Nova Scotia Assessment:
Mathematics in Grade 3
Lessons Learned

> "For learners to succeed, teachers must assess students' individual abilities and characteristics and choose appropriate and effective instructional strategies accordingly."
>
> – Helene J. Sherman

02-2020

Contents

Purpose of this document

This *Lessons Learned* document was developed based on an analysis of the Item Description Reports for the Nova Scotia Assessment: Mathematics in Grade 3 (2018–2019). It is intended to support all elementary classroom teachers (in particular grades Primary–3) and administrators at the school, region, and provincial levels, in using the information gained from this assessment to inform next steps for numeracy focus. The analysis of these items forms the basis of this document, which was developed to support teachers as they further explore these areas through classroom-based instruction and assessment across a variety of mathematical concepts.

After the results for each mathematics assessment become available, an Item Description Report is developed to describe each item of the mathematics assessment in relation to the curriculum outcomes and cognitive processes involved with each mathematical strand. The percentage of students across the province who answered each item correctly is also connected to each item. Item description reports for mathematics are made available to school regions for distribution to schools, and they include provincial, regional, and school data. Schools and regions should examine their own data in relation to the provincial data for continued discussions, explorations, and support for mathematics focus at the classroom, school, region, and provincial levels.

This document specifically addresses areas that students across the province found challenging based on provincial assessment evidence. It is essential that teachers consider assessment evidence from a variety of sources to inform the next steps most appropriate for their students. Effective classroom instruction and assessment strategies are responsive to the individual learners within a classroom.

The M3 assessment generates information that is useful in guiding classroom-based instruction and assessment in mathematics. This document provides an overview of the mathematics tasks included in the assessment, information about this year’s mathematics assessment results, and a series of Lessons Learned for mathematics. Suggested next steps for classroom instruction and assessment are included for each Lesson Learned.

Overview of the Nova Scotia Assessment: Mathematics in Grade 3

Nova Scotia Assessments are large-scale assessments that provide reliable data about how well all students in the province are learning the mathematics curricula. It is different from many standardized tests in that all questions are written by Nova Scotia teachers to align with curriculum outcomes and the results reflect a snapshot of how well students are learning these outcomes. These results can be counted on to provide a good picture of how well students are learning curriculum outcomes within schools, regions and in the province. Since the assessments are based on the Nova Scotia curriculum, and are developed by Nova Scotia teachers, results can be used to determine whether the curriculum, approaches to teaching and allocation of resources are effective. Furthermore, because individual student results are available, these, in conjunction with other classroom assessment evidence, help classroom teachers understand what each student has under control and identify next steps to inform instruction.

The assessment provides information about mathematics for each student and complements assessment data collected in the classroom. This assessment is administered at the beginning of Grade 3. It is designed to provide detailed information for every student in the province regarding his or her progress in achieving a selection of mathematics curriculum outcomes at the end of Grade 3. Information from this assessment can be used by teachers to inform their instruction and next steps in providing support and intervention for their students.

The design of the assessment includes the following:

- mathematical tasks that reflect a selection of outcomes aligned with the curriculum from the end of grade 1 to the end of grade 3 from across all strands of the mathematics curriculum
- Due to the timing of the administration in late spring, questions specific to Unit 11 and Unit 12 (multiplication and division) in the Yearly Plan for grade 3 would not be reflected in the Mathematics/Mathématiques Assessment or *Lessons Learned* document.
- all items are in selected response format
- all items are designed to provide a broad range of challenge, thereby providing information about a range of individual student performance

Table 1: Specific Curriculum Outcomes Assessed in 2018–2019

Strand	**Specific Curriculum Outcomes**
Number (N)	2N03, 2N09, 3N02, 3N03, 3N04, 3N05, 3N08, 3N09, 3N13
Patterns and Relations (PR)	2PR01, 3PR01, 3PR02, 3PR03
Measurement (M)	3M01, 3M02, 3M03, 3M04, 3M05
Geometry (G)	2G02, 3G01, 3G02
Statistics and Probability (SP)	3SP01, 3SP02

Table 2: Specific Curriculum Outcomes Assessed in 2018–2019 by Grade Level

SCO Assessed		
2N03, 2N09, 2PR01, 2G02	Grade 2	17.4%
3N02, 3N03, 3N04, 3N05, 3N08, 3N09, 3N13 3PR01, 3PR02, 3PR03 3M01, 3M02, 3M03, 3M04, 3M05 3G01, 3G02 3SP01, 3SP02	Grade 3	82.6%
Total = 23		100%

Cognitive levels of questions in mathematics are defined as:

- *Knowledge questions* may require students to recall or recognize information, names, definitions, or steps in a procedure.
- *Application questions* may require students to make connections, represent a situation in more than one way (translating between representations), or solve contextual problems.
- *Analysis questions* may require students to go beyond comprehension and application to higher order thinking skills, such as generalizations and non-routine problem-solving.

Table 3: Percentage of Cognitive Level Questions

Cognitive Level Table of Specifications	
Cognitive Levels	**Percentage**
Knowledge	20–30%
Application	50–60%
Analysis	10–20%

These percentages are also recommended for classroom-based assessments.

Please refer to Appendix A for further information about cognitive levels of questioning.

The Nova Scotia Assessment: Mathematics 3 includes 70 items distributed over two days for a duration of 60 minutes each day; 35 items on day one and 35 items on day two. The chart below shows the distribution, by mathematical strand and cognitive level, of items each day.

Table 4: Number of Items by Strand and Cognitive Level in 2018–2019

Number of Items Day 1				
	Knowledge	Application	Analysis	Total
Number	4	11	2	17
Patterns and Relations	1	2	1	4
Measurement	1	3	1	5
Geometry	1	3	1	5
Statistics and Probability	1	2	1	4
Number of Items Day 2				
	Knowledge	Application	Analysis	
Number	4	12	2	18
Patterns and Relations	1	3	1	5
Measurement	1	2	1	4
Geometry	1	2	1	4
Statistics and Probability	1	2	1	4
Total	16	42	12	70
Cognitive Level 2019-2020%	22.9%	60%	17.1%	100%
Table of Specifications	(20–30%)	(50–60%)	(10–20%)	

Performance Levels

Below are the Nova Scotia Assessment: Mathematics in Grade 3 Performance Levels

Level 1: Students at Level 1 can generally solve problems when they are simple and clearly stated or where the method to solve the problem is suggested to them. They rely on a limited number of strategies to solve problems. They can do addition and subtraction of whole numbers but may not understand when each operation should be used. They can recognize some mathematical terms and symbols, mainly from earlier grades. They may be able to pictorially and concretely represent a concept, such as place value.

Level 2: Students at Level 2 can generally solve problems similar to problems they have seen before. They depend on a few familiar methods to solve problems. They rely on strategies such as trial and error or guess and check rather than having a variety of strategies to choose from. They can do addition and subtraction of whole numbers and usually understand when each operation should be used. They can understand and use some mathematical terms and symbols, especially those from earlier grades. They can pictorially, concretely, and contextually represent a concept, such as place value.

Level 3: Students at Level 3 can generally solve problems that involve several steps and may solve problems they have not seen before. They can choose appropriate strategies to solve problems. They can apply number operations (+, –,) correctly and can judge whether an answer makes sense. They can understand and use many mathematical terms and symbols, including those at grade level. They pictorially, concretely, and contextually represent a concept, such as place value.

Level 4: Students at Level 4 can solve new and complex problems. They are consistent when choosing efficient strategies to solve problems. They can apply number operations (+, –) with confidence and ease. They can think carefully about whether an answer makes sense. They interpret and represents mathematical concepts using symbolic form with ease. They consistently use all representations with ease to represent a concept, such as place value.

Assessment Results

The Nova Scotia Assessment: Mathematics in Grade 3 was first administered in the 2018–2019 school year.

Mathematics in Grade 3 Lessons Learned

The assessment information gathered from the Nova Scotia Assessment: Mathematics in Grade 3 data has been organized into 8 ***Lessons Learned***:

- Translating Between and Among Representations
- Representing and Partitioning Whole Numbers
- Whole Number Operations
- Patterns and Relations
- Measurement
- Geometry
- Statistics and Probability.
- Problem Solving

Each Lesson Learned is divided into four sections that address the following questions

A. What conclusions can be drawn from the Nova Scotia Assessment: Mathematics in Grade 3?
B. Do students have any misconceptions or errors in their thinking?
C. What are the next steps in instruction for the class and for individual students?
D. What are the most appropriate methods and activities for assessing student learning?

Lessons Learned

1. **Translating Between and Among Representations:** Students did well when translating and moving flexibly between and among all representations of a concept. Students still need to be encouraged to translate between words, pictures or symbols. If asked to represent a question using representations, (words, symbols or pictures), most students provide words, pictures or symbols. Many students realize that they can use varied representations (words, pictures or symbols) when solving a question.

2. **Representing and Partitioning Whole Numbers:** Students were challenged when asked to apply their knowledge of basic facts and skills and to represent a situation or the steps in a procedure when given application questions. They struggled with translating between representations of a concept (contextually, concretely, pictorially, symbolically, and verbally). When students are working to partition whole numbers and to perform operations, it is very important for students to understand that numbers can be broken down into two or more parts in many different ways.

3. **Whole Number Operations:** Students were challenged when asked to apply basic skills, knowledge, and computational procedures to application and analysis questions. Students need to be able to apply the higher order thinking skills of problem solving, creativity, and reasoning to do application and analysis items. Students should have experiences with all the story structures for addition and subtraction. Students also need to be encouraged to estimate before calculating an answer to a question.

4. **Patterns and Relations:** Students were challenged when asked to transfer their visual representation of patterns to numerical patterns. Students should be able to describe either an increasing pattern or a decreasing pattern, and need to recognize that each term has a numeric value. Students seemed to forget that a pattern rule must have a starting point. Without the starting point, the pattern rule is incomplete. They need to be encouraged to provide all the information for a pattern rule. Students need to continue to work with representations of patterns, concretely, contextually, pictorially, symbolically, and verbally.

5. **Measurement**: Students were challenged with building conceptual understanding of what it means to measure with a ruler. Emphasis should be placed on counting the intervals between the numbers, rather than looking at the number on the ruler that is aligned with the end of the object. Students also need to use a ruler to measure the length of a pencil or other objects with and without using zero as the starting point. Students need to recognize which mass unit (gram or kilogram) is appropriate for measuring and comparing the mass of a specific item. Students need to be able to explain that perimeter is the distance around a shape or the length of the boundary of an enclosed region. Students need to find the perimeter of many different regular, and composite shapes, before being introduced to questions in pictorial form. Students need to work with perimeter in application and analysis questions.

6. **Geometry**: Students were challenged with developing their knowledge of 2-D shapes and 3-D objects by describing and sorting them according to their geometric attributes. Students need more experiences to identify the faces, edges, and vertices of 3-D objects including spheres, cones, cylinders, pyramids, cubes and other prisms. Students need to be provided with opportunities to explore these attributes through sorting and constructing activities. Students need to extend their knowledge of regular polygons. They need to focus on comparing the number of sides as the key attribute for classifying polygons. Students need more experiences with regular polygons, so that they begin to realize that a polygon, regardless of its dimensions, remains the same shape.

7. **Statistics and Probability:** Students were challenged using tally marks, lists, charts, line plots, and bar graphs to organize data relevant to their everyday life. Students need opportunities and experiences to interpret information collected, organized, and displayed in tally charts, charts, line plots and bar graphs. Students need to develop the skill of interpreting graphs, and answering questions and to draw conclusions from those tally charts, line plots and bar graphs. They need to be encouraged to ask or write questions that go beyond simplistic reading of a graph. Both literal and inferential comprehension questions need to be asked.

8. **Problem Solving:** Students were challenged with and require more exposure to application and analysis items in order to apply these higher order thinking skills when problem solving. Students need to be encouraged to understand that problems may have more than one entry point and there are many strategies to solve a problem. Students need to continue to work on translating between and among representations when problem solving. They do not seem to realize that they can use varied representations when asked to solve a word problem. These other representations may support their problem solving and their reasoning.

Key Messages

The following key messages should be considered when using this document to inform classroom instruction and assessment.

The Nova Scotia mathematics curriculum is based upon several key assumptions or beliefs about mathematics learning that have grown out of research and practice. These beliefs include the following:

- Mathematics learning is an active and constructive process.
- Learning is most effective when standards of expectation are made clear with ongoing assessment and feedback.
- Learners are individuals who bring a wide range of prior knowledge and experiences and who learn via various styles and at different rates.
- Learning is most likely to occur when placed in meaningful contexts and in an environment, that supports exploration, risk taking, and critical thinking and that nurtures positive attitudes and sustained effort.
- Students are curious, active learners with individual interests, abilities, and needs. They come to classrooms with varying knowledge, life experiences, and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.
- Students learn by attaching meaning to what they do and need to construct their own meaning of mathematics. This meaning is best constructed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of models and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students and enhance the formation of sound, transferable, mathematical concepts. At all levels, students benefit from working with and translating through a variety of materials, tools, and contexts when constructing meaning about new mathematical ideas. Meaningful discussions can provide essential links among concrete, pictorial, contextual, and symbolic representations of mathematics.
  (EECD, 2013b, p. 23)

Research has shown that assessment for learning (formative assessment) practices produce significant and often substantial learning gains, close achievement gaps, and build students' ability to learn new skills (Black & Wiliam 1998; OECD 2006). Student involvement in assessment promotes learning. Timely and effective teacher feedback and student self-assessment allow students to reflect on and articulate their understanding of mathematical concepts and ideas.

- Provincial assessment results form part of the larger picture of assessment for each student and complements assessment data collected in the classroom. Ongoing assessment for learning (formative assessment) is essential to effective teaching and learning. Assessment for learning can and should happen every day as part of classroom instruction. Assessment of learning (summative assessment) should also occur regularly and at the end of a cycle of learning. Both forms of assessment are necessary to guide teaching, stimulate learning, and produce achievement gains.
- It is important to construct assessment activities that require students to complete tasks across the cognitive levels. While it is important for students to be able to answer factual and procedural type questions, it is also important to embed activities that require strategic reasoning and problem-solving.

