LESSON LEARNED

Nova Scotia Assessment: Mathematics Grade 3

“For learners to succeed, teachers must assess students’ individual abilities and characteristics and choose appropriate and effective instructional strategies accordingly.”
– Helene J. Sherman

Contents

Purpose of this Document

This Lessons Learned document was developed based on an analysis of the Item Description Reports for the Nova Scotia Assessment: Mathematics in Grade 3 (NSA-M3). This document is intended to support all classroom teachers at grades Primary – 3, and administrators at the school, region, and provincial levels. The focus of the document is to help educators work through the process of taking in the information provided by the data analysis and see how it can inform lesson design and assessment in the classroom.

It is suggested that school teams make use of this resource in concert with their school's Item Description Report provided by the Department of Education and Early Childhood Development to all regional centres for education. These reports include student achievement data at the school, regional centre, and provincial level for all questions appearing on the Mathematics in Grade 3 Assessment. By analyzing their own performance on groupings of questions dealing with similar outcomes, schools can identify areas of strength and areas where changes in instruction and/or assessment might be made. This process is designed to foster continued discussions, explorations, and support for mathematics focus at the classroom, school, regional centre, and provincial levels that are all based on valid and reliable data.

This document specifically addresses some of the areas that students across the province found challenging based on provincial assessment data. It is essential that teachers consider assessment evidence from a variety of sources to inform the next steps most appropriate for their students. Effective classroom instruction and assessment strategies are responsive to the individual learners within a classroom.

This document highlights those outcomes where students seem to require additional support. It provides some information about student performance on the assessment in addition to suggested classroom instruction strategies. Sample assessment items are included for each topic explored.

Overview of the Nova Scotia Assessment: Mathematics in Grade 3

Nova Scotia Assessments are large-scale assessments that provide reliable data about how well all students in the province are learning the mathematics curricula. It is different from many standardized tests in that all questions are written by Nova Scotia teachers to align with curriculum outcomes and the results reflect a snapshot of how well students are learning these outcomes. These results can be counted on to provide a good picture of how well students are learning curriculum outcomes within schools, regions and in the province. Since the assessments are based on the Nova Scotia curriculum, and are developed by Nova Scotia teachers, results can be used to determine whether the curriculum, approaches to teaching and allocation of resources are effective. Furthermore, because individual student results are available, these, in conjunction with other classroom assessment evidence, help classroom teachers understand what each student has under control and identify next steps to inform instruction.

The assessment provides information about mathematics for each student and complements assessment data collected in the classroom. This assessment is administered at the end of Grade 3. It is designed to provide detailed information for every student in the province regarding their progress in achieving a selection of mathematics curriculum outcomes at the end of Grade 3. Information from this assessment can be used by teachers to inform their instruction and next steps in providing support and intervention for their students.

Lessons Learned Overview

Provincial assessments and examinations generate information that teachers can use to help inform classroom instruction and assessment. Following the analysis of each assessment or examination, patterns and trends are identified. These include areas of strength and areas for growth. The Lessons Learned documents specifically highlight concepts where growth is still needed.

There are four areas that have been identified as the areas of focus for this Lessons Learned document. They are:

- Solving whole number addition and subtraction questions in context.
- Measuring and Estimating length.
- Identifying and sorting irregular polygons.
- Interpreting data represented in tables and graphs.

Each section begins with an overview of the student errors and misconceptions identified through the provincial assessment. Strategies are then outlined that are designed to enhance student comprehension, drawing from researched best practices. The strategies emphasize the integration of essential models, tools, and interconnections to facilitate the transition between concrete, pictorial, and abstract representations of concepts, highlighted by the importance of deliberate planning and purposeful questioning. To support both assessment and instruction, sample lesson activities are presented alongside a series of cognitive-level questions, providing educators with ideas for addressing knowledge gaps and fostering strategic reasoning and problem-solving skills. Each section culminates with a selection of print and online resources, as well as recommended manipulatives to support professional learning and student understanding of that topic.

Solving Whole Number Addition and Subtraction Questions in Context

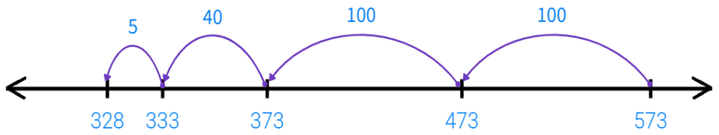
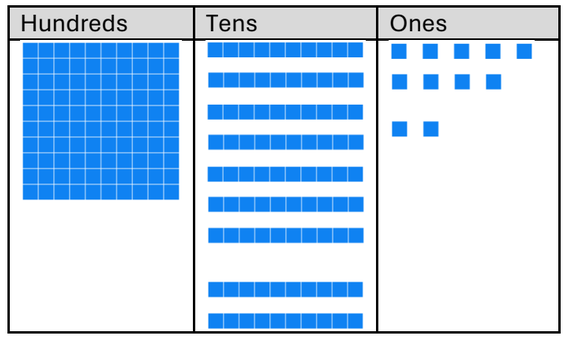
Alignment to previous Outcomes			Related Outcome
PN04: Students will be expected to represent and describe numbers 2 to 10 in two parts, concretely and pictorially.	**1N09:** Students will be expected to demonstrate an understanding of the addition of two one-digit numbers and the corresponding subtraction, concretely, pictorially, and symbolically, in join, separate, and part-part-whole situations.	**2N09:** Students will be expected to demonstrate an understanding of addition (limited to 1- and 2-digit numerals) with answers to 100 and the corresponding subtraction.	**3N09:** Students will be expected to demonstrate an understanding of addition and subtraction of numbers (limited to 1-, 2-, and 3-digit numerals) with answers to 1000.

What conclusions can be drawn from the NSA: Mathematics in Grade 3?

Students understand addition and subtraction of whole numbers with answers up to 1000, however some have difficulty when there is a need for regrouping. In questions where students were explicitly given all the information needed to do a knowledge question, students were successful. Yet, when given application and analysis items, that required higher order thinking skills when problem solving only half of the students were successful. This is evident in contextual problems within the assessment.

Students did well when translating and moving flexibly between and among representations of a concept. However, when presented with non-conventional formats, some students struggled. Therefore, students should be encouraged to use multiple representations to continue to be able to translate between words, pictures, or symbols. This includes, for example, various representations of numbers. When students are working to partition whole numbers and to perform operations, it is very important for students to understand that numbers can be broken down into two or more parts in many ways. This supports how students work with numbers and apply varying strategies when problem solving. Overall, helping them build their toolbox of strategies to solve contextual problems and further develop their reasoning abilities.

Why is this an area of need and how can we support students?

Place Value	
Misconceptions/Errors in Student Work	**Possible Next Steps in the Classroom**
When students are asked to solve addition or subtraction questions, you may get the following responses. **Sample question.** What is 573 – 245? A. 322 (student subtracts with trading in tens position, but still subtracts incorrectly in ones position) B. 328 (correct answer) C. 332 (student subtracts smaller digits from larger digits) D. 338 (student subtracts with trading – ones position, but forgets when subtracting in tens position)	Each of the different type of responses provides clues as to the root of the misconception. Before planning next steps, it is important to understand what the student understands well and tailoring their learning from their starting point. Regardless of what the student understands, using various forms of representation can only further their understanding and or add flexibility to their thinking. ***Using and Modelling with a Number Line*** Use an open number line to solve for addition and subtraction. Model and encourage counting up and counting back as strategies. In doing so, students focus on breaking numbers into friendlier chunks that support flexibility, efficiency, and accuracy.  573 – 100 = 473 473 – 100 = 373 373 – 40 = 333 333 – 5 = 328
Sample question. What is 179 + 22? A. 191 (student forgets to add in the ten when determining the sum the digits in the tens position) B. 201 (correct answer) C. 291 (student adds ten tens rather than one ten when determining the sum of the digits in the tens position) D. 1911 (student does not regroup)	***Modelling with Base 10 Material*** To help students focus on the place value rather than the digits, model and practice using base-ten blocks and place value charts. Align representations to help students identify how the strategies are similar.  ***Recording Place Value*** The traditional algorithm focuses on digits rather than place value. Have students break numbers into their place value and have them track their addition and subtraction. 179 + 22 179 is the same as 100 + 70 + 9 22 is the same as 20 + 2 So, 100 + 70 + 9 + + 20 + 2 100 + 90 + 11 100 + 90 = 190 190 + 11 OR 190 + 10 + 1 = 201 (Answer)

While not an error or a misconception perse, one could also infer that students may not be checking for reasonableness in their answers. They could be stuck in incorrectly using standard procedures negating what they know about number sense. Students may also not be aware of other, more helpful strategies that support how to work with numbers to solve what is unknown in a story problem. For example, when students respond with an answer of 291 to the question above (179 + 22), they may not be thinking that the number should be close to 200 based on the size of both numbers. They could be more caught up in using a procedure and not in the practice of checking for reasonability.	**To solve 179 + 22** (Hundreds) 100 (Tens) 70 + 20 = 90 (Ones) 9 + 2 = 11 → 10 + 1 (Tens) + (Ones) (Answer) 100 + 90 + 10 + 1 = 201 ***Estimation strategies*** Students need to be encouraged to estimate before calculating an answer to a question. Estimating sums and differences is valuable as it helps in checking the reasonableness of calculations, and predicting answers, and sometimes it is all that is required. The goal of estimation is not to get the exact answer but to get a close approximation quickly and efficiently. It is a valuable skill for mental math and for quickly checking the reasonableness of answers. To estimate sums and differences, students might use strategies that include rounding to a multiple of 10, 100, or 1000, rounding one number and not the other, front-end estimation, and rounding one number up and the other down. Students may also choose friendly or benchmark numbers that are easier to work with and are near the given numbers. Further examples can be found in the guides. At the core of estimating is understanding number relationships, benchmark numbers, and compatible numbers. ***Number Talks*** Number talks foster the development of number sense by encouraging students to think flexibly about numbers and mathematical operations. Through engaging in discussions and exploring various strategies, students deepen their understanding of mathematical concepts. Moreover, number talks promote mathematical discourse, as students articulate their reasoning, listen to their peers' strategies, and engage in collaborative problem-solving. Hence, number talks support the use of multiple strategies for problem-solving, reinforcing the idea that there are often multiple paths to a solution. They also facilitate the development of mental math skills by providing opportunities for regular practice. Furthermore, number talks can be adapted to meet the needs of diverse learners, making them a versatile tool for differentiated instruction. They offer valuable insights into students' mathematical understanding, enabling teachers to assess student progress and plan future instruction accordingly. ***Retrieval Practice Strategies*** It is essential that students know their addition and subtraction facts and understand how numbers relate to each other. Retrieval practice supports instruction and assessment by strengthening schemas that students have built, helping them commit the information to their long-term memory. Retrieval practice involves daily cumulative review and improves the efficiency of learning. Examples include making and using flashcards, concept maps, or grids, intentional games, choral response, and interleaved practice. All of which facilitate better learning than highlighting or re-reading questions.

