LESSON LEARNED

Focus: Measuring and Estimating Length

Nova Scotia Assessment: Mathematics Grade 3

"For learners to succeed, teachers must assess students' individual abilities and characteristics and choose appropriate and effective instructional strategies accordingly."
– Helene J. Sherman

NOVA SCOTIA

Purpose of this Document

This Lessons Learned document was developed based on an analysis of the Item Description Reports for the Nova Scotia Assessment: Mathematics in Grade 3 (NSA-M3). This document is intended to support all classroom teachers at grades Primary – 3, and administrators at the school, region, and provincial levels. The focus of the document is to help educators work through the process of taking in the information provided by the data analysis and see how it can inform lesson design and assessment in the classroom.

It is suggested that school teams make use of this resource in concert with their school's Item Description Report provided by the Department of Education and Early Childhood Development to all regional centres for education. These reports include student achievement data at the school, regional centre, and provincial level for all questions appearing on the Mathematics in Grade 3 Assessment. By analyzing their own performance on groupings of questions dealing with similar outcomes, schools can identify areas of strength and areas where changes in instruction and/or assessment might be made. This process is designed to foster continued discussions, explorations, and support for mathematics focus at the classroom, school, regional centre, and provincial levels that are all based on valid and reliable data.

This document specifically addresses some of the areas that students across the province found challenging based on provincial assessment data. It is essential that teachers consider assessment evidence from a variety of sources to inform the next steps most appropriate for their students. Effective classroom instruction and assessment strategies are responsive to the individual learners within a classroom.

This document highlights those outcomes where students seem to require additional support. It provides some information about student performance on the assessment in addition to suggested classroom instruction strategies. Sample assessment items are included for each topic explored.

Overview of the Nova Scotia Assessment: Mathematics in Grade 3

Nova Scotia Assessments are large-scale assessments that provide reliable data about how well all students in the province are learning the mathematics curricula. It is different from many standardized tests in that all questions are written by Nova Scotia teachers to align with curriculum outcomes and the results reflect a snapshot of how well students are learning these outcomes. These results can be counted on to provide a good picture of how well students are learning curriculum outcomes within schools, regions and in the province. Since the assessments are based on the Nova Scotia curriculum, and are developed by Nova Scotia teachers, results can be used to determine whether the curriculum, approaches to teaching and allocation of resources are effective. Furthermore, because individual student results are available, these, in conjunction with other classroom assessment evidence, help classroom teachers understand what each student has under control and identify next steps to inform instruction.

The assessment provides information about mathematics for each student and complements assessment data collected in the classroom. This assessment is administered at the end of Grade 3. It is designed to provide detailed information for every student in the province regarding their progress in achieving a selection of mathematics curriculum outcomes at the end of Grade 3. Information from this assessment can be used by teachers to inform their instruction and next steps in providing support and intervention for their students.

Lessons Learned Overview

Provincial assessments and examinations generate information that teachers can use to help inform classroom instruction and assessment. Following the analysis of each assessment or examination, patterns and trends are identified. These include areas of strength and areas for growth. The Lessons Learned documents specifically highlight concepts where growth is still needed.

There are four areas that have been identified as the areas of focus for this Lessons Learned document. They are:

- Solving whole number addition and subtraction questions in context.
- Measuring and estimating length.
- Identifying and sorting irregular polygons.
- Interpreting data represented in tables and graphs.

This section specifically addresses measuring and estimating length. It begins with an overview of the student errors and misconceptions identified through the provincial assessment. These include:

- Using and ruler
- Using personal referents

Strategies are then outlined that are designed to enhance student comprehension, drawing from researched best practices. The strategies emphasize the integration of essential models, tools, and interconnections to facilitate the transition between concrete, pictorial, and abstract representations of concepts, highlighted by the importance of deliberate planning and purposeful questioning. To support both assessment and instruction, sample lesson activities are presented alongside a series of cognitive-level questions, providing educators with ideas for addressing knowledge gaps and fostering strategic reasoning and problem-solving skills. Each section culminates with a selection of print and online resources, as well as recommended manipulatives to support professional learning and student understanding of that topic.

