbolic
- perform a specified procedure; for example, calculate a sum, difference, product, or quotient
- evaluate an expression in an equation or formula for a given variable
- draw or measure simple geometric figures
- read information from a graph, table, or figure

Application questions require students to make connections, represent a situation in more than one way, or solve contextual problems.

Application verbs: classify, sort, estimate, interpret, compare, and explain.

Application questions, items, and/or tasks:

- select and use different representations, depending on situation and purpose
- involve more flexibility of thinking
- solve a word problem
- use reasoning and problem-solving strategies
- may bring together skills and knowledge from various concepts or strands
- make connections between facts, terms, properties, or operations
- represent a situation mathematically in more than one way
- compare figures or statements
- explain and provide justification for steps in a solution process
- translate between representations
- extend a pattern
- use information from a graph, table, or figure to solve a problem
- create a routine problem, given data, and conditions
- interpret a simple argument

Analysis questions require students to go beyond comprehension and application to higher order thinking skills, such as generalizations and non-routine problem-solving.

Analysis verbs: analyze, investigate, justify, compare, explain, describe, and prove.

Analysis questions, items, and/or tasks:

- require problem solving, reasoning, planning, analysis, judgment, and creative thought
- thinking in abstract and sophisticated ways
- explain relationships among facts, terms, properties, or operations
- describe how different representations can be used for different purposes
- analyze similarities and differences between procedures and concepts
- generalize a pattern
- solve a novel problem, a multi-step, and/or multiple decision point problem
- solve a problem in more than one way
- justify a solution to a problem and/or assumptions made in a mathematical model
- describe, compare, and contrast solution methods
- formulate a mathematical model for a complex situation, such as probability experiments
- provide a mathematical justification and/or analyze or produce a deductive argument

Appendix B: From Reading Strategies to Mathematics Strategies

The following strategies are some examples to be used during the three-part lesson format (Investigate, Reflect & Share, Connect).

1. Concept Definition Map

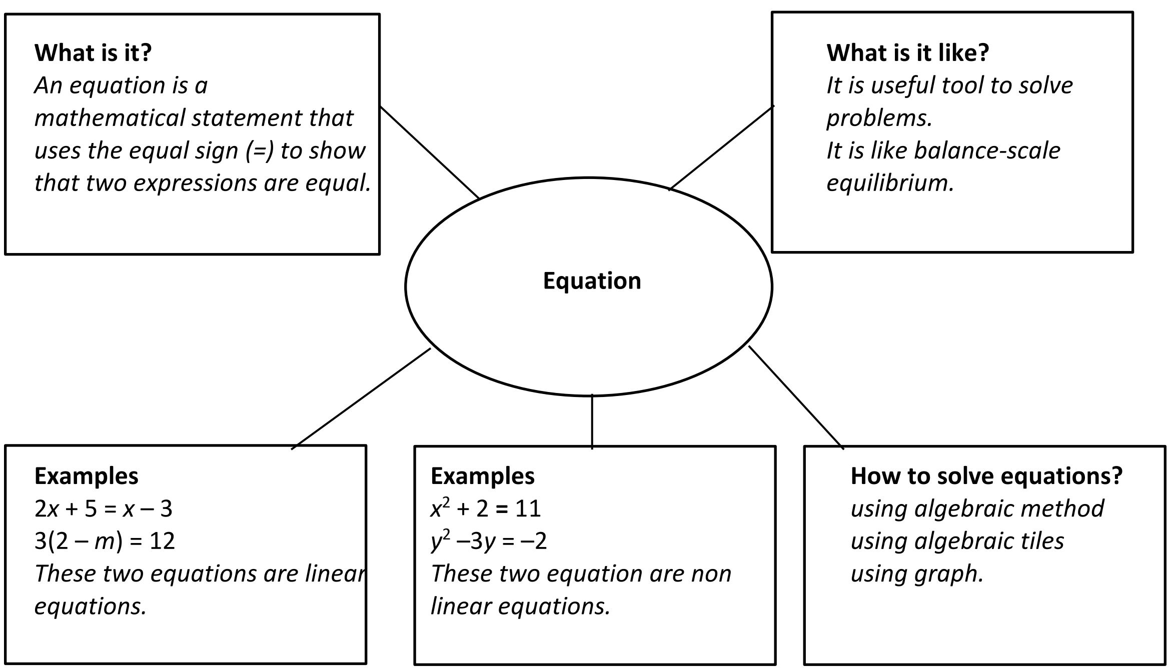
The purpose of a concept definition map is to prompt students to identify the main components of a concept, show the interrelatedness, and build vocabulary. Information is placed into logical categories, allowing students to identify properties, characteristics, and examples of the concept.