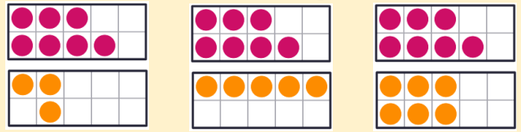
Activities to Support Lesson Planning

Number Talk and Number Strings
Use number talk routines to develop efficiency, flexibility, and accuracy with computations. They help to elicit specific strategies that focus on number relations and number theory rather than a series of steps found in traditional algorithms. Classroom conversations and discussions around purposefully crafted computation problems are at the very core of number talks.

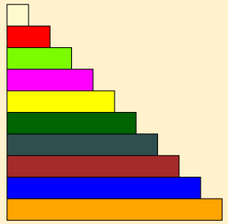
A few examples are outlined below, while further examples of strategies are outlined in the curriculum guides as well as the resources section.

Using manipulatives like counters, Cuisenaire rods, and open number lines support how students visualize counting and quantity along with adding and subtracting.

Begin with examples that require less cognitive load to support the development of effective strategies and models. These can include activities that focus on number relationships, compatible numbers, and benchmark numbers. Encourage flexibility and talk about estimation.

Grade P	Grade 1	Grade 2	Grade 3
Goal – recognizing different ways to make the value 5. Briefly show students one of the arrangement of dots. Discuss using one or more of the question prompts below. Then show the second image and then the third, discussing in between. Can students identify 5 in different ways?  **Question prompts:** Knowledge: How many dots do you see? How do you see them? Application: How does your strategy compare to the strategies shared by others in your group? Analysis: Create your own arrangement and show me how you determined how many dots. How does it compare to the ones we discussed as a group? (Note: this question can be used after the discussion to check for individual understanding.) **OR**	**Goal** – making and relating to tens using ten frames. Show students one of the pairs of dots in the ten frames. Discuss using one or more of the question prompts below. Show the second pair and then the third, discussing in between.  **Question prompts:** Knowledge: How many dots do you see? How do you see them? Application: How does your strategy compare to the strategies shared on the board? Analysis: Create your own arrangement and show me how you determined how many dots. How does it compare to the ones we discussed as a group? (Note: this question can be used after the discussion to check for individual understanding.) **OR**	**Goal** – break each number into its Place Value. You can select either the addition or the subtraction. Show each number sentence one at a time and have students solve it; discussing in between. What common strategies are students using and are they seeing the relationship between the values? 18 + 31 20 – 10 23 + 14 20 – 9 37 + 12 20 – 11 32 + 25 21 – 9 **Question Prompts:** Knowledge: Solve the number sentence. Explain your strategy or state your strategy. Application: How are these strategies the same? How does the previous number sentence help you solve this number sentence? Analysis: Create your own number sentence that relates to these number sentences and would use the same or similar strategy. **OR**	**Goal** – breaking each number into its Place Value You can select either the addition or the subtraction. Show each number sentence one at a time and have students solve it; discussing in between. What common strategies are students using and are they seeing the relationship between the values? 35 + 19 59 – 47 115 + 92 60 – 50 167 + 73 60 – 47 115 + 193 62 – 45 **Question Prompts:** Knowledge: Solve the number sentence. Explain your strategy or state your strategy. Application: How are these strategies the same? How does the previous number sentence help you solve this number sentence? Analysis: Create your own number sentence that relates to these number sentences and would use the same or similar strategy. **OR**

Show students the following arrangement of Cuisenaire rods.



Ask students:

Knowledge: What do you notice? What do you wonder?

Application: Investigate a wonder question. For example, how many whites make a green? How many reds make a purple? Have students build representations of their thinking.

Analysis: How many different ways can you show the same length as the yellow?

Show students the following arrangement of Cuisenaire rods. You may need to have students build the 'staircase' depending on how familiar they are with the tool.

Ask students:

Knowledge: Which rod is the same length as these two rods?

Next show the following arrangement:

Knowledge: Which rod(s) is the same length as these two rods?

Application: How did the first arrangement help you answer the second arrangement?

Next show students the following arrangement:

Knowledge: Which rod(s) is the same length as these two rods?

Application: How did the first two arrangement help you answer the second arrangement? What other rods can be used that would be the same length as these two rods?

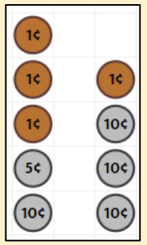
Analysis: Create your own length(s) and show me how you determined what other rod(s) matched the length. How does it compare to the ones we discussed as a group? What strategies help you know what lengths are easier or more difficult to compare?

Note: Some grade 2 students may start here to support readiness of breaking numbers in friendly or benchmark amounts when adding. A similar arrangement using smaller numbers can

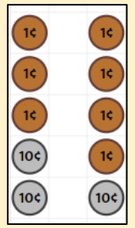
Instead of showing students the number sentences above, use an arrangement of coins or ten frames to show addition and/or subtraction. The coins (addition) and ten frames (subtraction) below represent the number sentences listed above.

The same prompting questions above, can be used with visual representations to support student thinking.

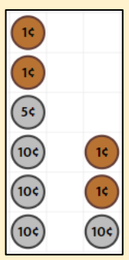
Addition:



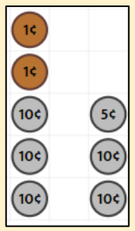
18 + 31



23 + 14

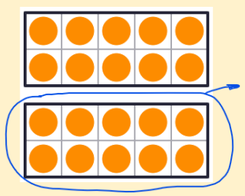


37 + 12

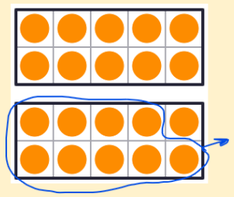


32 + 25

Subtraction:



20 – 10

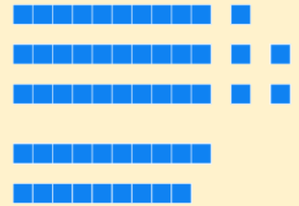


20 – 9

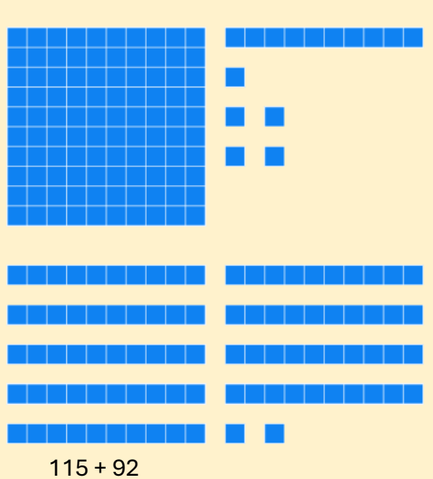
To help students visualize benchmark values and place value, use base ten blocks as a visual representation. Students can physically make groups of ten to transition into making groups of ten mentally. Some students may visualize and manipulate the materials in their head.

The same prompting questions can be used with visual representations to support student thinking. Here are a couple of examples for both addition and subtraction.

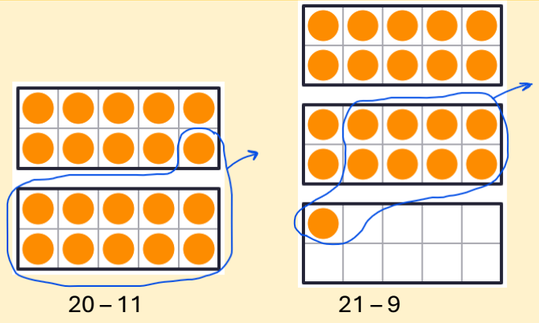
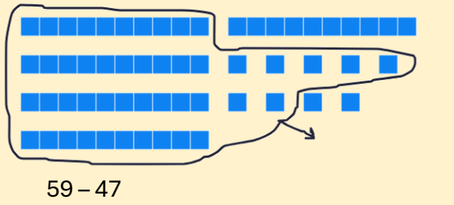
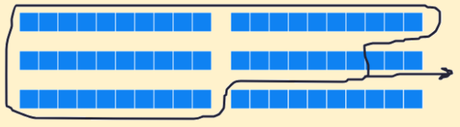
Addition:



35 + 19



115 + 92

	be used for subtraction as shown in the grade 2 example.	 20 – 11 21 – 9 Note: Some grade 3 students may start here to support readiness of breaking numbers into their place value when adding or subtracting.	Subtraction:  59 – 47  60 – 47

Number and Word Grabbing in Story Problems

Misconceptions/Errors in Student Work

Students use learned strategies to look for numbers and key words in a story problem and use these to create a number sentence. In doing so, they add or subtract using the two numbers without understanding the action taking place in the problem and checking if their solution is reasonable.

Students may:

- overgeneralize all stories to addition.
- not see part-part-whole relationships.
- subtract a whole from a part.
- not understand the relationship between addition and subtraction.

Here are two examples of story problems and possible student answers.

The first is a Separate Problem: Result Unknown.

Stacey went to the store and spent $18.
If Stacey had $50 to begin with, how much money is left?

- *$32 (correct answer)*
- *$42 (an error that occurs when a student incorrectly borrows and subtracts in the tens place)*
- *$48 (an error that occurs from a student that subtracts the smaller digits from the larger digits)*
- *$68 (an error that occurs when a student adds as opposed to subtracts)*

This second is a Join Problem: Change Unknown.

Stacey had $18 saved in a piggy bank at home. After mowing 3 lawns in the neighbourhood, Stacey now has $50.
How much money did she earn for moving the lawns?