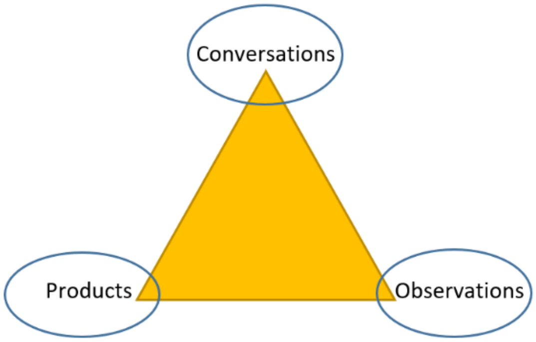
- Ongoing assessment for learning involves the teacher focusing on how learning is progressing during the lesson and the unit, determining where improvements can be made and identifying the next steps. "*Such assessment becomes 'formative assessment' when the evidence is actually used to adapt the teaching to meet learning needs," (*Black, Harrison, Lee, Marshall & Wiliam, 2003, p.2). Effective strategies of assessment for learning during a lesson include strategic questioning, observing, conversing (conferring with students to "hear their thinking"), analyzing student's work (product), engaging students in reviewing their progress, as well as, providing opportunities for peer and self-assessment.
- Assessment of learning involves the process of collecting and interpreting evidence for the purpose of summarizing learning at a given point in time, and making judgments about the quality of student learning on the basis of established criteria. The information gathered may be used to communicate the student's achievement to students, parents, and others. It occurs at or near the end of a learning cycle.
- All forms of assessment should be planned with the end in mind, thinking about the following questions:
    - What do I want students to learn? (identifying clear learning targets)
    - What does the learning look like? (identifying clear criteria for success)
    - How will I know they are learning?
    - How will I design the learning so that all will learn?
- Before planning for instruction using the suggestions for instruction and assessment, it is important that teachers review individual student results in conjunction with current mathematics assessment information. A variety of current classroom assessments should be analyzed to determine specific strengths and areas for continued instructional focus or support.

Balanced Assessment in Mathematics: Effective ways to gather information about a student's mathematical understanding

- Conversations/Conferences/Interviews: Individual, Group, Teacher-initiated, Child-initiated
- Products/Work Samples: Mathematics journals, Portfolios, Drawings, Charts, Tables, Graphs, Individual and classroom assessment, Pencil-and-paper tests, Surveys, Self-assessment
- Observations: Planned (formal), Unplanned (informal), Read-aloud (literature with mathematics focus), Shared and guided mathematics activities, Performance tasks, Individual conferences, Anecdotal records, Checklists, Interactive activities

(EECD, 2013a, p. 4)

"Triangulation increases the reliability and validity of student learning assessment and facilitates the implementation of pedagogical differentiation. Using triangulation, we take into account all learning styles and we engage all students, including those who have difficulty expressing themselves in writing and those who do not have the ability to undertake a written assessment task to demonstrate their learning."— Anne Davies (*Free Translation)*



Mathematics in Grade 3 Lesson Learned 1
Translating Between and Among Representations

Students did well when translating and moving flexibly between and among all representations of a concept. Students still need to be encouraged to translate between words, pictures or symbols. If asked to represent a question using representations, (words, symbols or pictures), most students provide words, pictures or symbols. Many students realize that they can use varied representations (words, pictures or symbols) when solving a question.

Mathematics should be viewed as an integrated whole rather than as the study of separate strands or units. Connections must also be made between and among the different representational modes – contextual, concrete, pictorial, linguistic/verbal, and symbolic. The process of making connections, in turn, facilitates a child's learning.

"Children who have difficultly translating a concept from one representation to another are the same children who have difficulty solving problems and understanding computations. Strengthening the ability to move between and among these representations improves the growth of children's concepts" (Van De Walle*,* John A. 2001*, Elementary and Middle School Mathematics*, Fourth Edition, p. 34).

One way to encourage children to use multiple representations is to explicitly ask for them.
Ask questions such as

- How many ways can you show the number 20 using words, pictures, models, and numbers?
- How many ways can you write 75?
- Can you represent a rectangle as a combination of other shapes?
- Can you represent this line plot as a bar graph?
- Can you use an equation to represent how you thought about this story problem?

A. What conclusions can be drawn from the Nova Scotia Assessment: Mathematics in Grade 3?

Historically, our students were challenged when asked to translate between and among representations of a concept. Students now have translating between and among representation well under control. We found that students have a good understanding of basic facts and procedures, but when given application items, they appear to want to rush to symbolic. For example, when problem solving, students are able to understand the context of the question, but many are not able to translate between the representations (translating from words to pictures or symbolic to pictures, etc.).

B. Do students have any misconceptions or errors in their thinking?

It is important to distinguish between misconceptions and errors. Misconceptions are, in effect, the misunderstandings about mathematical ideas which students entertain, and which usually lead to errors occurring. These are the most serious kind of errors which require urgent action on the part of the teacher. Errors may occur for a variety of underlying reasons, ranging from careless mistakes to errors resulting from misconceptions. - John SanGiovanni – (*Mine The Gap For Mathematical Understanding Grades K-2*)

If asked to represent a question using words, symbols or pictures, students usually only provided symbols. They do not seem to realize that they can use various representations (words, pictures or symbols) when solving a question. For example, when given an analysis question concerning the perimeter (distance around) a named shape, students struggle with knowing how many sides the named shape would have, unless the actual shape is shown in the question as a picture/diagram. Students should be encouraged to translate between the name of the shape (words) and a picture. Rather than relying on the question itself, students could draw the picture and check how many sides the named shape has.

The student results of the **2018–2019 Nova Scotia Assessment Mathematics in Grade 3** shows the following:

- most students answered questions correctly when translating between and among representations

C. What are the next steps in instruction for the class and for individual students?

Next steps in instruction should provide opportunities for students to use representations to communicate mathematical ideas. They should have experiences selecting, applying, modeling, and translating among mathematical representations to solve problems. The five representations of a concept are contextual, concrete, pictorial, symbolic, and verbal (written/oral). All five representations should remain a focus for students.

“Representational competence (Novick, 2004) for students is knowing how and when to use particular mathematical representations. A key aspect of understanding mathematics means not only knowing how to use a representation during problem-solving situations but also being able to make connections between representations” (*Teaching Children Mathematics*. NCTM, August 2010, p. 40).

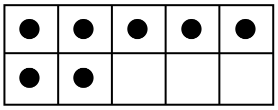
When students can move between and among the different representations of a concept, we say that they have concept attainment.

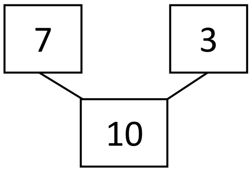
There are three specific strategies that provide opportunities to support students’ development of representational competency

- “Engaging in dialogue about the explicit connections between representations
- Alternating directionality of the connections made among representations
- Encouraging purposeful selection of representation”
  (*Teaching Children Mathematics*. NCTM, August 2010, p. 40)

For example, during instruction students focus on specific parts of one representation and think about the correspondence with parts in another. Teachers should ask questions that require students to translate between parts. Alternating directionality supports students’ thinking with various representations.
For example, consider the activity below, All About Ten (*Teaching Children Mathematics*. NCTM, August 2010, p. 44).

All About Ten: Fill in the box diagram to show the two parts of ten shown in the ten-frame. Write number sentences to match. [Below is the completed activity.]





7 + 3 = 10	10 = 7 + 3
3 + 7 = 10	10 = 3 + 7
10 – 7 = 3	3 = 10 – 7
10 – 3 = 7	7 = 10 – 3

Instructional Strategies

“Three specific instructional strategies create opportunities that may support students’ development of representational competence …” (*Teaching Children Mathematics*. NCTM, August 2010, p. 40).

1. Students need to **engage** in dialogue about the explicit connections between representations. Pose questions to students such as
   - How is the number 10 represented in each diagram?
   - How are the three representations of the number the same? How are they different?
   - Can you show the number 10 in a different way?

2. Students need to be encouraged to **alternate directionality** in order to make connections among representations. When discussing multiple representations, ask focused questions such as
    - Can you describe the 3 in the ten-frame, in the box diagram, and in each of the equations?
    - What meaning does the 3 have in each of the diagrams?

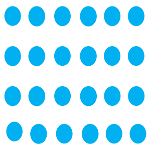
    “The **directionality** of the connections made between the representations and the problem situation is another important feature of representational competence. For example, translating from a box diagram to a ten-frame, and vice versa, promotes the use of different mathematical thought processes**.** An important aspect of developing understanding of mathematics means not only knowing how to use a representation during problem-solving situations but also being able to move flexibly between different representations, making connections from one representation to the other, and vice versa.” (*Teaching Children Mathematics*. NCTM, August 2010, p. 44)

3. Students need to be **encouraged** to purposefully select the most appropriate representation. “Encourage students to consider the suitability of a representation. Discuss a variety of reasons to use particular representations, including but not limited to the following: efficiency, accuracy, ease of use, appropriateness with respect to the problem context, and student preference. By comparing the use of multiple representations for the same problem, students can more easily see the suitablility of one representation over another” (*Teaching Children Mathematics*. NCTM, August 2010, p. 46).

Translating Between and Among Representations
It is important that student are exposed to the different types of representations and be able to translate between and among each one:

- concrete (two-sided counters, base-ten materials, etc.)
- pictorial (drawing, numerical line, ten-frame, etc.)
- contextual
- symbolic
- verbal

For example, the number 24 can be represented:



twenty-four 20 + 4 30 minus 6

D. What are the most appropriate methods and activities for assessing student learning?

Questions from the strands will be used to represent some of the appropriate methods and activities for assessing student learning.

Encourage students to share their thinking, their strategies, and their solutions. Rich discussion fosters the development of alternative problem-solving strategies.

Examples:

1. Peter found a stick that was more than a 1 m long.
How many centimetres long could the stick be?

○ 80 cm
○ 90 cm
○ 100 cm
○ 115 cm

Show what you know using words, pictures, or symbols.

2. The perimeter (distance around) of a pentagon is 20 cm.
How long is each side?

○ 4 cm
○ 5 cm
○ 20 cm
○ 25 cm

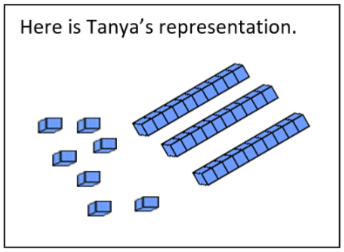
Show what you know using words, pictures, or symbols.

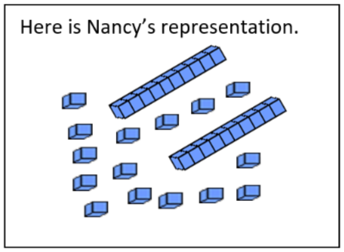
3. I am a 3-D object.
I have 5 faces.
I have 5 vertices.
I have 8 edges.
Which shape am I?

○ cube
○ sphere
○ square-based pyramid
○ triangular-based pyramid

Show what you know using words, pictures, or symbols.

4. Tanya and Nancy used base-ten blocks to represent numbers.


Here is Tanya's representation.

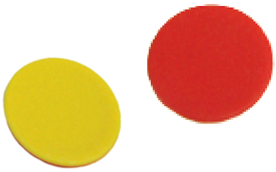

Here is Nancy's representation.

Write number sentences for each of the representations in the pictures above. Then compare the sum of the numbers using the equal sign (=) or the not equal sign (≠).

Show what you know using words, pictures, or symbols.

5. You have 29 counters.

You give your friend 14 counters.



How many counters do you have now?

Show what you know using pictures, words or numbers.

6. Show what you know using pictures, words or symbols for the following questions:

Materials required for this question: one full piece of paper, 20 cubes (cube-a-links)

Questions to the student:

a) Write the number 17 on a piece of paper. Now turn your paper over.

b) On the blank side of your paper that you turned over, show the number 17 using your cubes.

c) Now draw a picture of your arrangement of the cubes used to represent 17 on the blank piece of paper.

d) Can you show the number 17 represented another way using the cubes?

Mathematics in Grade 3 Lesson Learned 2
Representing and Partitioning Whole Numbers

Students have improved when asked to apply their knowledge of basic facts and skills and to represent a situation or the steps in a procedure when given application questions. There has also been improvement for students with translating between representations of a concept (contextually, concretely, pictorially, symbolically, and verbally). When students are working to partition whole numbers and to perform operations, it is very important for students to understand that numbers can be broken down into two or more parts in many ways.

A. What conclusions can be drawn from the Nova Scotia Assessment: Mathematics in Grade 3?

We noticed that students were able to do well when explicitly given all the information needed to do the question. Students performed well on knowledge questions that required them to use basic skills, symbolic procedures, and factual knowledge. For example, when asked to choose the number that is equal to thirty-one tens (310), most students were able to find the correct answer. Students were successful problem solvers and performed well on questions that required analysis and non-routine problem solving.

However, our assessment information also shows that many students experienced challenges with application questions. Our students were challenged when asked to apply their knowledge of basic facts and skills to a context. They also struggled with translating between representations of a concept (contextually, concretely, pictorially, symbolically, and verbally) when asked to solve a story problem.