The following steps illustrate how this organizer can be used. (e. g. Perpendicular bisector)

1. Display the template for the concept definition map.
2. Discuss the different headings, what is being sought, and the quality of work that is expected.
3. Model how to use this map by using a common concept.
4. Establish the concept(s) to be developed.
5. Establish the groupings (e.g. pairs) and materials to be used to complete the task.
6. Complete the activity by having the students write a complete definition of the concept.

Encourage students to refine their map as more information becomes available.

Example of a Concept Definition Map:



2. Frayer Model

The Frayer Model is a graphic organizer used to categorize a word and build vocabulary. It prompts students to think about and describe the meaning of a word by

- giving a definition
- describing main characteristics
- providing examples and non-examples of the word or concept

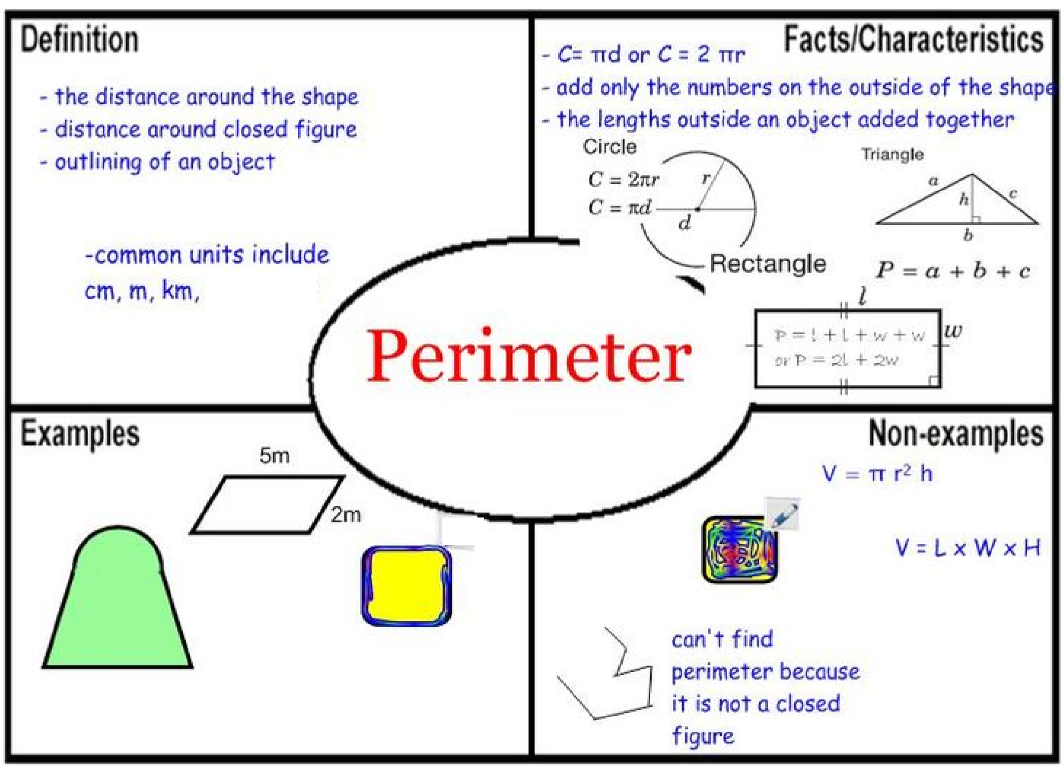
It is especially helpful to use with a concept that might be confusing because of its close connections to another concept.

The following steps illustrate how the organizer can be used.

1. Display the template for the Frayer model and discuss the various headings and what is being sought.
2. Model how to use this example by using a common word or concept. Give students explicit instructions on the quality of work that is expected.
3. Establish the groupings (e.g. pairs) to be used and assign the concept(s) or word(s).
4. Have students share their work with the entire class.