- *$32 (correct answer)*
- *$42 (an error that occurs when a student incorrectly borrows and subtracts in the tens place)*
- *$48 (an error that occurs from a student that subtracts the smaller digits from the larger digits)*
- *$68 (an error that occurs when a student adds as opposed to subtracts)*

Possible Next Steps in the Classroom

Developing an understanding of the relationship between addition and subtraction

In addition to strategies listed in the previous section, adding and subtracting must also be explored in meaningful contexts using a variety of concrete materials and pictures to model and compare contexts. Math tools and models like: Rekenreks, ten frame, linking cubes, number lines, base-ten blocks, Cuisenaire rods, and strip diagrams are helpful at visualizing quantity and developing personal strategies. When teaching both concepts, spend time on addition and subtraction separately, but also together to help students develop the understanding of how adding and subtracting are opposites of one another and how each is presented in questions.

Developing parts-whole relations

Model joining and separating problems using part-part whole mats (Strip Diagrams) and encourage students to do the same to help interpret story problems. Students must also experience addition and subtraction in comparison situations. They need to learn that addition and subtraction problems can be categorized based on the kinds of relationships they represent.
Further details can be found in the curriculum guides.

Join Problems	Separate Problems	Part-Part-Whole Problems	Compare Problems
Result Unknown		Whole Unknown	Difference Unknown
Change Unknown		Part Unknown	Larger Unknown
Initial Unknown			Smaller Unknown

Teaching About and Through Problem Solving

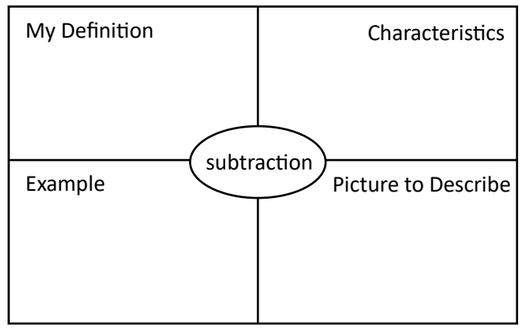
Provide direct instruction on problem-solving strategies as students share their own solutions and findings. Use students' methods to guide instruction. Elaborate on the methods used by students to solve and justify and encourage students to comment and ask questions of their peers.

Concepts and skills should be connected to everyday situations and other curricular areas. Encourage students to make connections to make mathematics come alive through math-to-world, math-to-math, and math-to-self connections.

Develop students' mathematical vocabulary, initiate effective ways to navigate informational text, and encourage students to reflect on what they have learned.

Embed strategies/tools such as the Frayer Model, Concept Circles, and Exit Cards to assess student learning.

Example of a Frayer Model



Modelling Thinking with Think Alouds

A teacher should model problem solving with students by sharing aloud their thinking as they read to understand a problem. Through the process, students learn how to verbalize to understand a question and to make sense of what is being asked and what is in their toolbox to answer the question.

Activities to Support Lesson Planning

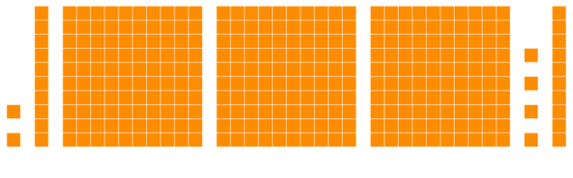
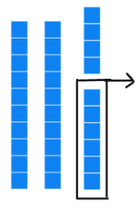
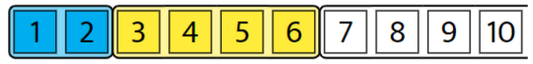
Open Tasks

Using open tasks and games offers ways for students to apply and demonstrate addition and subtraction in context and allow for multiple entry points. Cross-strand opportunities also help to reinforce and help students to see mathematics as holistic and not siloed into its strands. They can also be used across multiple grades and abilities. Games that include dice and cards also encourage students to determine count and quantity, and sums and differences of numbers in a fun and interactive way.

The following activities can be set up to support a problem-solving lesson. Using common literacy strategies that help students make sense of different types of texts can also be used to support understanding a story problem. This includes think-alouds to model and engage in conversation with students.

Grade P	Grade 1	Grade 2	Grade 3
Using dot plates or counters in small groups, pairs or individually. Have two groups of counters or dot plates (e.g., one with 3 dots and one with 4 dots). Knowledge: Show both groups and ask how many altogether. Watch how the student combines the count. Application: Which group has more? How do you know? Analysis: How can you arrange the dots/counters to help you know how many and how much more without counting?	Students can physically move around the room or use counters or Cuisenaire rods to work through the various questions. Knowledge: How many students in the class have brown eyes? How many have blue eyes? How many all together. Application: How many more students have brown/blue eyes? Analysis: If 4 new students joined the class, how many students would have blue eyes and brown eyes, now. What would be the new totals, and which one would be greater and by how much?	Have students use manipulatives like Cuisenaire rods, open number lines, base ten blocks, and strip diagrams to show their thinking. Knowledge: If there are 76 students with blue eyes and 47 students with brown eyes, how many all together? What is the difference in the number of students? Application: If there are 38 students in grade 2 and almost 20 students have blue eyes, how many have brown eyes? Analysis: Create and solve number sentences. How many students have blue eyes and how many have brown eyes? What is the sum? What is the different?	Have students use manipulatives like Cuisenaire rods, open number lines, base ten blocks, and strip diagrams to show their thinking. Knowledge: If there are 176 students with blue eyes and 217 students with brown eyes, how many all together? What is the difference in the number of students? Application: If there are 386 students in the school and almost 150 students have blue eyes, how many have brown eyes? Analysis: Create and solve number sentences. How many students have blue eyes and how many have brown eyes? What is the sum? What is the different?

Use of Different Representations

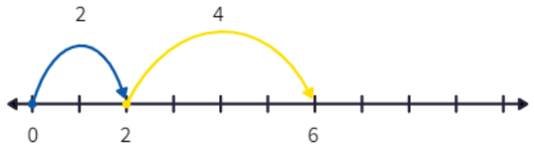
Misconceptions/Errors in Student Work	Possible Next Steps in the Classroom
If asked to interpret or represent a question where representation varies, some students have difficulty translating between pictures, words, or symbols. Students should recognize the various representations they can use when solving a number problem. Examples: ***Students need to demonstrate their knowledge of place value pictorially using non-conventional representations.*** *Choose the number represented by the base-ten blocks.*  • *11 (total number of blocks and not the value they represent)* • *326 (correct answer)* • *623 (a student has switched the values of the unit cubes with the flats)* • *14312 (reading the number of blocks from right to left, with no regard for place value)* Students need to demonstrate their knowledge of place value in words using non-conventional representations. *The number 642 is the same as:* • *5 hundreds, 2 tens, and 14 ones (regroups 14 ones incorrectly as 1 hundred and 4 tens)* • *64 tens and 2 ones (correct answer)* • *6 tens and 42 ones (reads 6 tens as 6 hundred but correctly identifies 42 ones)* • *6 hundreds, 20 tens, and 4 ones (misidentifies tens and ones place values)*	***Engage in dialogue and encourage purposeful selection of representations*** One way to encourage students to use multiple representations is to explicitly ask for them. Engage in dialogue about the explicit connections between representations and alternate between them. Encourage students to purposefully select various representations to check for understanding. Discuss a variety of reasons to use a particular representation, including but not limited to efficiency, accuracy, ease of use, appropriateness with the problem context, and student preference. By comparing and discussing the use of multiple representations for the same problem, students can more easily see the suitability of one representation over another. For example, representing a story problem with a strip diagram or using base-ten blocks or an open number path / number line to model addition and subtraction. Equality can be represented in both an equation, but also visually using a balance scale. When working with story problems, help students to identify whether the problem is joining, separating or part whole. This will help in how students use strip diagrams to identify what is given and what has to be determined. For example, the question Bobby had 98 stamps. Bobby was given more by a friend. Now Bobby has 137 stamps. How many stamps did Bobby get from the friend. 98 | ? 137 This is represented by an equation as follows: 98 + ___ = 137 but can then be solved using different strategies that may include subtraction or adding up. Other models include the examples below: Base-ten blocks  30 – 6 = Number Path  2 + 4 =

When students are more commonly presented with one version of a story problem, i.e. result unknown, they will typically misuse a strip diagram. See example:

Bobby had 98 stamps. Bobby was given more by a friend. Now Bobby has 137 stamps. How many stamps did Bobby get from the friend.

?	
98	*137*

Number Line



2 + 4 =

Balance Scale

63 + 25 = ? + 15

It is also important to present to students with non-conventional displays of whole numbers to support thinking and reasoning. These include base ten blocks and expanded notation.

Concrete Representational Abstract (CRA) Modeling

This approach is a system of learning that uses physical and visual tools and models to build students' understanding of abstract concepts.

Students are introduced to a new mathematical concept using concrete resources (e.g. base-ten blocks, ten-frames). When they are comfortable solving problems with physical aids, they are given problems with pictures – usually visual representations of the concrete objects they were using.

Then students are asked to solve problems where they only have the abstract, i.e. numbers or other symbols. Building these steps across a lesson can help students better understand the relationship between numbers and the real world, and therefore helps secure their understanding of the mathematical concept they are learning.

It is important to note that learning takes time and students should not be rushed into using abstract representations. Students should be using tools and representations everyday to support and check for understanding.

Activities to Support Lesson Planning

When designing instruction, use activities that encourage the use of tools and different representations that align to the development of the mathematical concept.

Games that utilize math manipulatives are a great way to integrate the use of various representations to build number sense and reinforce skills, while engaging students in the learning process. These should be intentional and target and reinforce skills and concept development.

Grade P

Knowledge: Using a Rekenrek or counters, have students represent a number of their choice. How do they know they are correct?

Application: How many ways can you show the number 5 using a Rekenrek or counters? How many of one colour and how many of the other do you use to show 5? How do you know they are the same? What number can you make using these two groups of cubes? (e.g., 3 red cubes and 2 green cubes make how many cubes all together)

Analysis: How can you arrange your counters to help you see the number 5 without counting? How does it help you?