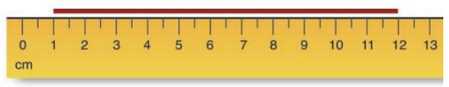
Measuring and Estimating Length

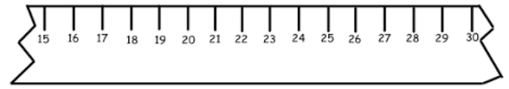
Alignment to previous Outcomes			Related Outcome
PM01: Students will be expected to use direct comparison to compare two objects based on a single attribute, such as length, mass, volume, and capacity	**1M01:** Students will be expected to demonstrate an understanding of measurement as a process of comparing by ▪ identifying attributes that can be compared ▪ ordering objects ▪ making statements of comparison ▪ filling, covering, or matching	**2M04:** Students will be expected to measure length to the nearest nonstandard unit by using multiple copies of a unit and using a single copy of a unit (iteration process).	**3M03:** Students will be expected to demonstrate an understanding of measuring length (cm, m).

What conclusions can be drawn from the NSA: Mathematics in Grade 3?

Students continue to have difficulty reading and interpreting measures on a ruler. Over half of students read the number on the ruler that is aligned with the end of the object rather than determine the number of intervals between the start and end of the object. Continued emphasis is also needed on the use of referents. Just under half of students still struggle to use personal referents to help visualize units of length (centimetres and metres). Continued experience is still needed in using and relating to various tools and measures to support an understanding of length.

Why is this an area of need and how can we support students?

Using a Ruler	
Misconceptions/Errors in Student Work	**Possible Next Steps in the Classroom**
When using rulers, students have a tendency to only look at the number on the ruler that is aligned with the end of the object.  For example, students read the line as being 12 cm instead of 11 cm. Looking at the end tick mark and not the distance between the start and end points. Some students also count the numbers or tick marks along the length of an object, rather than the intervals between the numbers.  For example, students read the line above as being 12 cm instead of 11 cm. They use the count of 12 tick marks beginning at 1 cm and using the distance between the start and end points or the jumps from 1 to 12.	**Measuring with a Ruler** The skill of learning how to use a ruler is introduced for the first time in grade 3. Emphasis should be on counting the intervals between the numbers, rather than looking at the number on the ruler that is aligned with the end of the object. In younger grades this can be modelled using a number line by highlighting jumps forwards and backwards between the numbers rather than just counting the tick marks. In addition, using a number path helps to highlight the space between the tick marks and when using the same jumping motions instead of just counting the number of units (squares, cubes) supports the transition to reading values on rulers. Have students use simple rulers that are created by students initially. Move onto tools that are easy for students to read. Students should use rulers (or the side of the ruler) that show only numbered centimetres and not millimetres.  Show students how to measure something that is longer than a ruler by marking, recording, and starting again. When using nonstandard units, help students to make sure there are no spaces between the units being used. When transitioning from nonstandard to standard units, demonstrate that the numbers on the ruler correspond to the number of small cubes by starting at 0 and lining up small cubes from base-ten materials along the ruler. Cuisenaire rods and base-ten blocks are a nice tool to show units of 1 cm and 1 dm or their relation to 1 m. To further support understanding of length, students should identify objects from around the classroom that would be an appropriate referent for a centimetre or a metre; for example, the width of a pencil (cm), the distance from the bottom of a door to the doorknob (1m). Practice with referents will support reasoning skills when working with larger and smaller scales of measurement.

Activities to Support Lesson Planning			
Grade P	**Grade 1**	**Grade 2**	**Grade 3**
In the classroom or outside on the playground, have students compare the length, width, or height of given objects. Use direct and indirect comparison. **Question prompts:** Knowledge: Which object is longer? Shorter? How do you know? Application: Find two object that have the same or similar length, width, or height. Compare your objects with another person's object. Which is shorter? Longer? Analysis: Sort all the collected objects from shortest to longest. How do you know you are correct? If I was to add one more object, where would you place the object in comparison to the others already lined up?	In the classroom or outside on the playground, have students compare the length, width, or height of given objects. Use direct and indirect comparison. **Question prompts:** Knowledge: Which object is longer? Shorter? How do you know? Application: Find two object that have the same or similar length, width, or height. Compare your objects with another person's object. Which is shorter? Longer? How do you know? What attributes are you using? Analysis: Sort all the collected objects from shortest to longest. How do you know you are correct? If I was to add one more object, where would you place the object in comparison to the others already lined up? What attributes are you using to make that decision?	In the classroom or outside on the playground, have students measure the length, width, or height of given objects using two different units (e.g., snap cubes, base-ten rods, short and long paperclips, string). **Question Prompts:** Knowledge: How long is each of your objects? Application: How does the length of the object compare to others you measured (shorter, longer)? By how much? Analysis: What happens when you don't use the same units to measure the object? Does it increase in size? Does it become shorter or longer? How does it compare to other objects measured? Explain what happens when you use different units?	In the classroom or outside on the playground, have students measure the length, width, or height of given objects. They can use a standard ruler. Have students complete the activity again using a ripped tape measure and ask them to measure items in the classroom. Observe how they attempt to measure items.  **Question Prompts:** Knowledge: How long are each of your objects? Did you get the same lengths as your partner or another group? Application: How do you use the ruler to make sure you get the same length as your partner? Analysis: If you are going to measure the length of the bookshelf, what strategy would you use if your ruler isn't long enough? How do you know your strategy is going to work?