B. Do students have any misconceptions or errors in their thinking?

It is important to distinguish between misconceptions and errors. Misconceptions are, in effect, the misunderstandings about mathematical ideas which students entertain, and which usually lead to errors occurring. These are the most serious kind of errors which require urgent action on the part of the teacher. Errors may occur for a variety of underlying reasons, ranging from careless mistakes to errors resulting from misconceptions. - John SanGiovanni – (*Mine The Gap For Mathematical Understanding Grades K-2*)

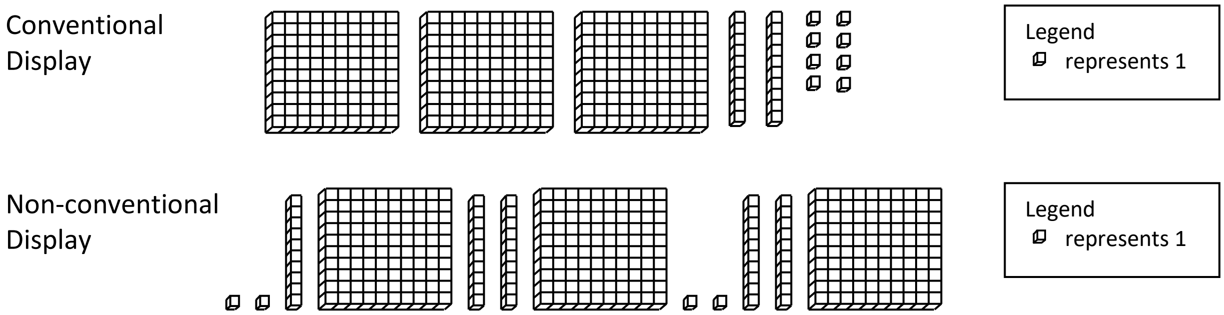
Students were very capable of performing well with partitioning numbers when given knowledge questions. For example, students represented a number using base-ten blocks in a conventional display. However, the difficulty appeared when students were asked to apply their knowledge or to represent a situation or the steps in a procedure when given application questions. For example, students could correctly represent a number, such as 75 using a conventional display of base-ten blocks but were challenged when asked to partition a number in a variety of ways, such as 70 + 5, 50 + 25, or 60 + 12 + 3, 25 + 25 + 20. Many students have the misconception that these are expressions that have an answer of 75, and do not understand that these also represent four ways of writing 75. An expression names a number. Sometimes an expression is a number such as 150. Sometimes an expression shows an arithmetic expression, such as 125 + 25. 150 may also be represented by its partitions, such as 80 + 70, 100 + 50, and 50 + 50 + 50. Numbers can also be represented by a difference expression, such 175 – 25.

The student results of the **2018–2019 Nova Scotia Assessment Mathematics in Grade 3** shows the following:

- most students answered questions correctly when representing and partitioning whole numbers
- 53% of the students had difficulty using place-value strategies, represent symbolically a number written in word

C. What are the next steps in instruction for the class and for individual students?

When students translate between and among the five representations of a concept (contextual, concrete, pictorial, symbolic and verbal), we say that they have concept attainment. Students need numerous experiences representing numbers to 1000 and translating between and among these representations of a concept to strengthen their knowledge. They need many experiences with base-ten materials, pictures such as number lines and tallies, ten-frames, words, and contexts to conceptualize a number being made up of two or more parts. It is extremely important that students have opportunities to view and create numbers using conventional and non-conventional displays of base-ten blocks.



Partitioning numbers using the models above supports students' ability to recognize that any number can be partitioned into two or more parts. It also helps students develop part-part-whole thinking. Although it is important for students to experience a variety of partitions including traditional expanded notation (425 = 400 + 20 + 5). They should also continue to experience partitions such as 424 + 1, 325 + 100, 200 + 200 + 10 + 10 + 5. This is the most important understanding that can be developed about number relationships. Develop critical thinking by asking students to explain how the representations are alike and why they are different.

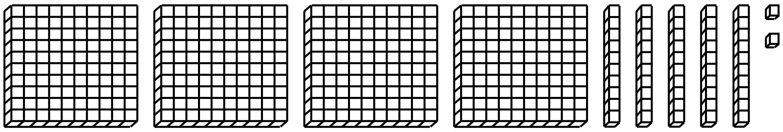
D. What are the most appropriate methods and activities for assessing student learning?

Below are some sample questions related to representing and partitioning whole numbers which will be used to represent some of the appropriate methods and activities for assessing student learning.

Encourage students to share their thinking, their strategies, and their solutions. Rich discussion fosters the development of alternative problem-solving strategies.

Examples:

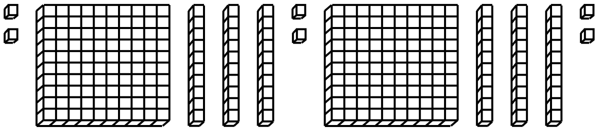
1. What number do these base-ten blocks represent?



Legend
◻ represents 1

Write the number, __________.

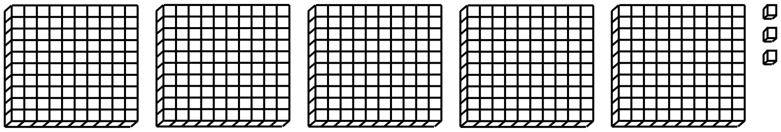
2. What number do these base-ten blocks represent?



Legend
◻ represents 1

Write the number, ___________.

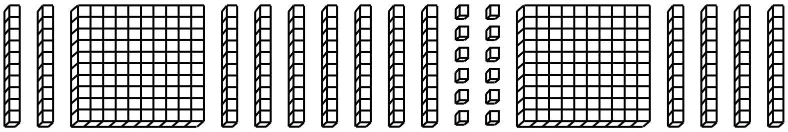
3. What number do these base-ten blocks represent?



Legend
◻ represents 1

Write the number, __________.

4. What number do these base-ten blocks represent?



Legend
◻ represents 1

Write the number, __________.

5. Draw a picture of base-ten blocks to show 236 in 3 different ways.

Legend
⧉ represents 1

6. The number 358 is the same as:

○ 100 + 100 + 50 + 8 + 100
○ 300 + 5 + 8
○ 400 – 58
○ 3 + 5 + 8

7. Choose the number that is equal to thirty-one tens.

○ 31
○ 301
○ 310
○ 3010

8. The number 642 is the same as:

○ 5 hundreds, 2 tens, and 14 ones
○ 64 tens and 2 ones
○ 6 tens and 42 ones
○ 6 hundreds, 20 tens and 4 ones

9. Write three expressions that can be used to represent 53.

53 is the same as ____

53 is the same as ____

53 is the same as ____

10. Write the following number in words:

263 ____

373 ____

487 ____

597 ____

Mathematics in Grade 3 Lesson Learned 3
Whole Number Operations

Students improved when asked to apply basic skills, knowledge, and computational procedures to application and analysis questions. Students need to be able to apply the higher order thinking skills of problem solving, creativity, and reasoning to do application and analysis items. Students should have experiences with all the story structures for addition and subtraction. Students also need to be encouraged to estimate before calculating an answer to a question and to consider the reasonableness of the answer. Place-value strategies to represent a number written in words symbolically, is still very difficult for many students. Developing personal strategies and alternative algorithms tend to focus on the meaning of the number, rather than on individual digits.

The Mathematics curriculum documents for grades Primary to 3 note that a true sense of number goes well beyond the skills of simply counting, memorizing facts, and the situational rote use of algorithms. Number sense develops when students connect numbers to real-life experiences and use benchmarks and referents. This results in students who are computationally fluent, flexible with numbers, and have intuition about numbers. The evolution of number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing rich mathematical tasks that allow students to make connections. Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects, and concepts.

A. What conclusions can be drawn from the Nova Scotia Assessment: Mathematics in Grade 3?

We noticed that students were able to do well when explicitly given all the information needed to do the question. Students performed well on knowledge questions that required them to use basic facts and skills, symbolic procedures, and factual knowledge. For example, students were able to solve 487 – 37 when the problem was presented symbolically.

However, when students were asked to apply basic skills, knowledge, and computational procedures to application and analysis questions, they were challenged. This appeared to be a theme throughout the assessment data. At times, students were not sure whether they should add or subtract when questions were presented in the context of a story problem. The assessment analysis also showed that our students did not understand the relationship between addition and subtraction. Many of the students were doing addition and subtraction questions as procedures and were not making any connection between these two operations.

B. Do students have any misconceptions or errors in their thinking?

It is important to distinguish between misconceptions and errors. Misconceptions are, in effect, the misunderstandings about mathematical ideas which students entertain, and which usually lead to errors occurring. These are the most serious kind of errors which require urgent action on the part of the teacher. Errors may occur for a variety of underlying reasons, ranging from careless mistakes to errors resulting from misconceptions. - John SanGiovanni – (*Mine The Gap For Mathematical Understanding Grades K-2*)

Addition and Subtraction

When solving computation questions, many students performed better when solving the computation using an alternative algorithm, such as presenting the digits to be added or subtracted horizontally. Students did less well, when the traditional algorithm was used. This may result because the traditional algorithm focuses on single digits within the computation rather than thinking about the number as a whole.

Examples:

$67 - 32 = 35$

$$\begin{array}{r} 650 \\ \underline{-\ 421} \\ 229 \end{array}$$

Many students have the misconception that they always subtract the smaller number from the larger number. They apply this thinking regardless of whether the position of the number is in the subtrahend (a number which is to be subtracted from another number) or the minuend (a number from which another number is to be subtracted) in the question. Below are two examples.

451 – 231 = 220 451 (minuend) 5**0**9 (minuend) 5**0**9 – 2**8**9 = 2**8**0
– 231 (subtrahend) – 3**8**9 (subtrahend)
220 (difference) 2**8**0 (difference)

In the first question, when subtracting the tens, 5 tens – 3 tens = 2 tens, (50 – 30 = 20) the student completes the question correctly using his/her understanding of subtracting the smaller number from the larger number. But in the second question, the student tries to use the same method (smaller number 0 subtracted from larger number 8) and they get an answer of 8 tens, which is incorrect.

At times, students forgot to regroup when adding. They often wrote a two-digit number where there should have only been one digit. Below is an example.

14**5**
+ 24**7**
38**12**

Use partitioning to solve 145 + 247 = 145 + 5 + 242 = 150 + 242 = 392 or the front-end strategy.

Some students misaligned the digits when recording their calculations and computed incorrectly. Ways to address these errors or misconceptions is to focus on developing personal strategies and alternative algorithms which tend to focus on the meaning of the number, rather than on individual digits.

The student results of the **2018–2019 Nova Scotia Assessment Mathematics in Grade 3** shows the following:

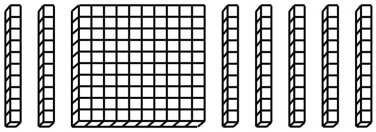
- 53% of the students had difficulty using place-value strategies, to represent a number written in words symbolically
- 52% of the students had difficulty to subtract a 3-digit whole number from a 3-digit whole number arranged vertically; one of the digits was zero (trading)
- 55% of the students had difficulty estimating the cost of objects in a context to the nearest hundred ($)
- 54% of the students had difficulty when solving a multi-step story problem involving whole number computation

C. What are the next steps in instruction for the class and for individual students?

Students were challenged when applying estimation strategies. They knew how to estimate numbers in isolation but could not estimate sums and differences in context. One of the first steps in instruction is making sure that children are exposed to and understand how to estimate sums and differences. Estimating allows students to predict answers, check their calculations, and ask themselves if their actual answer is reasonable. The factors that may influence estimating sums and differences, is the context, the numbers and operations involved.

Representation of Whole Numbers

It is important to note that there are students who misinterpret the representation of a number using base-ten blocks. Here is an example:



Students see this base-ten block arrangement as representing the number 215 instead of 170. This example shows that students count the base-ten blocks as: 2 hundreds, 1 ten, and 5 units. This misconception is because the students do not understand that the flat represents a hundred and the rod represents 10.

This is an unconventional representation of base-ten blocks. The students are do not recognize that the base-ten blocks are not arranged in the conventional way that they have become accustomed to seeing. Students need to be exposed to both conventional and unconventional arrangements of base-ten blocks representing numbers.

Addition and Subtraction

Students need to learn that addition and subtraction are related. They are inverse operations; they undo each other. The basic facts of addition and subtraction do not have to be learned as separate facts. Below are some examples.

6 + 5 = 11, so 5 + 6 = 11	11 – 5 = 6, so 11 – 6 = 5
11 = 6 + 5, so 11 = 5 + 6	6 = 11 – 5, so 5 = 11 – 6

Addition and subtraction problems can be categorized based on the kinds of relationships they represent. It is important that all the story problem structures are presented and developed from students' experiences. Initial work with the story structures will focus on join and separate types of problems because students associate the actions in these problems with the operations. However, they must also experience addition and subtraction in part-part-whole and comparison situations. Please refer to the grade level appropriate curriculum documents (*Mathematics 1*, p. 64; *Mathematics 2*, p. 68; and *Mathematics 3*, p. 71) for more information about the story structures and instructional strategies.

Students will be expected to use and describe strategies to determine sums and differences using manipulatives and visual aids. Initial strategies include

- counting on or counting backwards
- one more or one less
- making ten
- doubles
- near doubles

Other strategies are described in the curriculum documents, *Mathematics 1*, *Mathematics 2*, and *Mathematics 3*.

Manipulatives can and should be used to model not only the above strategies but also model the story structures. Examples of manipulatives that can be used for this purpose include

- two-sided counters
- linking cubes
- game materials (number cubes)
- ten-frames
- walk-on number line
- Rekenrek

Understanding Addition and Subtraction: Story Structures

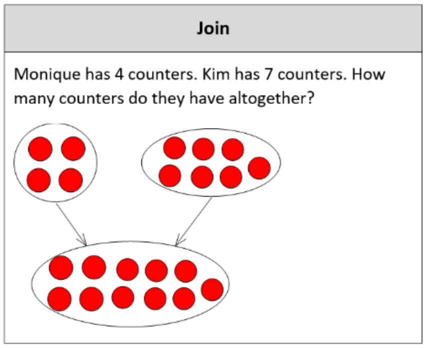
The development of the meaning of addition and subtraction cannot be rushed. It is essential that students explore adding and subtracting situations in meaningful contexts. Students should have extensive investigative experiences in which they use a variety of concrete materials and pictures to model and compare contexts, before recording the number sentences that represent their thinking. It is important that problems be personalized, but students also need experience interpreting how addition and subtraction are portrayed in print.

When working with addition and subtraction, include examples of active situations that involve the physical joining (join problems) and separating (separate problems) of sets. It is also important when working with addition and subtraction to include examples of static situations (part-part-whole and comparison) that involve the implied joining or separating of sets that are not physically joined or separated.

Addition and subtraction problems can be categorized based on the kinds of relationships they represent – joining, separating, part-part-whole, and comparison. Therefore, it is critically important that all the following structures of problems be presented in order that students develop a robust and complete understanding of addition and subtraction.

When students draw pictures to illustrate joining or separating situations, they must indicate the action of joining or separating. Often, students use arrows to indicate the joining or separating of sets.