Grade 1

Knowledge: Have students model a group of 3 then have them model a group of 4. Ask, how many items are there altogether?

Application: Hide 3 small items in your hand or under a piece of paper. Tell students that 3 items are hidden. Put out 4 more items where students can see them. Give students additional counters to model the 3 hidden items if they need them. How many items are there altogether?

Analysis: How many more items are there in this group than the hidden one?

You can also use the two colours on a Rekenrek to help students to notice different ways of making different numbers up to 10 or 20.

Grade 2

Knowledge: Using base-ten materials, show the following in different ways. Represent each value in different ways.

45 + 28 =

45 – 12 =

Application: Using base-ten materials, show all possible sums and differences using these values: 12, 28, 45. Are there calculations you found more challenging? Easier? Are there other ways you can represent the given numbers to help you solve the problems?

Analysis: Ask students to determine which numbers are missing. Hint: all rows, columns, and diagonals add to the same amount.

	17	22
19		23
20	25	

7	17	22
19	4	23
20	25	1

*To create additional magic squares, you can always begin with one that you have, like the one above, and add, subtract, multiply, or divide all entries using the same value. You can also rotate or reflect the square. Have students create their own.

Grade 3

Knowledge: Using base-ten materials, show the following in different ways. Represent each value in different ways.

128 + 82 =

145 – 82 =

Application: Using base-ten materials, show all possible sums and differences using these values: 82, 128, 145. Are there calculations you found more challenging? Easier? Are there other ways you can represent the given numbers to help you solve the problems?

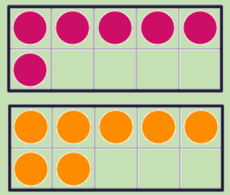
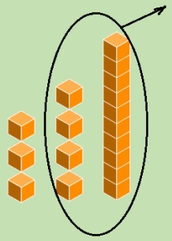
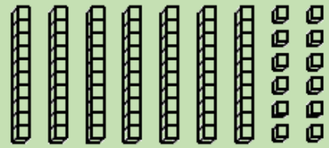
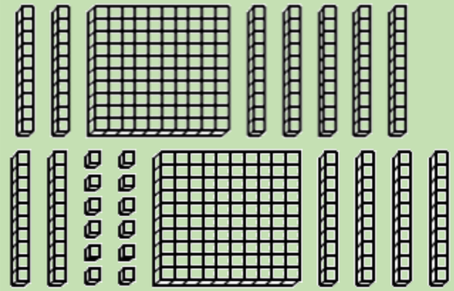
Analysis: Ask students to determine which numbers are missing. Hint: all rows, columns, and diagonals add to the same amount.

	51	66
57		69
60	75	

21	51	66
57	12	69
60	75	3

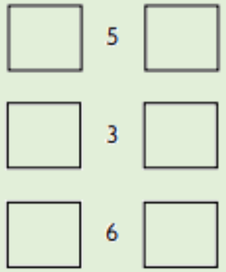
*To create additional magic squares, you can always begin with one that you have, like the one above, and add, subtract, multiply, or divide all entries using the same value. You can also rotate or reflect the square. Have students create their own.

What are some sample questions to help support assessment?

Cognitive Level	Grade P	Grade 1	Grade 2	Grade 3
Knowledge	How many counters altogether?  Show me with your fingers a way of making 6. What number comes next as I count...3, 4, 5, ...	How many dots are there altogether?  How many blocks am I taking away? What is left?  Write addition fact families that contain 5 as one of the numbers.	What number is represented by the base-ten blocks?  46 + 36 = 46 + ___ = 82 88 - 36 = 48 = ___ + 24 Write an expression that can be used to represent 53. Using an open number line, determine the sum of 23 and 36. Using an open number line, determine the difference between 78 and 46.	What number is represented by the base-ten blocks?  Write two different expressions that can be used to represent the 597. 127 - 38 = 326 + 48 = 48 + ____ = 326 127 = ___ + 38 What two numbers could you add to get a sum of 123? What two numbers could you subtract to get a difference of 56?

Application

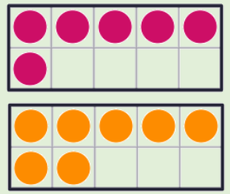
Fill in the number that come before and after the following numbers.



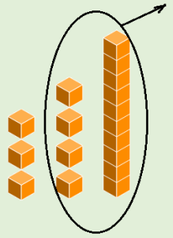
You have 4 crayons, and your friend has 4 crayons. How many crayons do you have altogether?

Represent the number 5 in different ways?

What number sentence do these dots represent?



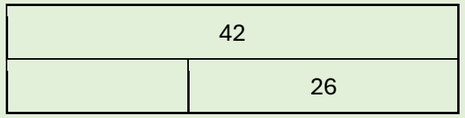
What number sentence do these base ten blocks show?



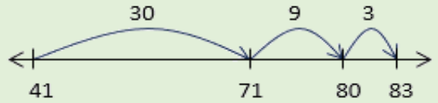
You have 8 cubes. There are 5 red cubes, and the rest are green. How many cubes are green?

Do you think the sum of 5 + 9 will be larger or smaller than 15? Explain.

What equation is represented by the strip diagram?



What equation is represented by the number line shown?

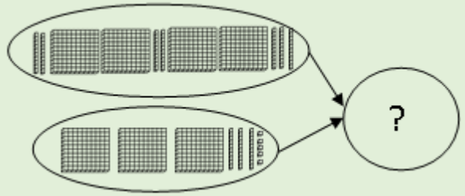


I made 43 cookies and gave 21 away to my classmates. How many cookies do I have left?

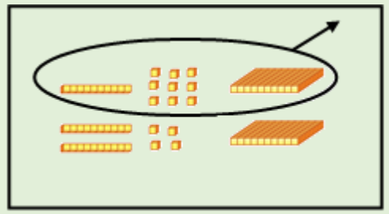
My friend has a book with 48 stickers, and I have a book with 32 stickers. How many more stickers does my friend have?

Do you think the sum of 34 + 57 will be larger or smaller than 100? Explain.

Which addition problem is represented by this set of base-ten blocks?



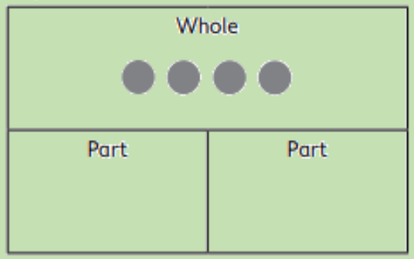
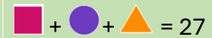
Which subtraction problem is represented by this set of base-ten blocks?



When travelling to our vacation spot, we drove 239 km and stopped for lunch. How much further do we have to drive if the total distance to our destination is 526 km?

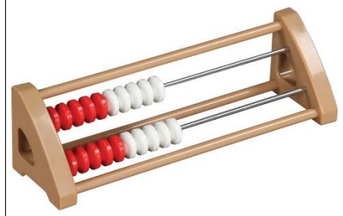
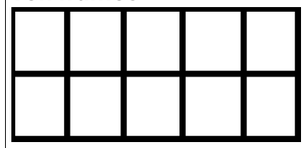
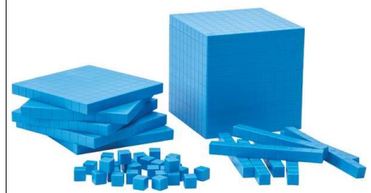
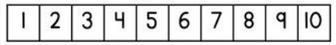
Create a strip diagram to model the following situation: You are saving money to buy a new bike. You are given $65 for your birthday and now you have $126. How much money did you start with?

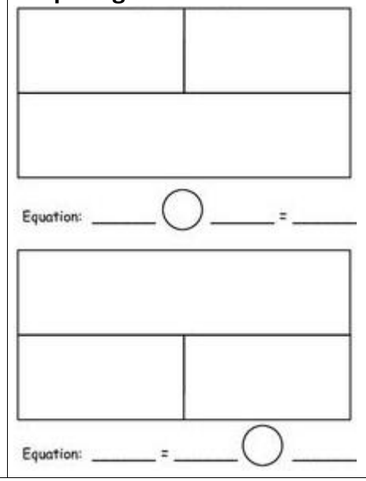
Do you think the sum of 134 + 458 will be larger or smaller than 600? Explain.

Analysis	Represent the counters shown in two parts.  You have 8 cubes. Some are green and some are red. How many of each do you have? You have 8 cubes. There are 5 red cubes, and the rest are green. How many cubes are green? Is 4 close to 0, 5, or 10? How do you know what number it is close to?	Create your own story problem using the following numbers: 6, 7, 13. How are these number sentences the same and how are they different? 6 + 5 = 11 11 - 5 = 6 11 = 5 + 6 5 = 11 – 6 You have 10 apples and eat 2 of them each day. What day will you have zero apples? You have 8 cubes. Some are green and some are red. How many of each do you have? Is 14 close to 0, 10, 15, or 20? How do you know what number it is close to?	The answer is 94, what is the question? Create your own story problem using the following numbers: 16, 47, 63. I made 43 cookies. Some are oatmeal cookies, and some are chocolate chip cookies. How many of each cookie did I make? I gave some cookies away to my friends. If I made 43 cookies and only had 12 left, how many cookies did I give away? Is 84 close to 0, 50, 80, 90, or 100? How do you know what number it is close to?	You have been given a handful of money. You notice you have a mixture of nickels, dimes, and quarters. You decide to use the money to buy a used book for 85 cents. If you use all the money to buy this book, how many nickels, dimes and quarters did you have? What do you notice about the types of coins you are using? Pete has 123 cards and gives some to his friend. Now Pete has 86 cards. How many more cards does Pete have than his friend? Each shape represents a different number. Find a number that each shape could represent. Select one or more to solve.    Is 184 close to 100, 150, 180, 190, or 200? How do you know what number it is close to?