Using Personal Referents	
Misconceptions/Errors in Student Work	**Possible Next Steps in the Classroom**
When students do not understand the size of a standard unit, using personal referents becomes harder to do. For example, students may overestimate and underestimate length. Students may have yet to develop familiar benchmarks they can apply everyday to estimate length. This leads to the lack of mental internalization of their personal referents so they cannot call upon these visual images when estimating. Errors when estimating or comparing length become difficult. For examples: *Which object is the best referent for 1 metre?* • *the length of a hallway (students understand that a hallway is longer than then them and a metre is long, but have limited relation between them – assume they are the same)* • *the width of a finger (students may have a lot of experience with this referent, but may confuse the lengths of a centimetre and a metre)* • *the height of a building (students understand that a building is taller than them and a metre is long, but have limited relation between them – assume they are the same)* • *the width of a door (correct answer)*	**Establishing Referents and Practicing Estimation** The best approach to improving estimation skills is to have students do a lot of estimating. Referents for useful measures can be developed and recorded on a class chart. Help students to develop strategies that enable them to use their referent lengths. For example, the height of the doorknob or bookshelf or the length of a book or pencil. Hand and finger widths are also helpful estimation tools. Verify with rulers of different lengths. Use chunking when appropriate. In other words, use smaller referents/lengths to help estimate longer ones. Help students to iterate mentally or physically. Be precise with your language, and do not use the word “measure” interchangeably with the word “estimate”.

Activities to Support Lesson Planning

Estimation activities do not have to be elaborate. Any measurement activity can have an "estimate first" component. For more emphasis on the process of estimation itself, simply think of measures that can be estimated, and have students estimate. Below are a few suggestions.

Grade P

Ask students to find something that is the same size, shorter or longer than a given object. Have them the object or a picture of the object to select something in the classroom.	OR	Conduct estimation scavenger hunts. Give teams a list of visual images of objects of different lengths and have students find items in the classroom that are close to having those lengths. Have students order the objects by length. Do not use measuring instruments.

Knowledge: Which object is longer? Shorter? How do you know?

Application: Find two object that have the same or similar length, width, or height. Compare your objects with another person's object. Which is shorter? Longer? How do you know? What attributes are you using?

Analysis: Sort all the collected objects from shortest to longest. How do you know you are correct? If I was to add one more object, where would you place the object in comparison to the others already lined up? What attributes are you using to make that decision?

Grade 1

Ask students to find something that is the same size, shorter or longer than a given object. Have them use a mental picture of an object known to them to select something in the classroom.	OR	Conduct estimation scavenger hunts. Give teams a list of visual images of objects of different lengths and have students find items in the classroom that are close to having those lengths. Have students order the objects by length. Do not use measuring instruments.

Knowledge: Which object is longer? Shorter? How do you know?

Application: Find two object that have the same or similar length, width, or height. Compare your objects with another person's object. Which is shorter? Longer? How do you know? What attributes are you using?

Analysis: Sort all the collected objects from shortest to longest. How do you know you are correct? If I was to add one more object, where would you place the object in comparison to the others already lined up? What attributes are you using to make that decision?

Grade 2

Ask students to find something that is about the length of a paperclip, 10 snap cubes, or other non-standard unit items students are used to using. Have them use a mental picture of an object known to them to select something in the classroom of the same length.	OR	Conduct estimation scavenger hunts. Give teams a list of nonstandard measurements and have them find things that are close to having those measurements. Do not use measuring instruments.

Knowledge: What did you do to help you estimate the length?

Application: How does your estimate compare to the nonstandard measurement given? How does your estimate compare to your partner's estimate?

Analysis: Did you use the same referent? Does it matter if your referents are different?

Grade 3

Ask students to find something that is about 1 cm, 10 cm, or 1 m long. Have them use a mental picture of an object known to them to select something in the classroom of the same length.	OR	Conduct estimation scavenger hunts. Give teams a list of standard measurements and have them find things that are close to having those measurements. Do not use measuring instruments.