Example:



Join

Monique has 4 counters. Kim has 7 counters. How many counters do they have altogether?

The arrows show the action of joining 4 counters with the 7 counters. This may be recorded symbolically as 4 + 7 = 11.

More information about developing part-part-whole relationships can be found in:

Mathematics Primary Curriculum Guide (Draft May 2013) on pages 40–44 and 89–90.

Mathematics 1 Curriculum Guide (Draft May 2013) on pages 63–64, 134–135, 42–46, and 126–127.

Mathematics 2 Curriculum Guide (Draft May 2013) on pages 42–46, 169–170, 66–72, and 178–182.

Mathematics 3 Curriculum Guide (Draft May 2013) on pages 36–40, page 171, 70–76 and 182–188.

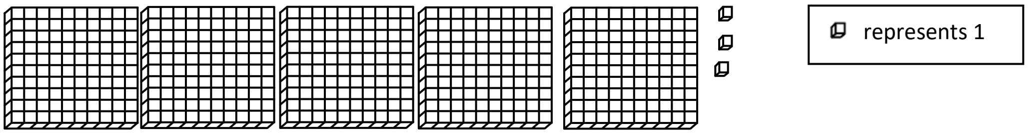
D. What are the most appropriate methods and activities for assessing student learning?

Below are some sample questions related to operations with whole numbers which will be used to represent some of the appropriate methods and activities for assessing student learning.

Encourage students to share their thinking, their strategies, and their solutions. Rich discussion fosters the development of alternative problem-solving strategies.

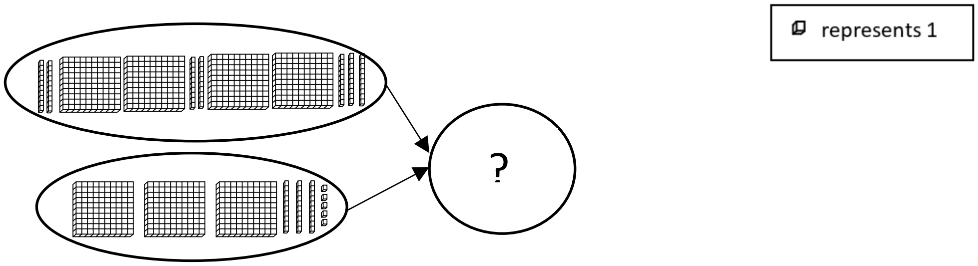
Examples:

1. What number is represented by this set of base ten blocks?



○ four hundred thirteen
○ forty tens, thirteen ones
○ four hundred, ten tens, three ones
○ five tens , 3 ones

2. Which addition is represented by this set of base ten blocks?



○ 407 + 315 = 722
○ 470 + 305 = 775
○ 470 + 315 = 715
○ 470 + 335 = 805

3. Choose the correct answer for the following addition 363 + 25

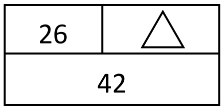
○ 308
○ 388
○ 618
○ 5113

4. Chose the correct answer for the following subtraction 809 – 489

○ 320
○ 420
○ 480
○ 1298

5. Forty-two students were in the gym. Twenty-six of them were girls. How many were boys?

Choose the equation that shows a way to work out this problem.



○ $42 = 26 + \triangle$

○ $26 + 42 = \triangle$

○ $\triangle - 26 = 42$

○ $26 - \triangle = 42$

6. Which number is missing in this equation? $\square = 7 + 9$

○ 19
○ 16
○ 13
○ 6

7. Which number is missing in this equation $3 + \square = 12 - 6$

○ 1
○ 3
○ 6
○ 9

8. The number 605 is the same as:

○ 500 + 150 + 5
○ 400 + 100 + 15
○ 500 + 100 + 5
○ 600 + 12 + 5

9. Choose the number that is equal to seventy-one tens

○ 71
○ 701
○ 710
○ 6011

10. Luke has 72 stickers. Lilly has 29 stickers.
About how many more stickers does Luke have than Lilly?
Choose the best estimate.

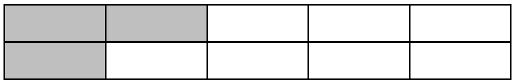
○ 100
○ 90
○ 40
○ 30
○

11. Marley did this subtraction.
675 – 346 = 329

Which expression could help her check her work?

- ○ 675 + 329
- ○ 675 + 346
- ○ 329 + 346
- ○ 346 – 329

12. Which fraction represents the shaded part?

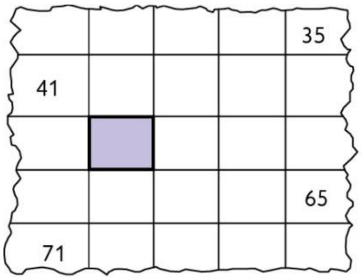


- ○ $\frac{3}{10}$
- ○ $\frac{3}{7}$
- ○ $\frac{7}{3}$
- ○ $\frac{10}{3}$

13. An apple was cut into equal pieces.
Choose the fraction that would represent the smallest piece of apple.

- ○ $\frac{1}{2}$
- ○ $\frac{1}{4}$
- ○ $\frac{1}{8}$
- ○ $\frac{1}{10}$

14. This part of a **hundred chart** has some numbers missing.



What number belongs in the shaded box?

- ○ 42
- ○ 43
- ○ 52
- ○ 63

Mathematics in Grade 3 Lesson Learned 4
Patterns and Relations

Students were challenged when asked to transfer their visual representation of patterns to numerical patterns. Students improved when asked to describe either an increasing pattern or a decreasing pattern, and need to recognize that each term has a numeric value. They still have difficulty when asked to identify in an increasing pattern a specific element. Students seemed to forget that a pattern rule must have a starting point. Without the starting point, the pattern rule is incomplete. They need to be encouraged to provide all the information for a pattern rule. Students need to continue to work with representations of patterns, concretely, contextually, pictorially, symbolically, and verbally.

Patterns are the foundation for many mathematical concepts. Patterns should be taught throughout the year in situations that are meaningful to students. Patterns are explored in all the strands and are also developed through connections with other disciplines, such as science, social studies, English language arts, physical education, and music. Providing students with the opportunity to discover and create patterns, and then describe and extend those patterns, will result in more flexible thinking across strands and across subjects. Students should initially describe non-numerical patterns, such as shape, action, sound, and then incorporate numerical patterns by connecting them to the non-numerical patterns.

A large focus in Mathematics 3 is the introduction and development of decreasing patterns. Students use their knowledge of increasing patterns to make connections to the concept of decreasing patterns, since similar understandings are developed.

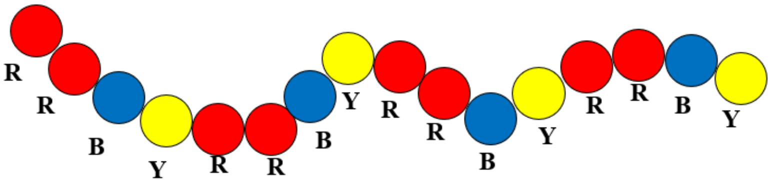
A. What conclusions can be drawn from the Nova Scotia Assessment: Mathematics in Grade 3?

We noticed that our students did very well recognizing simple errors in increasing number patterns, identifying a pattern rule used to create a given increasing pattern, and identifying the next term in an increasing pictorial pattern. These types of items were either knowledge questions or application questions. Analysis questions related to patterns challenged our students. For example, students experience difficulty when asked to create an increasing pattern in which a specific element is identified (e.g., the 7th element is 56).

Students did extremely well when the patterns that they were working with were a visual representation of a pattern. See below for examples:

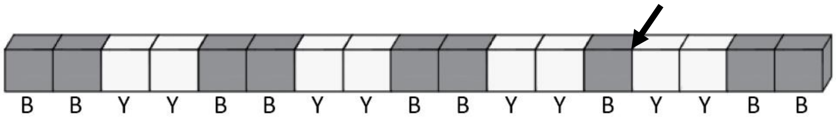
Example 1:

"The pattern for my beads is red (R), red (R), blue (B), yellow (Y)"



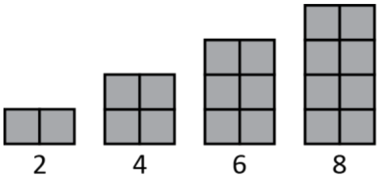
Example 2:

“I see a mistake in this block pattern. It needs another blue block here.”

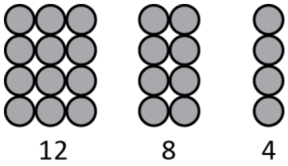


Students were challenged when they were transferring their knowledge of visual patterns to numerical patterns. Students should be able to describe an increasing pattern made of shapes but need to recognize that each term in the pattern also has a numeric value. For example,

Example 3:



increasing pattern made of shapes



decreasing pattern made of shapes

B. Do students have any misconceptions or errors in their thinking?

It is important to distinguish between misconceptions and errors. Misconceptions are, in effect, the misunderstandings about mathematical ideas which students entertain, and which usually lead to errors occurring. These are the most serious kind of errors which require urgent action on the part of the teacher. Errors may occur for a variety of underlying reasons, ranging from careless mistakes to errors resulting from misconceptions. - John SanGiovanni – (*Mine The Gap For Mathematical Understanding Grades K-2*)

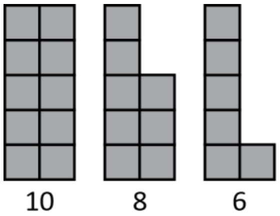
One of the most fundamental concepts in pattern work, but also one not clear to all students, is that, although the part of the pattern that they see is finite, when mathematicians talk about a pattern, they are talking about something that continues beyond what the student sees. (Small, 2009, p.568)

In Mathematics 2, students describe, reproduce, extend, and create repeating, increasing and, decreasing patterns. They use ordinal numbers (to tenth) to describe elements of repeating patterns. Students in Mathematics 3 explore increasing and decreasing patterns, both numerical patterns with numbers to 1000 and non-numerical patterns with concrete materials, pictures, sounds, and actions. They use ordinal numbers (to 100th) to refer to or to predict terms within a pattern.

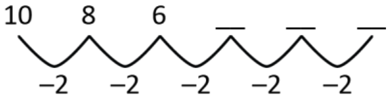
Students have difficulty extending an increasing number pattern or a decreasing number pattern. A suggested strategy is to have students locate the numbers in the pattern on a hundred chart and place a transparent counter over each number. Have students use the visual pattern in the counters to extend the pattern. Help students relate the visual pattern to the starting point and the number added each time in the number pattern. (Pearson, 2009b, p. 14)

For example, when skip counting by 3, use only starting points that are multiples of 3 (3, 6, 9, 12, ...). This will result in many diagonal representations on a hundred chart. Skip counting by 5, results in a pattern that is two vertical columns with numbers ending in the digits 5 (5, 15, 25, …) and 0 (10, 20, 30, …). Students should also explore hundred charts to 1000 (1–100, 101–200, 201–300, …) and look for patterns when counting by 2s, 5s, 10s, 25s, and 100s.

For example, in the pattern below, the pattern rule is to start with 10 squares and decrease by 2 squares each time.



As students describe decreasing shape patterns, help them recognize that each term has a numeric value. The above pattern can be expressed as 10, 8, 6, … by counting the number of squares in each term. Students may also find it useful to record the change from one term to the next as shown below:



Remind students that a pattern rule must have a starting point or the pattern rule is incomplete. For example, if a student describes the pattern 10, 8, 6, ... as a decrease by 2 without indicating that the pattern starts at 10, the pattern rule is incomplete.

The student results of the **2018–2019 Nova Scotia Assessment Mathematics in Grade 3** shows the following:

- most students correctly answered all questions related to patterns and relations

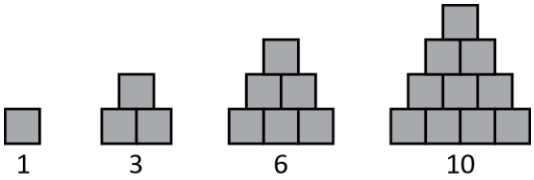
C. What are the next steps in instruction for the class and for individual students?

As students identify the core of a pattern, they should use appropriate patterning vocabulary, such as **core** (the repeating part of the pattern) and **elements** (the actual objects used in the pattern). It is important to create patterns that have the core repeated at least three times. To help students identify the pattern core, it is suggested students highlight, or isolate, the core each time it repeats. Remind students that repeating patterns can be extended in both directions. Encourage students to reference the position of the elements of the pattern using ordinal numbers. The core of the shape pattern below is: circle, square, triangle. There are three elements in this pattern, namely a circle, a square, and a triangle.

The pattern above is also a three-element pattern. The core of this three-element pattern is circle (1st element), square (2nd element), and triangle (3rd element).

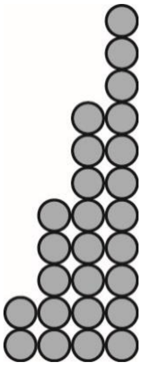
Increasing Numeric Patterns

Students should be able to describe an increasing pattern. An increasing pattern is a growing pattern where the size of the term increases in a predictable way. The terms in an increasing pattern grow by either a constant amount or by an increasing amount each time. Students need sufficient time to explore increasing patterns using various manipulatives, such as cube-a-links, tiles, toothpicks, counters, pattern blocks, base-ten blocks, ten-frames, bread tags, stickers, or buttons, to realize they increase in a predictable way. As students describe increasing shape patterns, help them recognize that each term has a numeric value. For example,

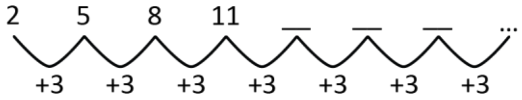


A counting sequence is an increasing pattern where each number represents a term in the pattern. For example, in the counting sequence 1, 2, 3, 4, …, 1 represents the first term, 2 the second term, 3 the third term … This counting sequence can be connected to ordinal numbers where students should be able to recognize that the 34th term is 34 and that 57 is the 57th term in the sequence. These ordinal number patterns should be investigated for numbers up to 100.