Supporting Resources

Manipulatives and Models to Support Learning

<table>
<tr>
<td>Counters
  </td>
<td>Linking cubes
  </td>
<td>Rekenreks
  </td>
<td>Ten frames
  </td>
<td>Base-ten blocks
  </td>
<td>Play money
  </td>
</tr>
<tr>
<td>Number path (P-1)
 

Open Number line (2-3)
  </td>
<td>Cuisenaire Rods
  </td>
<td>Balance Scales
  </td>
<td>Strip Diagrams
 
</td>
<td>Dominoes
  </td>
<td>Number Cubes and Cards
  </td>
</tr>
</table>

Print and Electronic Resources

Bay-Williams, J. M. and SanGiovanni, J. J.. (2021), *Figuring out Fluency in Mathematics Teaching and Learning, Grades K – 8*. Corwin Press.

Cameron, Antonia. (2020). *Early Childhood Math Routines: Empowering Young Minds to Think*. Portsmouth, New Hampshire, Stenhouse Publishers.

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Van de Walle, J.A. and Lovin, L. (2006). *Teaching student-centered mathematics grades 3–5*. Boston: Pearson Allyn & Bacon.

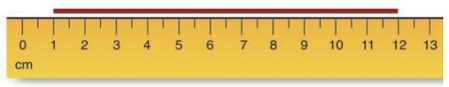
Measuring and Estimating Length

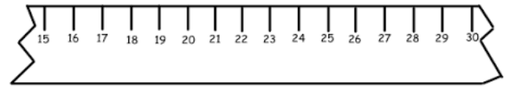
Alignment to previous Outcomes			Related Outcome
PM01: Students will be expected to use direct comparison to compare two objects based on a single attribute, such as length, mass, volume, and capacity	**1M01:** Students will be expected to demonstrate an understanding of measurement as a process of comparing by ▪ identifying attributes that can be compared ▪ ordering objects ▪ making statements of comparison ▪ filling, covering, or matching	**2M04:** Students will be expected to measure length to the nearest nonstandard unit by using multiple copies of a unit and using a single copy of a unit (iteration process).	**3M03:** Students will be expected to demonstrate an understanding of measuring length (cm, m).

What conclusions can be drawn from the NSA: Mathematics in Grade 3?

Students continue to have difficulty reading and interpreting measures on a ruler. Over half of students read the number on the ruler that is aligned with the end of the object rather than determine the number of intervals between the start and end of the object. Continued emphasis is also needed on the use of referents. Just under half of students still struggle to use personal referents to help visualize units of length (centimetres and metres). Continued experience is still needed in using and relating to various tools and measures to support an understanding of length.

Why is this an area of need and how can we support students?

Using a Ruler	
Misconceptions/Errors in Student Work	**Possible Next Steps in the Classroom**
When using rulers, students have a tendency to only look at the number on the ruler that is aligned with the end of the object.  For example, students read the line as being 12 cm instead of 11 cm. Looking at the end tick mark and not the distance between the start and end points. Some students also count the numbers or tick marks along the length of an object, rather than the intervals between the numbers.  For example, students read the line above as being 12 cm instead of 11 cm. They use the count of 12 tick marks beginning at 1 cm and using the distance between the start and end points or the jumps from 1 to 12.	**Measuring with a Ruler** The skill of learning how to use a ruler is introduced for the first time in grade 3. Emphasis should be on counting the intervals between the numbers, rather than looking at the number on the ruler that is aligned with the end of the object. In younger grades this can be modelled using a number line by highlighting jumps forwards and backwards between the numbers rather than just counting the tick marks. In addition, using a number path helps to highlight the space between the tick marks and when using the same jumping motions instead of just counting the number of units (squares, cubes) supports the transition to reading values on rulers. Have students use simple rulers that are created by students initially. Move onto tools that are easy for students to read. Students should use rulers (or the side of the ruler) that show only numbered centimetres and not millimetres.  Show students how to measure something that is longer than a ruler by marking, recording, and starting again. When using nonstandard units, help students to make sure there are no spaces between the units being used. When transitioning from nonstandard to standard units, demonstrate that the numbers on the ruler correspond to the number of small cubes by starting at 0 and lining up small cubes from base-ten materials along the ruler. Cuisenaire rods and base-ten blocks are a nice tool to show units of 1 cm and 1 dm or their relation to 1 m. To further support understanding of length, students should identify objects from around the classroom that would be an appropriate referent for a centimetre or a metre; for example, the width of a pencil (cm), the distance from the bottom of a door to the doorknob (1m). Practice with referents will support reasoning skills when working with larger and smaller scales of measurement.

Activities to Support Lesson Planning			
Grade P	**Grade 1**	**Grade 2**	**Grade 3**
In the classroom or outside on the playground, have students compare the length, width, or height of given objects. Use direct and indirect comparison. **Question prompts:** Knowledge: Which object is longer? Shorter? How do you know? Application: Find two object that have the same or similar length, width, or height. Compare your objects with another person's object. Which is shorter? Longer? Analysis: Sort all the collected objects from shortest to longest. How do you know you are correct? If I was to add one more object, where would you place the object in comparison to the others already lined up?	In the classroom or outside on the playground, have students compare the length, width, or height of given objects. Use direct and indirect comparison. **Question prompts:** Knowledge: Which object is longer? Shorter? How do you know? Application: Find two object that have the same or similar length, width, or height. Compare your objects with another person's object. Which is shorter? Longer? How do you know? What attributes are you using? Analysis: Sort all the collected objects from shortest to longest. How do you know you are correct? If I was to add one more object, where would you place the object in comparison to the others already lined up? What attributes are you using to make that decision?	In the classroom or outside on the playground, have students measure the length, width, or height of given objects using two different units (e.g., snap cubes, base-ten rods, short and long paperclips, string). **Question Prompts:** Knowledge: How long is each of your objects? Application: How does the length of the object compare to others you measured (shorter, longer)? By how much? Analysis: What happens when you don't use the same units to measure the object? Does it increase in size? Does it become shorter or longer? How does it compare to other objects measured? Explain what happens when you use different units?	In the classroom or outside on the playground, have students measure the length, width, or height of given objects. They can use a standard ruler. Have students complete the activity again using a ripped tape measure and ask them to measure items in the classroom. Observe how they attempt to measure items.  **Question Prompts:** Knowledge: How long are each of your objects? Did you get the same lengths as your partner or another group? Application: How do you use the ruler to make sure you get the same length as your partner? Analysis: If you are going to measure the length of the bookshelf, what strategy would you use if your ruler isn't long enough? How do you know your strategy is going to work?

Using Personal Referents	
Misconceptions/Errors in Student Work	**Possible Next Steps in the Classroom**
When students do not understand the size of a standard unit, using personal referents becomes harder to do. For example, students may overestimate and underestimate length. Students may have yet to develop familiar benchmarks they can apply everyday to estimate length. This leads to the lack of mental internalization of their personal referents so they cannot call upon these visual images when estimating. Errors when estimating or comparing length become difficult. For examples: *Which object is the best referent for 1 metre?* • *the length of a hallway (students understand that a hallway is longer than then them and a metre is long, but have limited relation between them – assume they are the same)* • *the width of a finger (students may have a lot of experience with this referent, but may confuse the lengths of a centimetre and a metre)* • *the height of a building (students understand that a building is taller than them and a metre is long, but have limited relation between them – assume they are the same)* • *the width of a door (correct answer)*	**Establishing Referents and Practicing Estimation** The best approach to improving estimation skills is to have students do a lot of estimating. Referents for useful measures can be developed and recorded on a class chart. Help students to develop strategies that enable them to use their referent lengths. For example, the height of the doorknob or bookshelf or the length of a book or pencil. Hand and finger widths are also helpful estimation tools. Verify with rulers of different lengths. Use chunking when appropriate. In other words, use smaller referents/lengths to help estimate longer ones. Help students to iterate mentally or physically. Be precise with your language, and do not use the word “measure” interchangeably with the word “estimate”.

Activities to Support Lesson Planning

Estimation activities do not have to be elaborate. Any measurement activity can have an "estimate first" component. For more emphasis on the process of estimation itself, simply think of measures that can be estimated, and have students estimate. Below are a few suggestions.

Grade P

Ask students to find something that is the same size, shorter or longer than a given object. Have them the object or a picture of the object to select something in the classroom.	OR	Conduct estimation scavenger hunts. Give teams a list of visual images of objects of different lengths and have students find items in the classroom that are close to having those lengths. Have students order the objects by length. Do not use measuring instruments.

Knowledge: Which object is longer? Shorter? How do you know?

Application: Find two object that have the same or similar length, width, or height. Compare your objects with another person's object. Which is shorter? Longer? How do you know? What attributes are you using?

Analysis: Sort all the collected objects from shortest to longest. How do you know you are correct? If I was to add one more object, where would you place the object in comparison to the others already lined up? What attributes are you using to make that decision?

Grade 1

Ask students to find something that is the same size, shorter or longer than a given object. Have them use a mental picture of an object known to them to select something in the classroom.	OR	Conduct estimation scavenger hunts. Give teams a list of visual images of objects of different lengths and have students find items in the classroom that are close to having those lengths. Have students order the objects by length. Do not use measuring instruments.

Knowledge: Which object is longer? Shorter? How do you know?

Application: Find two object that have the same or similar length, width, or height. Compare your objects with another person's object. Which is shorter? Longer? How do you know? What attributes are you using?

Analysis: Sort all the collected objects from shortest to longest. How do you know you are correct? If I was to add one more object, where would you place the object in comparison to the others already lined up? What attributes are you using to make that decision?

Grade 2

Ask students to find something that is about the length of a paperclip, 10 snap cubes, or other non-standard unit items students are used to using. Have them use a mental picture of an object known to them to select something in the classroom of the same length.	OR	Conduct estimation scavenger hunts. Give teams a list of nonstandard measurements and have them find things that are close to having those measurements. Do not use measuring instruments.

Knowledge: What did you do to help you estimate the length?

Application: How does your estimate compare to the nonstandard measurement given? How does your estimate compare to your partner's estimate?

Analysis: Did you use the same referent? Does it matter if your referents are different?

Grade 3

Ask students to find something that is about 1 cm, 10 cm, or 1 m long. Have them use a mental picture of an object known to them to select something in the classroom of the same length.	OR	Conduct estimation scavenger hunts. Give teams a list of standard measurements and have them find things that are close to having those measurements. Do not use measuring instruments.

Knowledge: What referent did you use to help you estimate the length?

Application: How does your estimate compare to the actual measurement? How does your estimate compare to your partner's estimate?