Knowledge: What referent did you use to help you estimate the length?

Application: How does your estimate compare to the actual measurement? How does your estimate compare to your partner's estimate?

Analysis: Did you use the same referent? Does it matter if your referents are different?

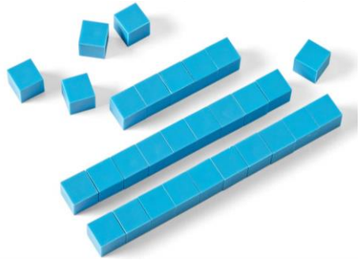
What are some sample questions to help support assessment?

Cognitive Level	Grade P	Grade 1	Grade 2	Grade 3
Knowledge	Have a few objects available and a piece of string. Which is shorter than this piece of string? Which is longer than this piece of string? Have students explain their thinking. Give students two objects (e.g., crayon, pencil, paper clip) and ask to predict which item is longer or shorter. Have them explain their thinking.	Can you tell me which of these two objects is longer? Shorter? How do you know? Show students five different objects one at a time. For each object, ask students if the length of the object is less than, greater than, or about the same as the length of a sheet of paper. After students record the estimation for each object, do a direct comparison of that object to a piece of paper. After students have estimated and compared the length of each object, ask them to place the objects in order from shortest to longest.	What are some things that are easy to measure? Hard to measure? Why? What are some tools we can use to measure the length of an object?	What could you use to measure if you don't have a ruler? What object could you use as a referent for 1 m? The width of your thumb is about what unit of measure?
Application	Give students a piece of string and ask them to find two objects that are the same length, two that are shorter and two that are longer. Sort them into "shorter", "same", and "longer" groups. Show students a length of string. Ask them if they think they are taller than the string without allowing them to stand beside it. After making a prediction, students should measure themselves against the string.	Give each student a true and a false card. Make comparative statements and ask students to hold up either the true or false card in response. For example, "My desk is longer than the white board." "The white board eraser is shorter than this paper clip." Have students explain their thinking. How can you compare these objects? (e.g. pencil and book; block and a counter; bookshelf and chair Ask students to order objects from shortest to longest, shortest to tallest. Include situations in which students are dealing with an independent variable, such as objects that are not straight and objects that are also wide or thick.	Show students measurements with non-standard units, some of which are correct and others of which have obvious gaps and overlapping and ask them to explain which measurements are accurate and which are not. Students should fix the incorrect measurements. Before students make any measurement of length, have them examine the object that they are going to measure and the non-standard unit they will use, and get them to commit in writing their estimates of the number of units they will use. After they measure the length, get them to compare it to their estimates, and have them discuss strategies that could be used to get closer estimates.	Estimate the height of a doorknob from the floor. Is this pencil 15 cm long? Explain your thinking.  Using a broken tape measure, measure length of your desk. Have a partner measure the same length. Are the results the same? Verify with another group.

Analysis	Ask students if it is possible or impossible for some of the following: my arm is longer than my foot, my hand is longer than this crayon, or my finger is longer pencil. Am I taller when I stand up compared to when I lie down? Ask students to describe the steps, in order, that one would take to decide which of two objects is longer.	Have students participate in “dramas” in which someone measures incorrectly, and the other students figure out what is wrong. For example, one student could line up pencils of different lengths to measure an item, or could use uniform units, but counts, “1, 2, 4, 5, … Ask two students to perform standing long jumps. Encourage them to find a way to determine who jumped farther. Emphasize afterwards, with the students, the importance of a common starting point.	Give students common objects found in the classroom that can be easily bent into curvy lines, such as pipe cleaners and wool/string. Have students first estimate and measure the objects straight and then curvy. They could also measure all around an object, such as their desk or a picture frame. Explain your strategies when measuring. What do you do when the objects do not have straight lines to measure? How did you determine your lengths?	How can you use a piece of string to identify objects that are about half a metre in length? Explain the relationship between 1 mm, 1 cm and 1 m. When would you use each of these units to measure?

Supporting Resources

Manipulatives and Models to Support Learning

String, Yarn, or Rope 	Paper clips or Plastic links 	Markers or Crayons 	Craft Sticks 	Linking Cubes 	Centimetre cubes 
Cuisenaire Rods 	Base-ten blocks (units, rods) 	Ruler 	Measuring Tape 	Trundle Wheel 	Finger, hand, and arm lengths 

Printed and Electronic Resources

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