Students should be able to describe a given increasing pattern by stating the pattern rule. A pattern rule tells how to make the pattern and can be used to extend an increasing pattern. Give students the first three or four terms of an increasing pattern. Ask them to state the pattern rule by identifying the term that represents the starting point and describing how the pattern continues. For example, in the pattern below, the pattern rule is, “Start with 2 counters and add 3 counters each time”.



As students describe concrete or pictorial patterns, help them recognize that each term has a numeric value. For example, the above pattern can be expressed as, 2, 5, 8, 11, … by counting the number of counters in each term. Students may also find it useful to record the change from one number to the next as shown below.



Students need opportunities to compare numeric patterns, discussing how they are the same and how they are different. When comparing increasing patterns, compare the starting points and how each term increases. For example, one-way students may address this is by using a page with four small hundred charts. Ask them to skip count starting a 0 and shade one chart by 2s, one chart by 5s, one chart by 10s, and one chart by 25s. Then, discuss the pattern rule in each chart comparing the starting points and the amount of increases.

Students should be able to create increasing patterns, concretely, pictorially, and symbolically, and be able to describe the pattern rule they used to create their representations. When creating increasing patterns, initially students need to choose a starting point and then decide on the amount of increase. The amount of increase may be either a constant amount or an increasing amount. Students should be able to describe their pattern by clearly explaining how it changes from one term to the next. Students may share their patterns and the strategies they used to create their pattern.

Students should have frequent experiences using increasing patterns to solve real-world problems that interest and challenge them. They should use concrete materials or pictures to model the problem before determining the pattern rule. A variety of strategies may be used to solve the problem such as using a number line, a hundred chart, a picture, concrete materials, or skip counting.

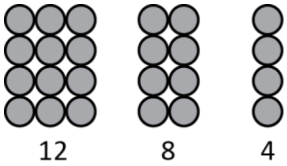
Decreasing Numeric Patterns

Students should be able to describe a decreasing pattern. A decreasing pattern is a shrinking pattern that decreases by a constant amount each time. Students need sufficient time to explore decreasing patterns using various manipulatives, such as cube-a -links, tiles, toothpicks, counters, pattern blocks, base-ten blocks, ten-frames, bread tags, stickers, and buttons. Sometimes students are more comfortable during the exploration stage if they can experiment first, using manipulatives, then pictures, and eventually numbers.

Students should be able to identify and describe various decreasing patterns such as horizontal, vertical, and diagonal patterns found on a hundred chart. Working with decreasing patterns can be connected to skip counting in outcome N01. Provide copies of hundred charts. Ask students to begin at 100 and skip count backward by a given number, shading in the number for each count all the way to 1. Then they write a description of the pattern. For example, if they chose to skip count by 10s, the pattern results in one vertical column, regardless of the starting point.

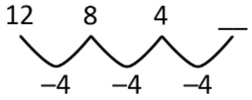
As students begin to investigate patterns, they sometimes confuse repeating patterns with decreasing patterns. Remind them to look for a core first. If they cannot find a core, then the pattern is not a repeating pattern.

Earlier, students became familiar with assigning a numeric value to each element in an increasing pattern. This expectation also applies to decreasing patterns.



Students should be able to describe a given decreasing pattern by stating the pattern rule. A pattern rule includes a term representing a starting point and a description of how the pattern continues. A pattern rule tells how to make the pattern and can be used to extend a pattern. For example, in the pattern above, the pattern rule is to start with 12 squares and decrease by 4 squares each time.

As students describe decreasing shape patterns, help them recognize that each term has a numeric value. The above pattern can be expressed as 12, 8, 4, … by counting the number of circles in each term. Students may also find it useful to record the change from one term to the next as shown below.



Remind students that a pattern rule must have a starting point, or the pattern rule is incomplete. For example, if a student describes the pattern 12, 8, 4, ... as a decrease by 4 patterns without indicating that it starts at 12, the pattern rule is incomplete.

Students need opportunities to compare numeric patterns and to discuss how they are the same and how they are different. When comparing decreasing patterns, compare the starting points and how each term decreases using a variety of representations such as shape patterns, hundred charts, and number patterns.

For example, give students a page with four small hundred charts. Ask them to skip count backward starting at 100 and shade one chart by 2s, one chart by 5s, one chart by 10s, and one chart by 25s. Then discuss the pattern rule in each chart indicating the starting point and the amount of decrease.

D. What are the most appropriate methods and activities for assessing student learning?

Below are some sample questions related to pattern and relations which will be used to represent some of the appropriate methods and activities for assessing student learning.

Encourage students to share their thinking, their strategies, and their solutions. Rich discussion fosters the development of alternative problem-solving strategies.

Examples:

1. What are the two missing numbers in the number pattern Below?
66, 61, 56, 51, _____, 41, 36, _____, …

- ○ 45 and 35
- ○ 46 and 31
- ○ 52 and 31
- ○ 52 and 37

2. Natalie created the following decreasing pattern:
546, 536, 526, 516, 506, 496, …
What is the rule for this pattern?

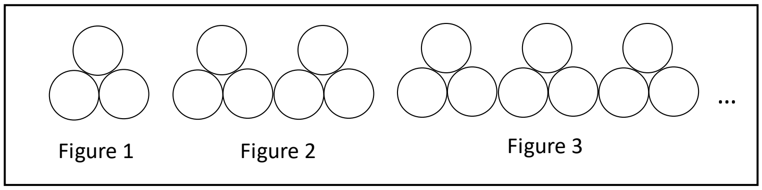
- ○ Subtract 10.
- ○ Start at 546 subtract 5 each time.
- ○ Start at 496 and subtract 10 each time.
- ○ Start at 546 and subtract 10 each time.

3. Monique created the following decreasing number pattern. Two numbers are missing in this pattern.
55, 50, 45, 35, 30, 25, 20, 10, 5, …

What are the two missing numbers?

- ○ 40 and 30
- ○ 15 and 25
- ○ 40 and 15
- ○ 45 and 15

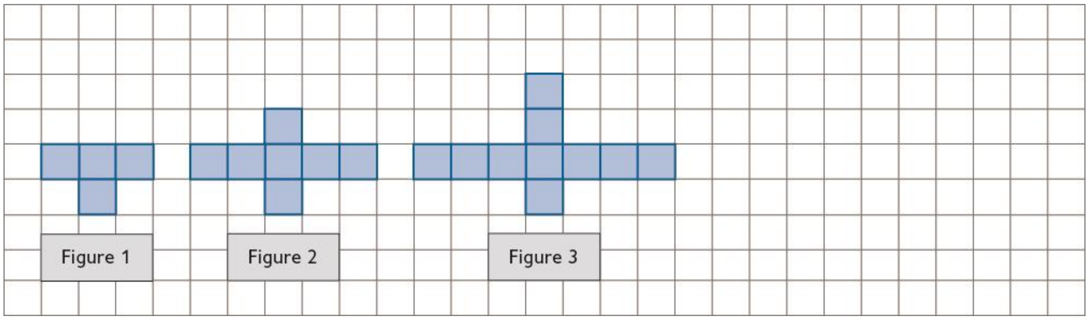
4. Simon created the following pattern.



How many small circles are there in the fifth figure?

- ○ 15 circles
- ○ 12 circles
- ○ 10 circles
- ○ 5 circles

5. Examine the following pattern:



How many squares are there in Figure 4?

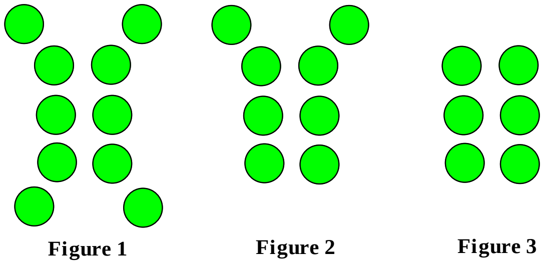
○ 14 squares
○ 13 squares
○ 12 squares
○ 11 squares

6. Which statement about the two following patterns is true?

62, 74, 86, 98, … and 62, 50, 38, 26, …

○ They have the same starting point and increase in the same way.
○ They have the same starting point and they are increasing patterns.
○ They have the same starting point and they are decreasing patterns.
○ They have the same starting point and they do not change in the same way.

7. Examine the following pattern:



How many circles are there in Figure 5?

○ 2
○ 3
○ 4
○ 5

8. Examine the following pattern:

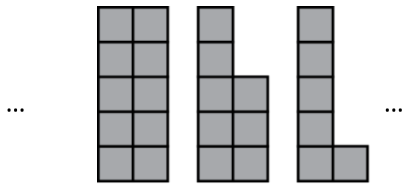


Figure 3 Figure 4 Figure 5

How many squares are there in Figure 1?

○ 1 square
○ 4 squares
○ 12 squares
○ 14 squares

9. Marthe created the following pattern using yellow and red counters:

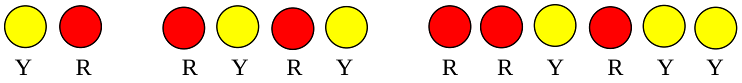


Figure 1 Figure 2 Figure 3

What is the rule for this pattern?

○ Figure 1: add one yellow counter to the left, and one red counter to the right each time.
○ Figure 1: add two red counters to the left, and two yellow counters to the right each time.
○ Figure 1: add one yellow counter, and one red counter to the left, and one yellow counter, and one red counter to the right each time.
○ Figure 1 add one red counter to the left, and one yellow counter to the right each time.

10. Forty-two students in the second and third grades are in the school gymnasium.
Twenty-six students are in the second grade.

What equation do you use to determine the number of third-year students who are in the gymnasium?

○ 42 = 26 + □
○ 26 + 42 = □
○ □ – 26 = 42
○ 26 – □ = 42

Mathematics in Grade 3 Lesson Learned 5
Measurement

Students are expected to build conceptual understanding of what it means to measure with a ruler. Emphasis should be placed on counting the intervals between the numbers, rather than looking at the number on the ruler that is aligned with the end of the object. Students also need to use a ruler to measure the length of a pencil or other objects with and without using zero as the starting point. Students need to recognize which mass unit (gram or kilogram) is appropriate for measuring and comparing the mass of a specific item. Students need to be able to explain that perimeter is the distance around a shape or the length of the boundary of an enclosed region. Students need to find the perimeter of different regular, and composite shapes, before being introduced to questions in pictorial form. Students need to work with perimeter in application and analysis questions.

A. What conclusions can be drawn from the Nova Scotia Assessment: Mathematics in Grade 3?

We noticed that students did very well when presented with questions related to direct measure, time referents, and approximate measure when using personal referents. For example, students were able to identify how many centimetres are in one metre (Knowledge question). They were also able to solve a given problem involving the number of seconds in a minute, the number of minutes in an hour, and the number of hours in a day (Knowledge question).

Students were able to use their personal referents for 1 g and 1 kg to estimate the mass of common objects, such as a bag of sugar or a paper clip (Application question). Students also did well when estimating the length or height of an object using personal referents. For example, students used the height of a doorknob from the floor as a personal referent for 1 m (Application question).

Having these personal referents helps students visualize measurements and estimate more accurately. Personal referents also help students identify the units required for specific measurements.

A big idea developed in Grade 3 is perimeter. Students appeared to not understand the concept of perimeter even though a definition was given in parentheses in a question on the assessment. Students need to be able to explain that perimeter is the distance around a shape or the length of the boundary of an enclosed region.

B. Do students have any misconceptions or errors in their thinking?

It is important to distinguish between misconceptions and errors. Misconceptions are, in effect, the misunderstandings about mathematical ideas which students entertain, and which usually lead to errors occurring. These are the most serious kind of errors which require urgent action on the part of the teacher. Errors may occur for a variety of underlying reasons, ranging from careless mistakes to errors resulting from misconceptions. - John SanGiovanni – (*Mine The Gap For Mathematical Understanding Grades K-2*).
Often teachers assume that students know how to use a ruler. This skill is introduced in Grade 3.

A common error that many students make is the placement of the ruler when measuring an object. This may indicate that they do not understand that they are counting the intervals between the numbers to determine length. Some students fail to consider the gap between the end of the ruler and the zero mark. Other students may begin to measure at other points on a ruler, other than the 0 cm mark when they first start to use a standard measurement tool such as a centimetre ruler. Some students ignore the 0 cm mark and begin at the 1 cm mark. This may indicate that students do not realize that the scale of the ruler begins at 0 cm, especially if it is not labelled at the beginning of the ruler.

Note however, that some students may start at 1 on the ruler but still use the ruler accurately by taking this into consideration as shown in the picture found below. The pencil being measured starting at the 0 cm mark measures 4 cm. The pencil being measured starting at 1 cm and ends at 5 cm, still measures 4 cm.

Students have misconceptions related to estimating mass and comparing the kilogram and the gram. Students need to recognize which mass unit (kilogram or gram) is appropriate for measuring and comparing the mass of a specific item. Students have difficulty matching an item with its estimated mass. When you measure how heavy an object is, you're measuring its mass. Students may not understand that mass and size are not necessarily related. Different sized objects can have the same mass, while objects of the same size can have different masses.

Another common misconception that students have is related to perimeter and area. Some students have the misconception that perimeter and area of an object are always the same measures. Students should recognize that area and perimeter are independent of one another. Students often do not make the distinction between area and perimeter and do not understand when to use each one in a problem. They may calculate the area instead of the perimeter or vice versa. Others think that perimeter and area are interchangeable. This is seen when students are finding the perimeter of many different regular, irregular, and composite shapes drawn on 1 cm paper. They find it difficult to count the units along the perimeter.

The student results of the **2018–2019 Nova Scotia Assessment Mathematics in Grade 3** shows the following:

- 61% of students made a mistake when measuring a pencil length using a ruler
- 51% of students had difficulty when calculating the perimeter of a regular polygon in a context

C. What are the next steps in instruction for the class and for individual students?

Standard Units

Working with standard units is integral to students' understanding of measurement. Students may start using standard units to measure length when they realize that non-standard units mean different things to different people. For example, if someone says a book is 15 cm long, everyone knows how long that is, but to say the book is 15 cards long would be more difficult to interpret. Students need to develop a familiarity with standard units and explore the relationship between them.