Analysis: Did you use the same referent? Does it matter if your referents are different?

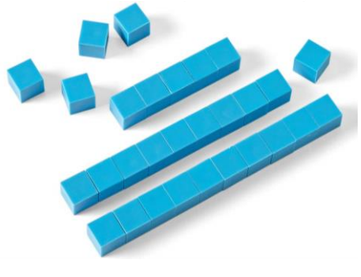
What are some sample questions to help support assessment?

Cognitive Level	Grade P	Grade 1	Grade 2	Grade 3
Knowledge	Have a few objects available and a piece of string. Which is shorter than this piece of string? Which is longer than this piece of string? Have students explain their thinking. Give students two objects (e.g., crayon, pencil, paper clip) and ask to predict which item is longer or shorter. Have them explain their thinking.	Can you tell me which of these two objects is longer? Shorter? How do you know? Show students five different objects one at a time. For each object, ask students if the length of the object is less than, greater than, or about the same as the length of a sheet of paper. After students record the estimation for each object, do a direct comparison of that object to a piece of paper. After students have estimated and compared the length of each object, ask them to place the objects in order from shortest to longest.	What are some things that are easy to measure? Hard to measure? Why? What are some tools we can use to measure the length of an object?	What could you use to measure if you don't have a ruler? What object could you use as a referent for 1 m? The width of your thumb is about what unit of measure?
Application	Give students a piece of string and ask them to find two objects that are the same length, two that are shorter and two that are longer. Sort them into "shorter", "same", and "longer" groups. Show students a length of string. Ask them if they think they are taller than the string without allowing them to stand beside it. After making a prediction, students should measure themselves against the string.	Give each student a true and a false card. Make comparative statements and ask students to hold up either the true or false card in response. For example, "My desk is longer than the white board." "The white board eraser is shorter than this paper clip." Have students explain their thinking. How can you compare these objects? (e.g. pencil and book; block and a counter; bookshelf and chair Ask students to order objects from shortest to longest, shortest to tallest. Include situations in which students are dealing with an independent variable, such as objects that are not straight and objects that are also wide or thick.	Show students measurements with non-standard units, some of which are correct and others of which have obvious gaps and overlapping and ask them to explain which measurements are accurate and which are not. Students should fix the incorrect measurements. Before students make any measurement of length, have them examine the object that they are going to measure and the non-standard unit they will use, and get them to commit in writing their estimates of the number of units they will use. After they measure the length, get them to compare it to their estimates, and have them discuss strategies that could be used to get closer estimates.	Estimate the height of a doorknob from the floor. Is this pencil 15 cm long? Explain your thinking.  Using a broken tape measure, measure length of your desk. Have a partner measure the same length. Are the results the same? Verify with another group.

Analysis	Ask students if it is possible or impossible for some of the following: my arm is longer than my foot, my hand is longer than this crayon, or my finger is longer pencil. Am I taller when I stand up compared to when I lie down? Ask students to describe the steps, in order, that one would take to decide which of two objects is longer.	Have students participate in “dramas” in which someone measures incorrectly, and the other students figure out what is wrong. For example, one student could line up pencils of different lengths to measure an item, or could use uniform units, but counts, “1, 2, 4, 5, … Ask two students to perform standing long jumps. Encourage them to find a way to determine who jumped farther. Emphasize afterwards, with the students, the importance of a common starting point.	Give students common objects found in the classroom that can be easily bent into curvy lines, such as pipe cleaners and wool/string. Have students first estimate and measure the objects straight and then curvy. They could also measure all around an object, such as their desk or a picture frame. Explain your strategies when measuring. What do you do when the objects do not have straight lines to measure? How did you determine your lengths?	How can you use a piece of string to identify objects that are about half a metre in length? Explain the relationship between 1 mm, 1 cm and 1 m. When would you use each of these units to measure?

Supporting Resources

Manipulatives and Models to Support Learning

String, Yarn, or Rope 	Paper clips or Plastic links 	Markers or Crayons 	Craft Sticks 	Linking Cubes 	Centimetre cubes 
Cuisenaire Rods 	Base-ten blocks (units, rods) 	Ruler 	Measuring Tape 	Trundle Wheel 	Finger, hand, and arm lengths 

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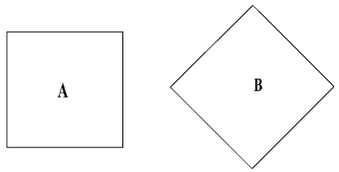
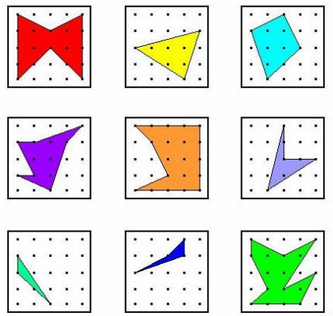
Identifying and Sorting Irregular Polygons

Alignment to previous Outcomes			Related Outcome
PG01: Students will be expected to sort 3-D objects using one attribute. **PG02:** Students will be expected to build and describe 3-D objects.	**1G01:** Students will be expected to sort 3-D objects and 2-D shapes using one attribute and explain the sorting rule. **1G02:** Students will be expected to replicate composite 2-D shapes and 3- D objects. **1G03**: Students will be expected to identify 2-D shapes in 3-D objects.	**2G03:** Students will be expected to recognize, name, describe, compare and build 2-D shapes, including triangles, squares, rectangles, and circles.	**3G02:** Students will be expected to name, describe, compare, create, and sort regular and irregular polygons, including triangles, quadrilaterals, pentagons, hexagons, and octagons according to the number of sides.

What conclusions can be drawn from the NSA: Mathematics in Grade 3?

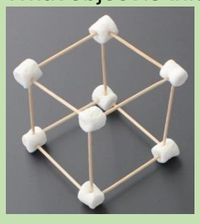
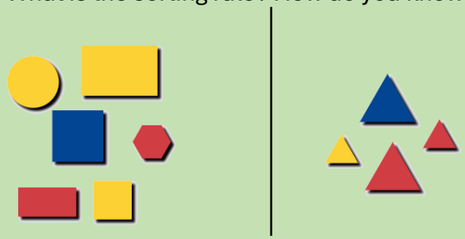
Students need to continue developing their knowledge of shapes by describing and sorting them according to their geometric attributes. Over half of the students had difficulty sorting and classifying irregular shapes. They need to focus on comparing the number of sides as the key attribute for classifying polygons. Students need more experience with irregular polygons, so that they begin to realize that a polygon, regardless of its dimensions, or position in space, remains the same shape. Vocabulary is also important as it associates with common shapes. For example, when students were asked to determine the perimeter of a polygon when the image was not provided, over half of the students responded incorrectly.

Why is this an area of need and how can we support students?

Developing Schema alongside Attribute Vocabulary	
Misconceptions/Errors in Student Work	**Possible Next Steps in the Classroom**
Some students incorrectly believe that the orientation of a geometric figure, changes the figure itself. Students recognize that shape A is square but think that shape B is not a square.  Unfamiliarity with irregular shapes and their names also pose difficulty for students in identifying and comparing the shapes. Common attributes are difficult to identify. For example, students misidentify the following are all pentagons. 	Provide students with various sizes of a polygon. Have students count the number of sides and identify the polygon. By having a variety of these experiences with different polygons, students should begin to realize that a polygon, regardless of its dimensions, remains the same shape. Use geoboards to create irregular polygons. Some examples include:  Students should also find examples of polygons in the world around them. Sort the shapes according to the number of sides as the key attribute for classifying polygons. This can also be turned into a game where pairs of students must guess a sorting rule. Venn diagrams or Carroll diagrams may be used to help with the sorting.

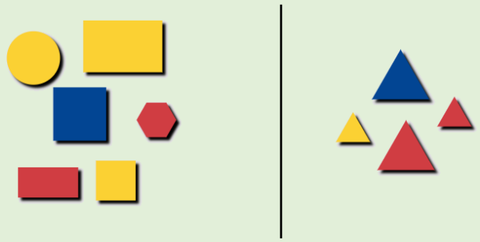
Activities to Support Lesson Planning			
Opportunities for students to look at, touch, compare, and create various types of regular and irregular polygons and objects supports the development of an understanding of the 2D shapes and 3D objects in the world around us. Simple search and find activities, building blocks and construction activities, along with drawing activities that include specific polygons helps in engaging students with the attributes and vocabulary of shapes and objects.			
Grade P	**Grade 1**	**Grade 2**	**Grade 3**
Knowledge: Have students identify objects around the classroom or in nature that resemble a specific given object (e.g., cylinder, cone, sphere, cube). Ask, for example, what makes it a cube? Application: Sort the objects into two groups. What is the sorting rule? What makes the objects the same/different? Analysis: Play eye-spy: I see an object that has 5 faces. What is it?	Knowledge: Identify/Draw all the shapes you see on this prism. Application: Sort the following objects/shapes using your own sorting rule. What is the rule? How are the objects/shapes the same/different? Analysis: A certain object is made up of 2 squares. What could it be?	Knowledge: Provide images of a series of regular and irregular shapes from the real world. Have students identify the shapes. Application: Provide students with two objects/shapes (e.g., cube and triangular prism or square and rectangle). How are these prisms/shapes different? How are these prisms/shapes the same? Analysis: Copy this shape (e.g., square). Draw a shape that is different from the shape in one way, but the same in another way. How are they different? How are they alike?	Knowledge: Provide images of a series of regular and irregular shapes from the real world. Have students identify the shapes. Application: Create an image using the following shapes: quadrilaterals, triangles, and circles. The image must also include at least three shapes that have 5 or more sides. Analysis: Using tangrams or pattern blocks, create a large square. Create a shape with at least one triangle and one quadrilateral. What do you call this shape?

What are some sample questions to help support assessment?

Cognitive Level	Grade P	Grade 1	Grade 2	Grade 3
Knowledge	Sort the objects.  What object is this called? How do you know? 	What is the sorting rule? How do you know?  Construct a triangular prism. What shapes did you combine to make your prism?	Name and describe the shapes you see in the diagram.  Construct a diagram using the following shapes: circle, square, rectangle and triangle.	Which shape is a quadrilateral? Hexagon? Name the other shapes?  Which figure is not a polygon? 