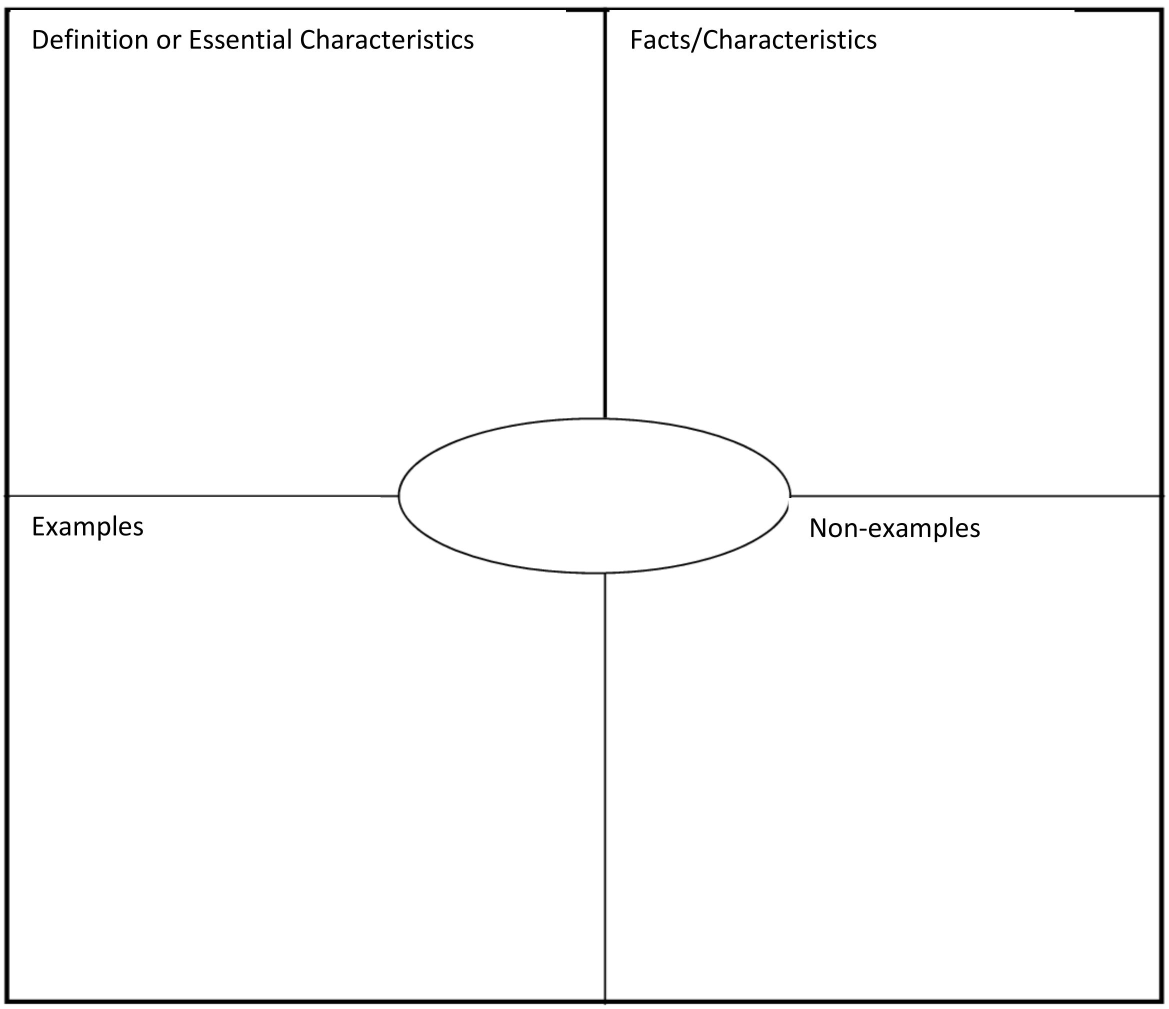
This is an excellent activity to do in poster form to display in class. Each group might do the same word or concept, or different words or concepts could be assigned (i.e., Perimeter, Area, Equations, Probability, etc.)

Here is an example of a Frayer Model.



Frayer Model Template
(Math Makes Sense 7, p. p 290 – 291)

Frayer Model



3. K-W-L (Know/Want to Know/Learned)

K-W-L is an instructional strategy that guides students through a text or mathematics word problem and uses a three-column organizer to consolidate the important ideas. Students brainstorm what they know about the topic and record it in the K column. They then record what they want to know in the W column. During and after the reading, students' record what they have learned in the L column. The K-W-L strategy has several purposes:

- to illustrate a student's prior knowledge of a topic
- to give a purpose to the reading
- to help a student monitor his or her comprehension

The following steps illustrate how the K-W-L can be used.