Students begin to use a standard tool to measure length in Mathematics 3. They learn about two basic standard units of length - centimetre and metre. It is valuable to initially use simpler rulers that are created by the students. Then, they should move on to tools that are easy for them to read. Students should use rulers (or the side of the ruler) that show only numbered centimetres and not millimetres. Students should identify objects from around the classroom that would be an appropriate referent for a centimetre or a metre; for example, a pencil, a garbage can, a teacher's desk, or a glue stick.

Measuring using a Ruler

When students begin to use rulers to measure in Grade 3, it is important for students to line up the 0 mark with one end of the shape being measured. Emphasis should be placed on counting the intervals between the numbers, rather than looking at the number on the ruler that is aligned with the end of the object. Lining up small cubes from base-ten materials along the ruler will demonstrate that the numbers on the ruler correspond to the number of small cubes, starting at 0. Observe how students use a ruler to measure a shape that is longer than the ruler. Show students how to measure something that is longer than a ruler by marking, recording, and starting again.

Using a centimetre ruler, students should measure the length, width, or height of a given 3-D object in the classroom, such as a lunch box, their desk, or a cereal box. Students can record their measurements using both the number and the measurement unit; for example, 3 cm or 3 centimetres. Ensure students are clear about the distance they should be measuring. Instruction should continue to support students in learning to use rulers accurately. A strategy often used to help students is to break up plastic rulers. Give a piece of broken ruler to each student and ask them to measure items in the classroom. Observe how they attempt to measure items, especially if there is not a familiar starting point for them, such as 0 cm or 1 cm on their piece of ruler.

Another important strategy to help students learn to measure accurately is to encourage them to estimate measurements before verifying them using a measurement tool.

Mass and Weight

The terms mass and weight are similar, but they are not the same. Weight measures how heavy an object is and depends upon gravity, so it will vary with height above sea level. On the other hand, mass measures the amount of matter in an object and will be the same at all heights above sea level.

As with all measurement units, it is important that students have a personal referent for a gram and a kilogram. Students should recognize which mass unit (gram or kilogram) is appropriate for measuring the mass of a specific item. It is helpful for students to investigate how everyday items, such as food items, are measured. Include items that are small and dense, such as a golf ball, as well as those that are large and hollow or porous, such as a beach ball. Students need to understand that grams are used to measure very light objects and kilograms are more appropriate units for heavier objects.

Using their understanding of a kilogram, ask students to brainstorm items that may have a mass of 1 gram. They may also use a small base-ten cube as a personal referent of a gram. You may wish to provide students with items such as a raisin, bean seed, jellybean, or a paper clip, to conceptualize the sense of how a gram feels.

It would be beneficial for students to have an opportunity to make a kilogram mass of their own. Provide students with materials such as sand, flour, sugar, and small cubes from base-ten materials to fill a container until it exactly balances with a 1 kg mass on a balance scale. Using this kilogram container, they can now compare its mass to items in the classroom to help them find a personal referent for 1 kg.

Using objects from the classroom, for example a counter, a raisin, a paper clip, a textbook, a sneaker, or a lunch box, ask students to identify whether the object is an appropriate referent for a gram or a kilogram. Once students have established a personal referent for 1 g and 1 kg, they can now use their referents to estimate the mass of common objects such as an eraser, an apple, a juice box, or a textbook, or to estimate whether an object is heavier or lighter than 1 kg.

Estimating mass is more difficult than estimating other measures, as the object’s size and shape is not directly related to its mass. A strategy often used to help students with this misconception is to encourage them to use the known mass of one object to estimate the mass of another. For example, a tennis ball and an orange are about the same size, but the orange is much heavier than a tennis ball.

It is important for students to know that 1000 grams is equal to a kilogram. Model for students how 1000 g is equal to 1 kg using a balance scale: use food items of various benchmark masses, such as 2 bags of 500 g, 4 boxes of 250 g, or a pre-counted bag of 1000 jellybeans.

Students should use the word mass rather than weight to say whether an object is heavier or lighter than another object. Mass and weight are two different physical quantities. There are three main differences between these two quantities:

- The mass of an object is the quantity of matter contained in this object. The weight of an object is the force of attraction exerted by the Earth on this object.
- The mass of an object does not change when its position changes. The weight of an object changes when its position changes. For example, the mass of an astronaut is 80 kg on Earth, on the Moon and at any point between Earth and the Moon because the matter of his body does not change. As the astronaut moves away from Earth, to move towards the moon, the force of attraction of the Earth on him decreases and he arrives at a certain distance from the Earth where his weight becomes zero. So, it floats.

- The mass of an object is measured using a balance. The weight of an object is measured using a dynamometer.

Perimeter

Students learn perimeter best when they can make connections to real life examples. Ideas include fencing a yard, measuring the perimeter of the classroom or gym using metre sticks.

It is important that students have many opportunities to find the perimeter of many different regular shapes concretely before being introduced to pictorial forms. Pentominoes may be used to illustrate measuring and recording the perimeter of a given composite shape. Pentominoes are shapes each made up of five squares, all of which must have at least one side matching up with the side of another.
In addition to composite shapes with straight sides, it is important to expose students to other shapes such as their handprint. Working with a partner, ask students to trace around their closed hand. Using string they can outline their handprint and then cut the string to determine the perimeter of their handprint by measuring the length of the string with their ruler. Again, students need to be able to explain the strategies they used for finding perimeter as they proceed.

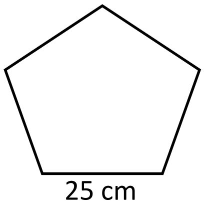
In the early grades, students should explore their own methods for determining the perimeter of a shape and should not develop or follow a formula for calculating perimeter. For example, provide students with various regular and irregular polygons (e.g., squares, rectangles, and triangles), some string, and a ruler. Small groups of students should be asked to find the perimeter of the various regular and irregular polygons in a variety of ways. Some may use the string, while others go directly to measuring the sides with the ruler.

Students need to draw more than one shape for the same given perimeter. Students may use a geo-region or centimetre grid paper to explore various shapes with the same perimeter. They may explore various rectangles before exploring other shapes.

Students should be given opportunities to construct shapes of a given perimeter. Discuss with students that when constructing shapes for a given perimeter, they must remember that their shapes should be completely enclosed. It would be easier for students to begin their constructions drawing rectangles using centimetre grid paper and horizontal and vertical lines only.

Example 1: Answer – Page 76

Each side of the pentagon shown measures 25 cm.



What is the perimeter of this pentagon?

A. 25 cm

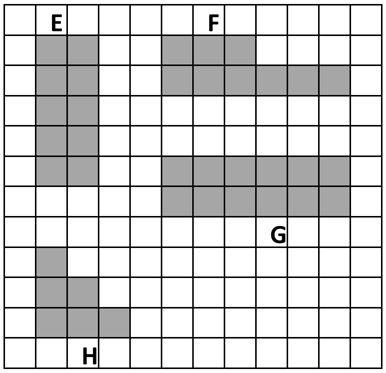
B. 50 cm

C. 100 cm

D. 125 cm

Example 2: Answer – Page 76

Samir draws these 4 shaded shapes on grid paper.



Which shaded shapes have the same perimeter?

A. E and F

B. F and G

C. F and H

D. H and G

Example 3: Answer – Page 76

Students need to be exposed to word problems with a context about perimeter rather than simply finding the perimeter of many different regular shapes.

Farmer Bill has 24 metres of fencing.

How many different rectangular chicken coops can he make?

A. 3

B. 5

C. 6

D. 24

D. What are the most appropriate methods and activities for assessing student learning?

Below are some sample questions related to measurement which will be used to represent some of the appropriate methods and activities for assessing student learning.

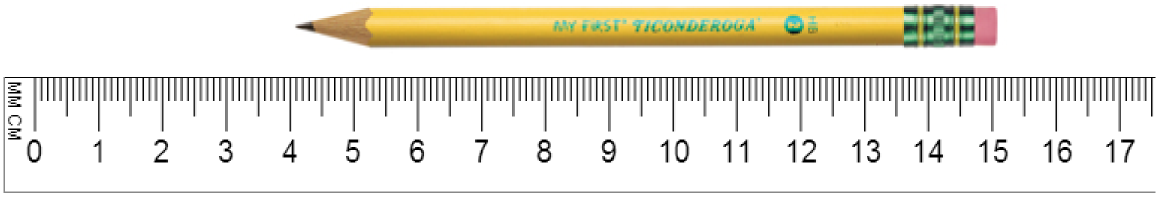
Encourage students to share their thinking, their strategies, and their solutions. Rich discussion fosters the development of alternative problem-solving strategies.

Examples:

1. What is the best unit to choose to measure the length of a season?

○ Days
○ Weeks
○ Months

2. What is the length of the pencil below?



○ 4 cm
○ 11 cm
○ 12 cm
○ 15 cm

3. The width of your thumb is about

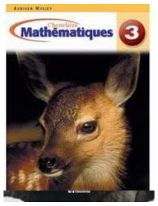
○ 1 cm
○ 10 cm
○ 100 cm
○ 1 m

4. The mass of a box of table salt is about

○ 1 g
○ 100 g
○ 1 kg
○ 100 kg

5. The mass of your mathematics book is about

○ 10 g
○ 100 g
○ 200 g
○ 1000 g

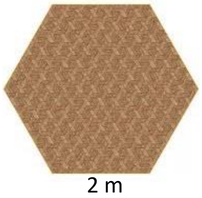


6. Which of the following statements is **true**:

○ My chair is lighter than my marker.
○ The mass of an orange is about 200 grams.
○ The length of my arm is about 5 m.
○ My pencil is longer than my mathematics book.

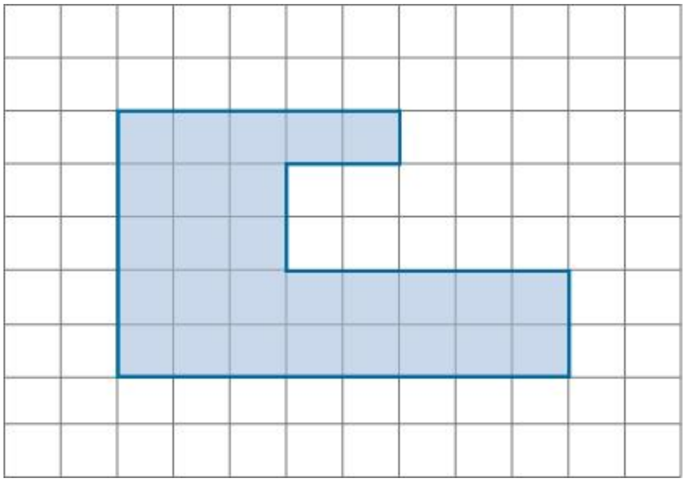
7. A carpet is in the shape of a hexagon. Each side of the carpet is 2 m long.
What is the perimeter of this carpet?

○ 12 m
○ 10 m
○ 8 m
○ 6 m



8. What is the perimeter of the figure below? It is drawn on the 1 cm grid paper.

○ 37 cm
○ 30 cm
○ 29 cm
○ 28 cm



9. How many rectangles can you draw that have a perimeter of 16 cm each?

○ 4
○ 3
○ 2
○ 1

Mathematics in Grade 3 Lesson Learned 6
Geometry

Students need to continue developing their knowledge of 2-D shapes and 3-D objects by describing and sorting them according to their geometric attributes. Students need experiences to identify the faces, edges, and vertices of 3-D objects including spheres, cones, cylinders, pyramids, cubes and other prisms. Students need to be provided with opportunities to explore these attributes through sorting and constructing activities. Students need to extend their knowledge of both regular and irregular polygons. They need to focus on comparing the number of sides as the key attribute for classifying polygons. Students need more experiences with regular polygons, so that they begin to realize that a polygon, regardless of its dimensions, or position in space, remains the same shape.

A. What conclusions can be drawn from the Nova Scotia Assessment: Mathematics in Grade 3?

Students did very well when presented with questions related to 3-D objects, such as cylinders and spheres, found in their everyday life. They were able to identify the object being described. These were all application questions.

Students need to continue to develop their knowledge of geometry by describing and sorting 3-D objects according to their geometric attributes. Students require more experiences to identify the faces, edges, and vertices of 3-D objects including spheres, cones, cylinders, pyramids, and cubes and other prisms.

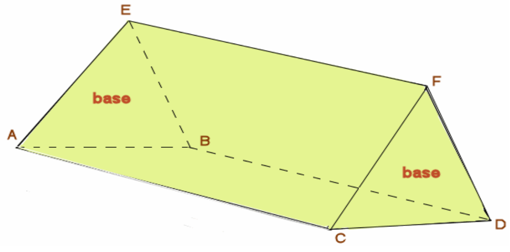
Students need to focus on comparing the number of sides as the key attribute for classifying polygons. Students need be able to name the specific polygons including the triangle, quadrilateral, pentagon, hexagon, and octagon.

B. Do students have any misconceptions or errors in their thinking?

It is important to distinguish between misconceptions and errors. Misconceptions are, in effect, the misunderstandings about mathematical ideas which students entertain, and which usually lead to errors occurring. These are the most serious kind of errors which require urgent action on the part of the teacher. Errors may occur for a variety of underlying reasons, ranging from careless mistakes to errors resulting from misconceptions. - John SanGiovanni – (*Mine The Gap For Mathematical Understanding Grades K-2*)

A common misconception that many students make is they confuse faces, edges, or vertices on a geometric object. They have difficulty distinguishing what determines the faces, edges, or vertices of an object.

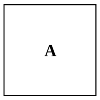
Another common error or misconception that students have is understanding that the base of a geometric object is whatever face it is sitting on. A pyramid or prism is named according to the shape of its base. A pyramid has one base and triangular faces. A prism has two matching bases that are polygons. The other faces are rectangles.

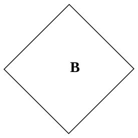


The two congruent triangles ABE and CDF are the two bases. The rectangles ABDC, ACFE and BDFE are the lateral faces.