Application

What is the sorting rule? How do you know?

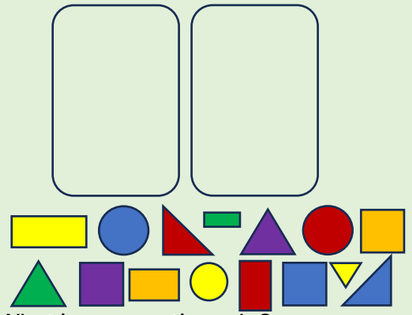


Construct a triangular prism. What shapes did you combine to make your prism?

Cut out the following images. Sort them. Have a partner determine your sorting rule. Switch.

Construct a house using toothpicks and marshmallows. What shapes did you construct/combine to build your house? How many different shapes did you construct?

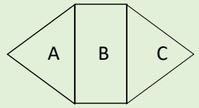
Sort the following shapes.



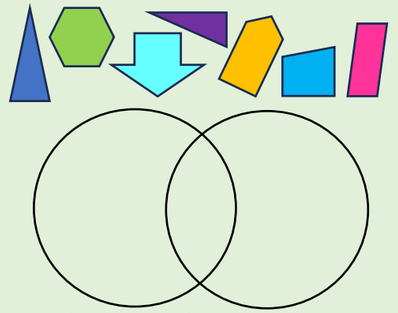
What is your sorting rule?

Fill in the image using pattern blocks. What shapes are you using to fill in the image? Can you recreate it using different shapes? What would be the shape you would use the most? Least? Are there shapes you cannot use? Why?

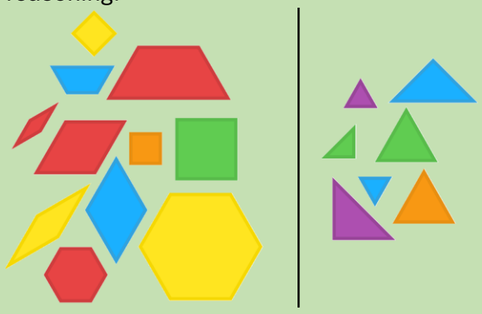
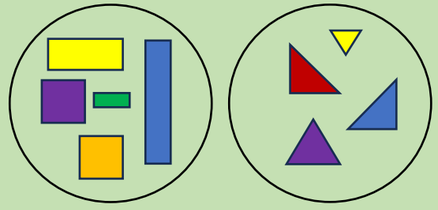
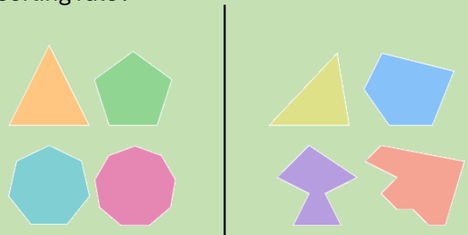
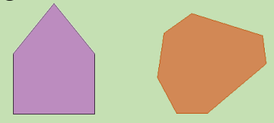
Which polygon results when the geometric figures of A, B, and C are joined in this way?



Sort the following shapes.

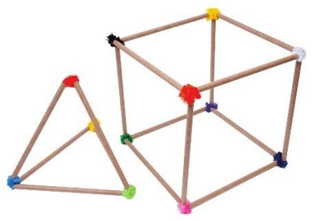
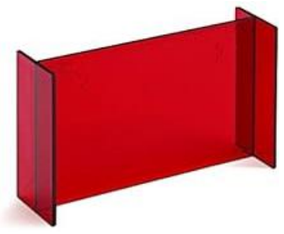
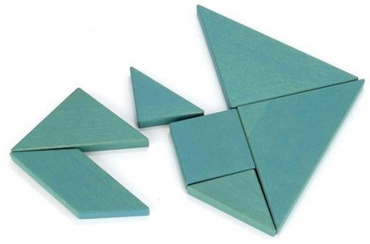


What is your sorting rule?

Analysis				
	Cut out the following images. Sort them. Have a partner determine your sorting rule. Switch.  Construct a house using toothpicks and marshmallows. What shapes did you construct/combine to build your house? How many different shapes did you construct?	Sort the shapes in a different way. Explain your reasoning.  Construct a pyramid and a prism using toothpicks and marshmallows. What shapes did you construct/combine to build each object? How many different shapes did you construct? How are the objects the same and different?	These shapes have been sorted. What is the sorting rule?  Where would you place the following shapes for sorting? 	These shapes have been sorted. What is the sorting rule?  Where would you place the following shapes for sorting. 

Supporting Resources

Manipulatives and models to Support Learning

Pattern Blocks	Straws and Pipe Cleaners	Geoboards	Miras	Tangrams
				

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Interpreting Data Represented in Tables and Graphs

Alignment to previous Outcomes	Related Outcome
2SP02: Students will be expected to construct and interpret concrete graphs and pictographs to solve problems.	**3SP02:** Students will be expected to construct, label, and interpret bar graphs to solve problems.

What conclusions can be drawn from the NSA: Mathematics in Grade 3?

Students were challenged in determining necessary information to include on graphs and when reading information from tally charts, line plots and bar graphs. Students also need to develop the skill of interpreting graphs, answering questions, and drawing conclusions from tally charts, line plots and bar graphs.

Why is this an area of need and how can we support students?

Common Attributes

Misconceptions/Errors in Student Work

A common misconception or error that many students make is concerning the common attributes of line plots, horizontal bar graphs, pictographs, and vertical bar graphs with the same given set of data.

Students sometimes do not connect attributes between representations; there could be different titles, different use of the horizontal axis, and different labels.

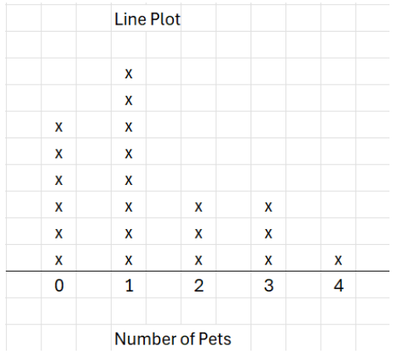
For example, while the representations below show the same data, they are presented in slightly different ways, and students may see this as different data and different results. Any missing labels may also go unnoticed.

Movie Types	Number of students
Action	𝍸
Comedy	𝍸 𝍸 |||
Drama	|||
Science Fiction	||||

The attributes that are commonly forgotten to identify include a title and labels on axes or items.

Possible Next Steps in the Classroom

Students should be encouraged to collect, organize, and record their data using a tally system, line plots, charts, and lists to answer questions relevant to their everyday life.



Provide students with opportunities to use tally marks and lists to keep track of information as they collect it outside of math class.

Provide students with opportunities to organize data on a line plot(s) (first using grid paper). A line plot provides a bridge from tally charts to bar graphs.

Ensure students include a title or heading and labels on constructed charts and graphs to inform the reader about the meaning of the data.

More importantly, provide opportunities for students to analyze graphical texts presented in different ways and found in different sources. Draw students' attention to how difficult it is to make sense of information when titles or labels are not provided, such as in the line plot pictured.

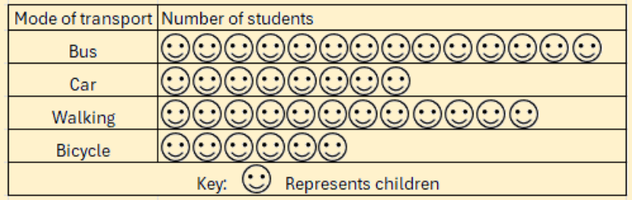
Encourage discussions about how data is presented in different ways. Data talks can be used as short classroom discussion to help students develop data literacy. It is similar in structure to a number talk, but instead of numbers, students are shown a data visual.

Activities to Support Lesson Planning

Incorporating data talks into classroom instruction supports not only data literacy, but also encourages critical thinking, communication, and relevance to real-world contexts. In a data talk, the focus of learning and discussion is around a presentation of data. The discussions can cover a wide range of skills, including analysis, visualization, interpretation, and decision-making where students are sharing findings, insights, and best practices related to managing and utilizing data effectively. Data talks provide opportunities for students to voice their opinions and perspectives based on evidence. This empowers them to take ownership of their learning and contribute meaningfully to discussions. Teachers can use data talks to assess student understanding and tailor instruction accordingly. They can identify misconceptions, address gaps in knowledge, and provide targeted support to individual students. Below are a couple of examples of how data talks can be used in the classroom.

Grade 2

Mode of transport	Number of students
Bus	𝍸𝍸 ||||
Car	𝍸 |||
Walking	𝍸𝍸 ||
Bicycle	𝍸 ||

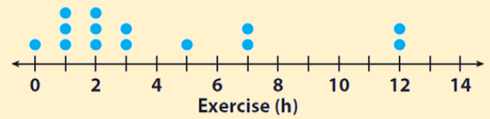


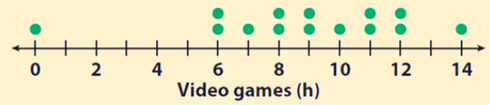
Show students the information presented above. Have them think about what they are seeing by using the following question prompts.

Question Prompts:
Knowledge: What do you notice?
Application: What is going on in this data visualization? How do the data presentations compare?
Analysis: What makes it easy / difficult to read the information presented in the graph? What would you change to make it better?

Use student responses to co-create success criteria and have them recreate the graph based on their input.