1. Present a template of the organizer to students, explain its features, and be explicit about the quality of work that is expected.
2. Ask them to fill out the first two sections, what they know and what they want to know before proceeding.
3. Check the first section for any misconceptions in thinking or weakness in vocabulary.
4. Have the students read the text and take notes as they look for answers to the questions they posed.
5. Have students complete the last column to include the answers to their questions and other pertinent information.
6. Discuss this new information with the class, and address any questions that were not answered.

The following is a student's example.

Pythagorean Theorem

K	W	L
- *It is about triangles.* - *It is named for the Greek mathematician, Pythagoras*	- *Can I apply it to any triangle?* - *What am I supposed to do with it?* - *How do I use it to solve what I am trying to solve?* - *Why is it important?* - *What does this theorem say?* - *When was it created?* - *What's hypotenuse?* - *How do I use it in real life?*	- *Now, I can apply it only to the right triangles.* - *I need it to find a hypotenuse or the length of a missing side on a triangle.* - *I replace variables with numbers and then find the third side.* - *It important because it is a way to find side c.* - $a^2 + b^2 = c^2$. *The sum of the squares of legs is equal to the square of the hypotenuse.* - *It was created long time ago, B. C.* - *In real life, if I want to find out a right triangle's missing side, I can find it now using this theorem.*

Appendix C: Cognitive Levels of Sample Questions

Lesson Learned 1 Problem Solving		Lesson Learned 2 Number		Lesson Learned 3 Patterns and relations		Lesson Learned 4 Measurement and Geometry		Lesson Learned 5 Statistics and probability	
Question	**Type of question**	**Question**	**Type of question**	**Question**	**Type of question**	**Question**	**Type of question**	**Question**	**Type of question**
1	Application	1	Application	1	Analysis	1	Knowledge	1	Knowledge
2	Application	2	Knowledge	2	Application	2	Application	2	Knowledge
3	Analysis	3	Application	3	Application	3	Analysis	3	Application
4	Analysis	4	Knowledge	4	Application	4	Analysis	4	Analysis
5	Application	5	Knowledge	5	Application	5	Application	5	Application
6	Application	6	Knowledge	6	Analysis	6	Analysis	6	Application
7	Application	7	Application	7	Application	7	Application	7	Application
8	Analysis	8	Application	8	Application	8	Analysis	8	Application
9	Application	9	Application	9	Knowledge	9	Analysis	9	Application
10	Analysis	10	Application	10	Application	10	Analysis	10	Application
		11	Application			11	Knowledge	11	Analysis
		12	Analysis			12	Application	12	Application
		13	Application			13	Application		
		14	Analysis			14	Application		
		15	Analysis			15	Analysis		

Appendix D: Sample Question Answers

Lesson Learned 1 Problem Solving		Lesson Learned 2 Number		Lesson Learned 3 Patterns and relations		Lesson Learned 4 Measurement and Geometry		Lesson Learned 5 Statistics and probability	
Question	**Answer**	**Question**	**Answer**	**Question**	**Answer**	**Question**	**Answer**	**Question**	**Answer**
1	41	1	159	1	241	1	9, 30, 35	1	60 and 11
2	4:9	2	–2	2	D	2	$y^2 = 64 - 36$	2	25%
3	$\frac{1}{6}$	3	3	3	$e = 35t + 35$	3	16.2 m	3	97º
4	24 min	4	216	4	$P = 6x + 100$	4	72 cm^2	4	59
5	15	5	0.0065	5	$-3x + 9 = 2x - 6$	5	113 cm^2	5	D
6	7	6	D	6	2, 4, 1, 3	6	$x = 8$	6	$\frac{2}{3}$
7	12	7	275%	7	1	7	30.3 m^2	7	2
8	D	8	1	8	20	8	10 km	8	$\frac{3}{5}$
9	9 tickets	9	$4.25	9	D	9	A	9	72
10	72	10	11.1%	10	$y = -2x + 4$	10	40º	10	15%
		11	$\frac{1}{12}$ m			11	D	11	$\frac{2}{80}$
		12	28%			12	36 cm	12	$\frac{1}{12}$
		13	20%			13	130º		
		14	80 min			14	B (2, -3)		
		15	5 cm			15	Q (0, 0)		