When students are working with 3-D objects such as cylinders, cones, and spheres, they consider the rounded surfaces on these 3-D objects to be faces. This is an error or misconception. Another important concept for students to understand is that an edge occurs where two faces of a 3-D object meet. A vertex (vertices) is a point where three or more edges meet. (**Note:** On a cone and a pyramid the highest point above the base is called the **apex**. In a pyramid the apex is also a vertex, but for a cone, it is a mistake to refer to the apex as a vertex as there are no edges that meet.) As they become more familiar with identifying the attributes, students can determine the number of faces, edges, and vertices. Students will use informal language at this stage rather than precise mathematical language.

Some students incorrectly believe that the orientation of a geometric figure, changes the figure itself. Students recognize that shape A is square but think that shape B is not a square.



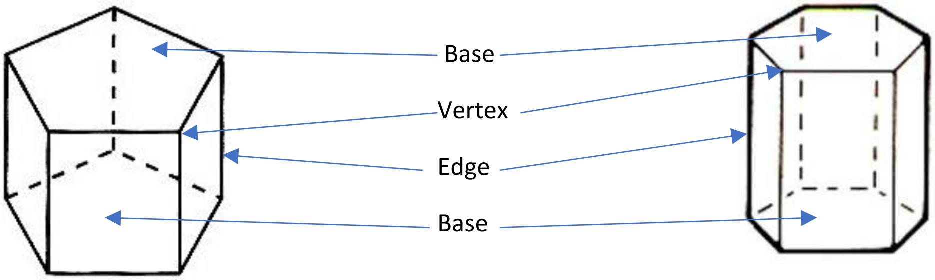


The student results of the **2018–2019 Nova Scotia Assessment Mathematics in Grade 3** shows the following:

- 63% of students had difficulty determining the rule for sorting a set of three-dimensional objects consisting of prisms and pyramids

C. What are the next steps in instruction for the class and for individual students?

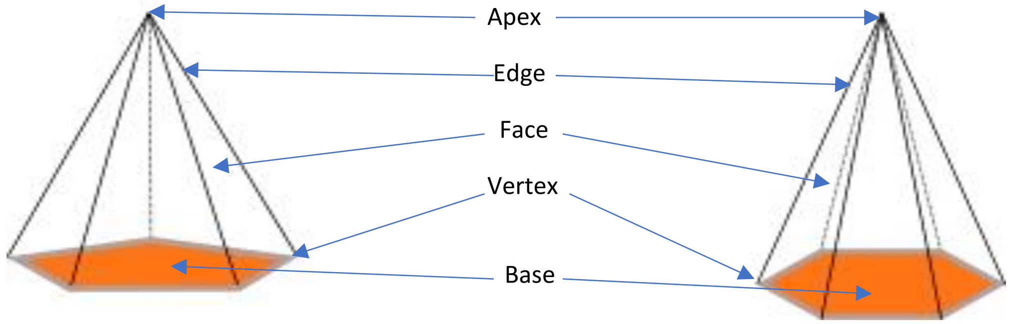
Students should be able to identify the faces, edges, and vertices as well as the shape of the faces of a given 3-D object.



This pentagonal prism has 7 faces (5 rectangular lateral faces and 2 pentagonal faces which are the 2 bases), 15 edges and 10 vertices.

This hexagonal prism has 8 faces (6 rectangular lateral faces and 2 hexagonal faces which are the 2 bases), 18 edges and 12 vertices.

Note: The lateral faces of a prism are **rectangles**. The two bases, that are also faces, are **polygons**. A prism is named by its bases.

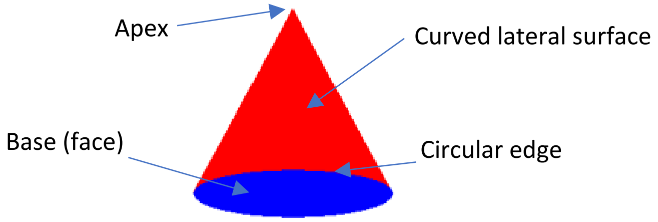


This pentagonal pyramid has 6 faces (5 lateral triangular and 1 pentagonal face, the base), 10 edges, 6 vertices (5 base vertices and 1 vertex or apex).

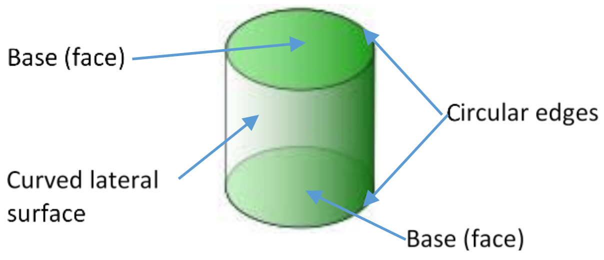
This hexagonal pyramid has 7 faces (6 lateral triangular and 1 hexagonal face, the base), 12 edges, 7 vertices (6 base vertices and 1 vertex or apex).

Note: The lateral faces of a pyramid are **triangles**. The base, that is also a face, is a **polygon**. A pyramid is named by its bases.

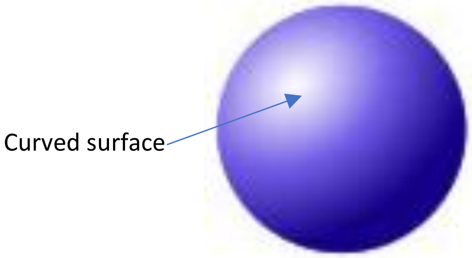
It is essential that students explore the attributes of a cone, a cylinder, and a sphere using manipulatives and concrete geometric materials. They must visualize these 3-D objects in order to understand their attributes.
A cone has a circular flat face which is its base, a lateral curved surface starting from the base to the apex and a circular edge.



A cylinder has two circular flat faces which are its bases, a lateral curved surface joining the two bases and two circular edges.



A sphere has a curved surface. It does not have faces, edges or vertices.



Note: The curved surface of a cone, a cylinder or a sphere is not a face.

A suggested strategy to help students identify faces, edges, and vertices is to put a different colour of a small ball of modelling clay on each face, edge or vertex as they count. This should help students describe 3-D objects according to the shape of the faces and the number of edges and vertices.

A pyramid or prism is named according to the shape of its base. A strategy that may help with identifying the base of a geometric object is to remember that the base of a pyramid may not be a triangle. For a rectangular prism, any of the faces can be the base.

When working with 3-objects such as cylinders, cones, and spheres, students consider the rounded surfaces on these 3-D objects to be faces. The strategy suggested to help with this error or misconception is to help students with the conceptual understanding that a face is a flat surface on geometric objects.

Show students models and real-life objects of cylinders, cones, and spheres. Ask students what the difference is between these solids and the prisms and pyramids already studied. Show students the faces, edges, and vertices of each solid. Brainstorm, with the students, what each term means.

Students should be able to determine the number of faces, edges, and vertices of a given 3-D object.

- A cylinder is a 3-D object with 2 faces, 1 curved surface, 2 edges, and 0 vertices.
- A cone is a 3-D object with 1 face, 1 curved surface, 1 edge, and 1 apex.
- A sphere is a 3-D object with 1 curved surface, 0 faces, 0 edges, and 0 vertices.

Students should compare and sort 3-D objects by observing the number of faces, edges, and vertices. A student may sort objects in various ways, such as those that have all square faces, those that have circular faces, those that have 8 vertices, or those that have straight edges. Students should play games with their peers in which they sort objects and ask their peers to guess the sorting rule according to the number of faces, edges, and vertices. It is essential to have a large collection of 3D solids available for students to explore.

Through many experiences with identifying polygons in a variety of positions, students should begin to realize that a polygon, regardless of its position, remains the same shape.

Provide students with various sizes of a particular polygon. Have students count the number of sides and identify the polygon. Having a variety of these experiences with different polygons, students should begin to realize that a polygon, regardless of its dimensions, remains the same shape.

Students should find examples of polygons in the world around them, perhaps even collect as many types of a shape as they can find. Sort the shapes according to the number of sides. By sorting polygons according to the number of sides, students can learn the names for the polygons. The following are examples:

D. What are the most appropriate methods and activities for assessing student learning?

Below are some sample questions related to geometry which will be used to represent some of the appropriate methods and activities for assessing student learning.

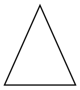
Encourage students to share their thinking, their strategies, and their solutions. Rich discussion fosters the development of alternative problem-solving strategies.

Examples:

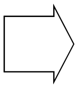
1. I am 3-D object. I have 5 faces, 8 edges and 5 vertices.
Which object am I?

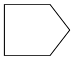
○ A rectangular prism
○ A triangular pyramid
○ A square prism
○ A square pyramid

2. Which figure **is not** a polygon?

○ 

○ 

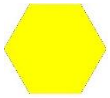
○ 

○ 

3. Which figure is a quadrilateral?

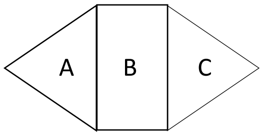
○ 

○ 

○ 

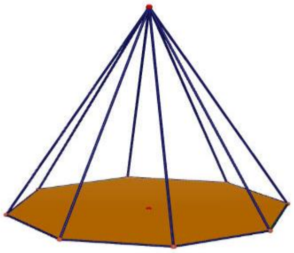
○ 

4. Which polygon is formed by joining the geometric figures A, B and C together?



○ a hexagon
○ a pentagon
○ an octagon
○ a quadrilateral

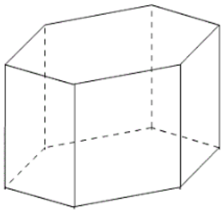
5. Look at the following pyramid:



Which statement is **true**?

○ The pyramid has 9 faces, 9 edges and 9 vertices.
○ The pyramid has a hexagonal base and 8 triangular faces.
○ The pyramid has 8 triangular faces, 8 edges and 8 vertices.
○ The pyramid has an octagonal base and 16 edges.

6. Look at the following prism:



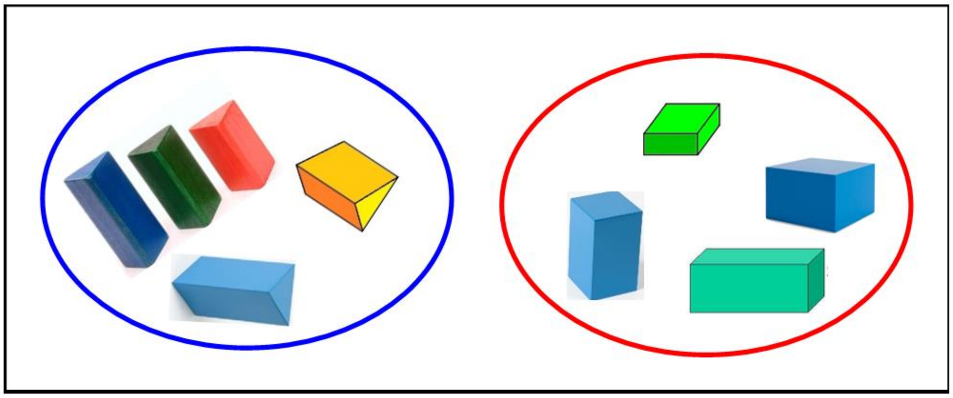
Which statement is **true**?

○ The prism has 8 faces, 18 edges and 12 vertices.
○ The prism has 6 faces, 18 edges and 12 vertices.
○ The prism has a hexagonal base and 8 rectangular faces.
○ The prism has an octagonal base, 8 faces and 12 vertices.

7. What objects have no vertices?

○ the prism and the sphere
○ the cylinder and the pyramid
○ the prism and the pyramid
○ the cylinder and the sphere

8. Robert has sorted the following prisms using a Venn diagram:

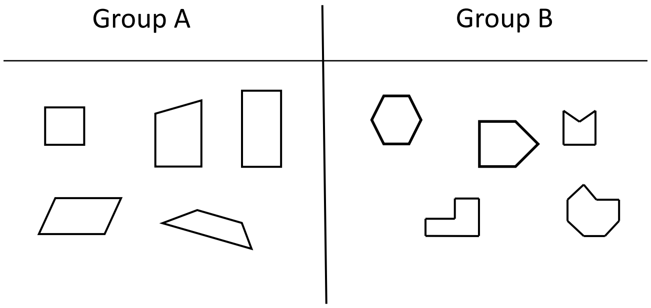


Which sorting rule did he use?

○ Prisms that have 6 faces and prisms that have 8 vertices.
○ Prisms that have 6 edges and prisms that have 6 faces.
○ Prisms that have 6 vertices and prisms that have 12 edges.
○ Prisms that have 6 vertices ad prisms that have 8 edges.

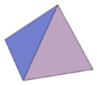
9. Lucie sorted these polygons.

What sorting rule did Lucie use?



○ Polygons that have 4 sides and polygons that have 5 sides.
○ Polygons that have 4 sides and polygons that have more than 4 sides.
○ Polygons that have 4 sides and polygons that have 6 sides.
○ Polygons that have more than 4 sides and polygons that have 8 sides.

10. Which object resembles a cone?

○ 
○ 
○ 
○ 

Mathematics in Grade 3 Lesson Learned 7
Statistics and Probability

Students were challenged using tally marks, lists, charts, line plots, and bar graphs to organize data relevant to their everyday life. Students need opportunities and experiences to interpret information collected, organized, and displayed in tally charts, charts, line plots and bar graphs. Students need to develop the skill of interpreting graphs, and answering questions and to draw conclusions from those tally charts, line plots and bar graphs. They need to be encouraged to ask or write questions that go beyond simplistic reading of a graph. Both literal and inferential comprehension questions need to be asked.

A. What conclusions can be drawn from the Nova Scotia Assessment: Mathematics in Grade 3?

Students did very well interpreting data from line plots. Line plots contain information that can be used to answer questions using a visual comparison, counting, and reading labels. It offers students a visual comparison of the different quantities of every piece of data. The cognitive level of these questions is application and analysis.

We noticed that students did very well when interpreting the data from a vertical bar graph when asked to answer a straightforward literal comprehension question about the data. They understood that they had to add all the data together to find the correct answer. This was an application question.

Students had difficulty when asked to interpret a table showing the results of a survey using tally marks. They understood the concept of the tally table and what it represented but could not interpret what they were being asked to answer for the data displayed. They did not realize that to answer the question, they had to compare two quantities and then subtract the smallest quantity from the largest in order to determine their answer. This was a knowledge question.