Grade 3





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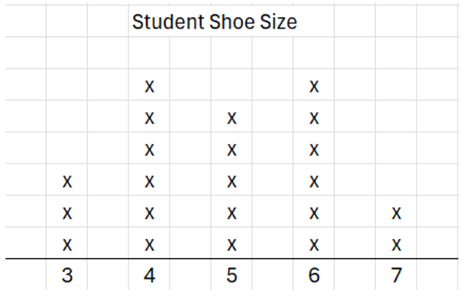
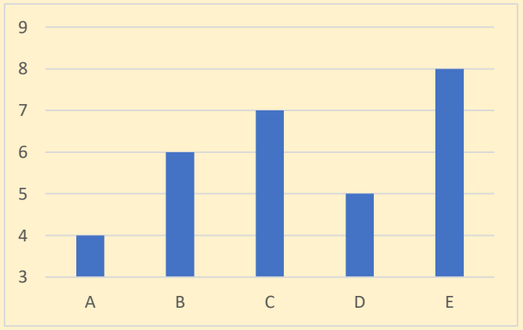
Interpretating Graphical Texts

Misconceptions/Errors in Student Work	Possible Next Steps in the Classroom
The difficulty with bar graphs, whether horizontal or vertical, is that students interpret data from the wrong bar on the graph. Therefore, when asked to answer questions concerning the bar graph data, it is not correct. For example, on the provincial assessment, although students did well when working with line plots, they did make errors when reading or counting the X's to draw conclusions. For example:  *What conclusion can be drawn from this line plot?* • *There are more students with size 5 shoes, then size 4 shoes. (students select the larger shoe size rather than the amount)* • *There are more students with size 7 shoes, then size 3 shoes. (students select the larger shoe size rather than the amount)* • *There are fewer students with size 6 shoes, then size 7 shoes. (students select the smaller shoe size rather than the amount)* • *There are just as many students with size 4 shoes, as students with size 6 shoes. (correct answer)*	As discussed through data talks, students need opportunities and experiences to interpret information collected, organized, and displayed in various ways like tally charts, charts, line plots and bar graphs. They need to be encouraged to ask or write questions that go beyond simplistic reading of a graph. Both literal and inferential comprehension questions need to be asked. Students should be provided with opportunities to discuss the information obtained from a display of data and be encouraged to work together to formulate questions that can be answered by other students using the data. Examples, include presenting students with vertical and horizontal bar graphs that represent two different sets of data and discuss the similarities and differences found between the two bar graphs, such as title, axes, and labels for the axes, numerical scale, and bars. Have students draw conclusions from the information presented in graphs. They should be encouraged to ask questions that go beyond simplistic reading of a graph. Teachers should ask both literal and inferential comprehension questions, such as: • What can you tell about _____ by looking at this graph? • How many more/less than …? • Based on the information presented in the graph, what other conclusions can you make? • Why do you think _____? Encourage data talks to pique students' curiosity and encourage question asking, and to help them understand and "read" the data-filled world in which they live.

Activities to Support Lesson Planning

Further data talk examples that can be leveraged to focus on how students are interpreting the data and drawing appropriate conclusions including inferring.

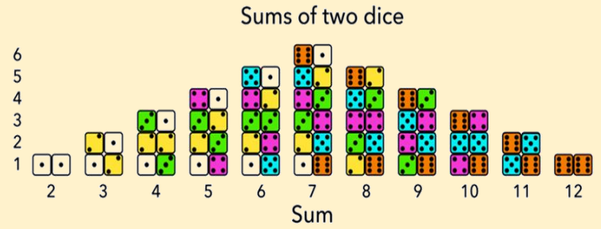
Grade 2



Question Prompts:
Knowledge: What do you notice?
Application: What conclusions can you make about the data?
Analysis: What makes it easy / difficult to read the information presented in the graph? What would you change to make it better?

Use student responses to co-create success criteria and have them recreate the graph based on their input.

Grade 3

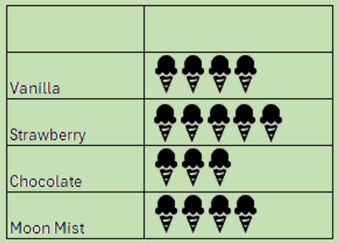
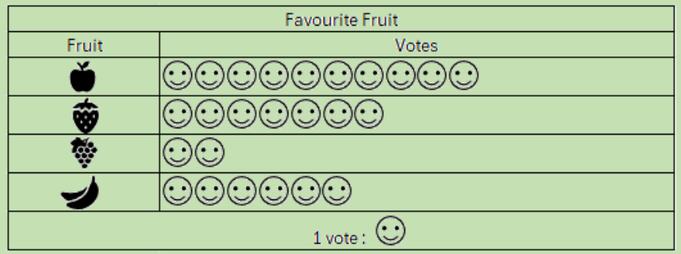
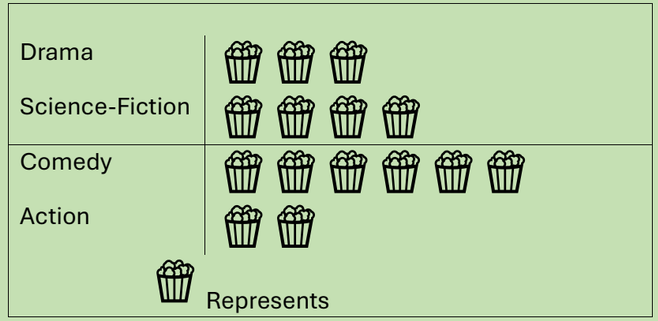


*Youcubed.org

Question Prompts:
Knowledge: What do you notice?
Application: What conclusions can you make about the data?
Analysis: What makes it easy / difficult to read the information presented in the graph? Why do you think the data was presented this way?

Use student responses to co-create success criteria and have them recreate the graph based on their input.

What are some sample questions to help support assessment?

Cognitive Level	Grade 2	Grade 3
Knowledge	Use the following 2 pictographs to identify what attributes are needed when presenting information this way? What are your success criteria for creating a pictograph?   Create a people graph to show the number of people with long sleeves and the number of people with short sleeves.	What information should be included when constructing a bar graph to make sure the reader understands the data being presented? The following pictograph was constructed to present the collected data. What is missing from the pictograph? 

Application

The following pictograph was constructed to present the collected data. Some errors were made.

Drama	
Science-Fiction	
Comedy	
Action	
Represents	

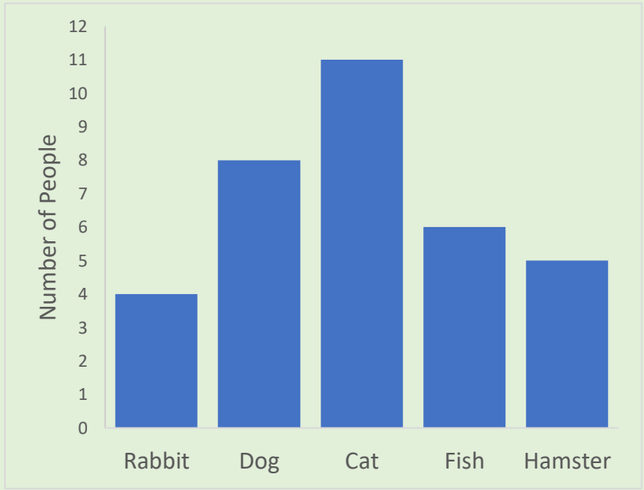
What are the errors?

I asked the Grade 2 students about their favourite season. This pictograph shows the results of the survey.

Favourite Season of Grade 3 Students	
Summer	
Fall	
Winter	
Spring	
Represents 1 student	

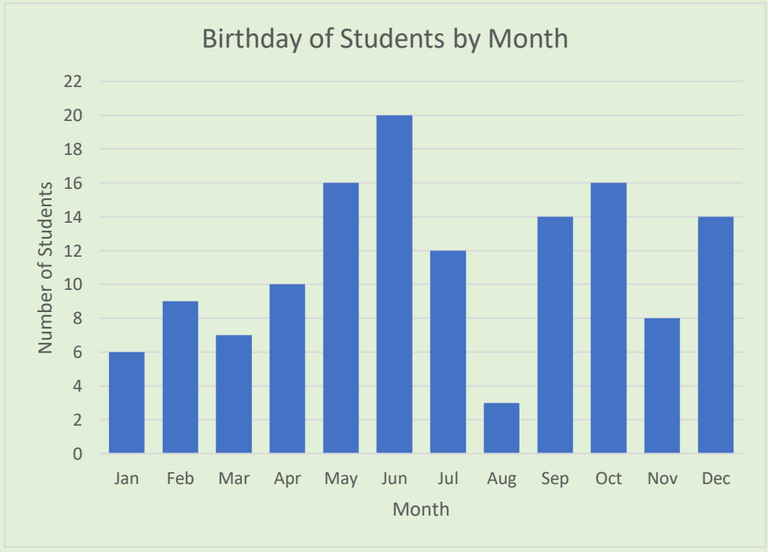
What conclusions can you make about the graph?

The following bar graph was constructed. Some errors were made.



What are the errors?

What conclusions can you make about the graph?



Analysis

When would a tally chart be useful and when would a pictograph be useful to show your data? When would they both be useful and when would only one be useful?

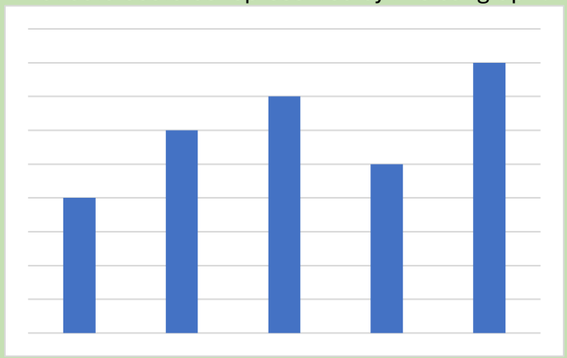
One pictograph shows the number of letters written by students. Another pictograph shows the number of books read by students. If 18 books have been read this week, which pictograph is which and why? How would you fix the pictographs to make them better at showing the data?

Monday	
Tuesday	
Wednesday	
Thursday	
Friday	
Saturday	

Monday	
Tuesday	
Wednesday	
Thursday	
Friday	
Saturday	
Sunday	

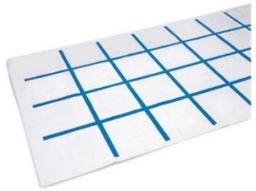
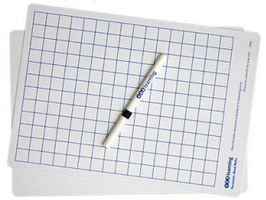
When would a bar graph be useful and when would a pictograph be useful to show your data? When would they both be useful and when would only one be useful?

What data could be represented by this bar graph?



Supporting Resources

Manipulatives and Models to Support Learning

Graphing mat or a learning carpet 	**Linking cubes** 	**Counters** 	**Square tiles** 	**Grid paper** 	**Stickers or Stamps** 

Print and Electronic Resources

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