Students also had difficulty when asked to interpret a question displaying information in a bar graph. Again, they understood the concept of the bar graph and what it represented, but they did not realize that in order to answer the question, they had to perform an operation of addition with all the information given in each bar being displayed. This was an application question.

Students found it difficult to draw conclusions by comparing data represented in different types of displays. They were asked to draw a conclusion concerning the common attributes of line plots, horizontal bar graphs, pictographs, and vertical bar graphs with the same given set of data. They could not determine which data display did not represent the data correctly.

B. Do students have any misconceptions or errors in their thinking?

It is important to distinguish between misconceptions and errors. Misconceptions are, in effect, the misunderstandings about mathematical ideas which students entertain, and which usually lead to errors occurring. These are the most serious kind of errors which require urgent action on the part of the teacher. Errors may occur for a variety of underlying reasons, ranging from careless mistakes to errors resulting from misconceptions. - John SanGiovanni – (*Mine The Gap For Mathematical Understanding Grades K-2*)

Although, students did well when working with line plots, they did make errors when reading or counting the Xs on a line plot.

The difficulty with bar graphs, whether horizontal or vertical, is that students interpret data from the wrong bar on the graph. Therefore, when asked to answer questions concerning the bar graph data, it is not correct.

A misconception or error that students have is concerning the common attributes of line plots, horizontal bar graphs, pictographs, and vertical bar graphs with the same given set of data. They should notice that the attributes that are common include the title, the labels, the horizontal axis, and the use of dots or crosses. They should also notice that the attributes can differ; for example, there could be different titles, different use of the horizontal axis, and different labels.

The student results of the **2018–2019 Nova Scotia Assessment Mathematics in Grade 3** shows the following:

- 64% of students had difficulty identifying the bar graph that best represents a set of given data

C. What are the next steps in instruction for the class and for individual students?

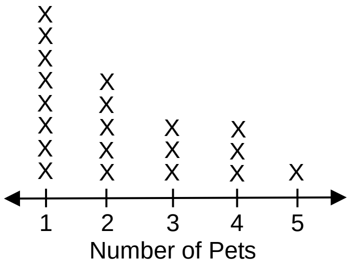
Students should be encouraged to collect, organize, and record their data using a tally system, line plots, charts, and lists to answer questions relevant to their everyday life. Using tally marks is a simple way for students to keep track of information as they collect it. Lists are a way for students to record the objects collected. A list can be made into a chart on which students would record their tally marks. Grouping the tally marks in fives makes it easier for students to total the numbers in each category by skip counting. When making a chart, students should always give it a title or heading to inform the reader about the meaning of the data. Students could then organize the data on a line plot(s).

A line plot provides a bridge from tally charts to bar graphs. At first, students should create their line plots using grid paper, with one dot or cross per grid paper square. The dots or crosses are placed one above the other for each tally mark for each item in the list or chart. Students should then count and write the total number of Xs under each column shown on the line plot. Students should notice that the attributes that are common include the title, the labels, the horizontal axis, and the use of dots or crosses. They should also notice that the common attributes can differ; for example, there could be different titles, different use of the horizontal axis, and different labels. They might also notice that when a line plot does not have a title as in the following picture, it is hard to make sense of the graph.

Tally system

Students Who Have Pets	
Number of Pets	Number of Students
1	𝍸 III
2	𝍸
3	III
4	III
5	I

Line plot



Chart/List

Students Who Have Pets	
Number of Pets	Number of Students
1	8
2	5
3	3
4	3
5	1

After a display of the data is constructed, discussing the information that can be obtained from the display is a valuable exercise. Students should work together to formulate questions that can be answered by other students using the data in the line plot, chart, or list. For example, from a line plot that displays the number of letters in your last name, students might formulate questions such as, What is the most common number of letters in a name? How many letters does the longest name in the class have? Shortest? A good graph should communicate some overall impressions of the data to a reader. Students should be able to answer questions using the display of the collected data. It is also important that each graph accurately represents the data and includes clear labelling and a title.

Present students with vertical and horizontal bar graphs that represent two different sets of data. Discuss what common attributes the two bar graphs have, such as title, axes, and labels for the axes, numerical scale, and bars. Discuss how the two bar graphs are different; for example, the titles of the graphs, labels for the axes, lengths and widths and spacing of the bars, and how some graphs have horizontal bars and others have vertical bars.

Students also experienced some difficulty drawing conclusions from graphs. To develop the skill of interpreting graphs, students should be given bar graphs and be asked to draw conclusions. They should be encouraged to ask or write questions that go beyond simplistic reading of a graph. Both literal and inferential comprehension questions should be asked, such as, What can you tell about _____ by looking at this graph? How many more/less than …? Based on the information presented in the graph, what other conclusions can you make? Why do you think _____? For example, when trying to solve the problem about what foods should be offered in the cafeteria, students would examine a bar graph that has the title, What Foods Should Be Available on the Cafeteria Menu? and be able to tell what food selections were considered the favourites by noticing which bars are the highest or longest. They may also draw conclusions that more students want healthy food for lunch than not healthy food.

Students should understand that to solve some problems, collecting and organizing graphs can help people to reach conclusions. Data is usually collected to answer questions, to discover something of interest, or most importantly, to solve a problem. Some examples of problems students might be interested in include, What should students be allowed to do during the lunch break? What foods should be available on the cafeteria menu? What foods should be removed from the cafeteria menu? What activities would you like to do in the gym? To answer questions like these, or to solve these kinds of problems, students could collect and display data, then interpret it.

D. What are the most appropriate methods and activities for assessing student learning?

Below are some sample questions related to statistics and probability which will be used to represent some of the appropriate methods and activities for assessing student learning.

Encourage students to share their thinking, their strategies, and their solutions. Rich discussion fosters the development of alternative problem-solving strategies.

Examples:

1. Emma asked her classmates a question to collect data. She recorded their answers in the following list:

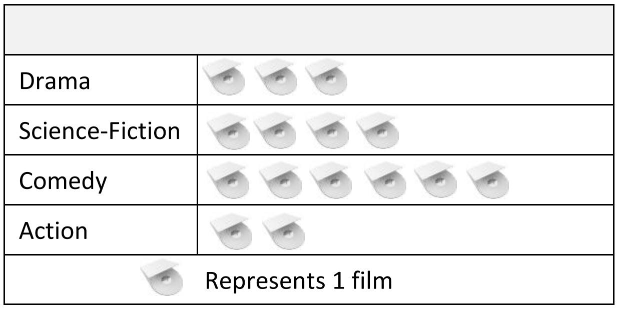
Favourite Films of the Students

Drama	3
Science-Fiction	4
Comedy	6
Action	2

Which question did Emma ask her classmates?

○ Do you prefer films about the police?
○ Do you prefer comedy more than action films?
○ What kind of films do you prefer?
○ Do you prefer drama more than action films?

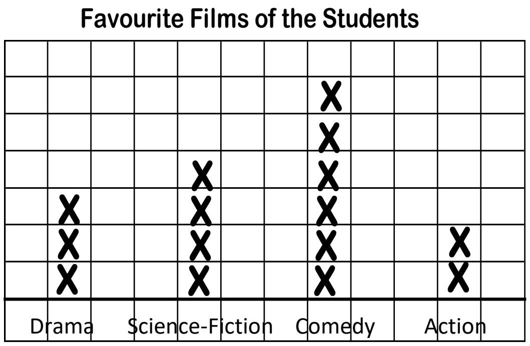
2. Emma constructed the following pictograph to present the collected data about her classmates. She made some errors.



What error did Emma make?

○ She forgot the title.
○ The number of comedy films is not equal to the number recorded in the list above.
○ She forgot the legend.
○ She forgot the title and the legend.

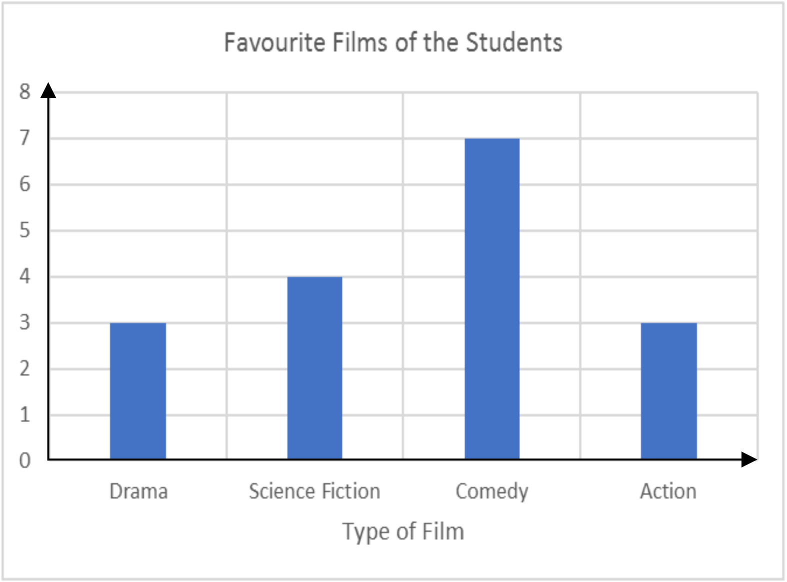
3. Emma presented the collected data above in the following line plot:



How many more comedy films than action films did the students choose?

- ○ 2
- ○ 4
- ○ 8
- ○ 9

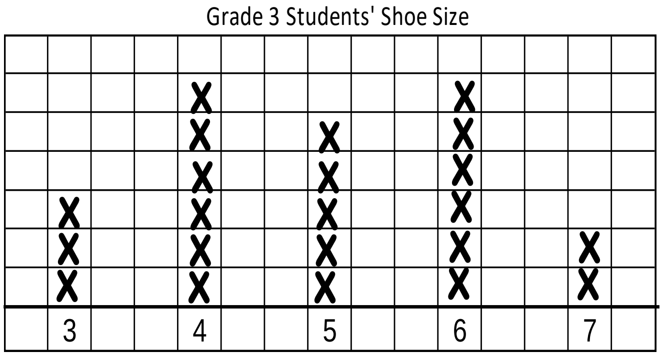
4. Tanya used the data collected by Emma to construct the following bar graph:



Examine the bar graph. What errors did Tanya make?

- ○ The axes labels are missing.
- ○ The vertical axis label and the title are missing.
- ○ The bars for comedy and action are not correct.
- ○ The vertical axis label and the bar for comedy are missing.

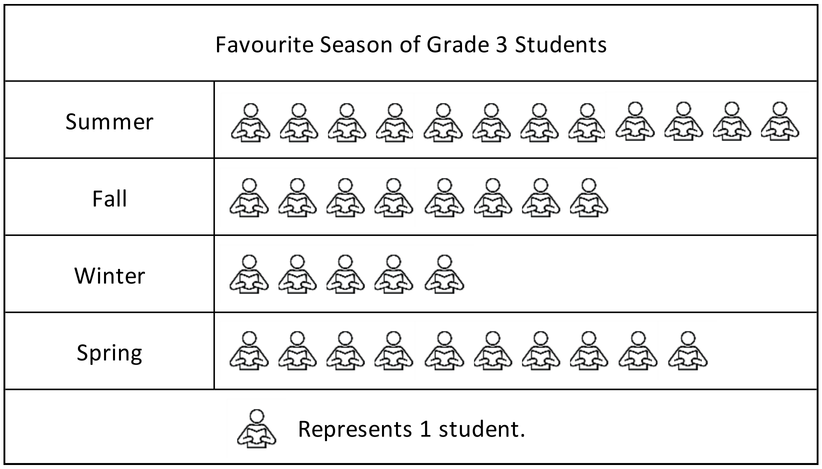
5. The line plot below shows the shoe size of Grade 3 students.



What conclusion can you draw from this line plot?

- ○ There are more students with size 5 shoes, then size 4 shoes.
- ○ There are more students with size 7 shoes, then size 3 shoes.
- ○ There are fewer students with size 6 shoes, then size 7 shoes.
- ○ There are just as many students with size 4 shoes, as students with size 6 shoes.

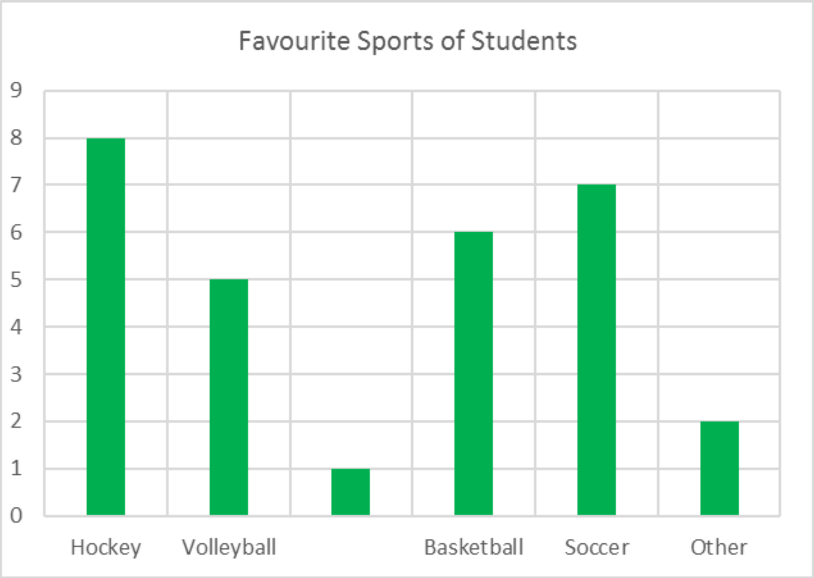
6. Tony asked the Grade 3 students about their favourite season. The pictograph below shows the results of Tony's survey.



Which statement is **true**?

- ○ Spring is the most popular season.
- ○ Fall is the least popular season.
- ○ Fewer students prefer fall to winter.
- ○ Summer is the most popular season.

7. Samir asked her 29 classmates to name their favourite sport. The bar graph below shows the results of Samir’s survey.



What errors did Samir make?

○ The labels of the axes and the title is missing.
○ The labels of the axes and the name of a sport are missing.
○ The bars are not the same width.
○ The total number of students in the graph is not equal to 29.

8. The table below shows the results of a survey of students favourite snacks.

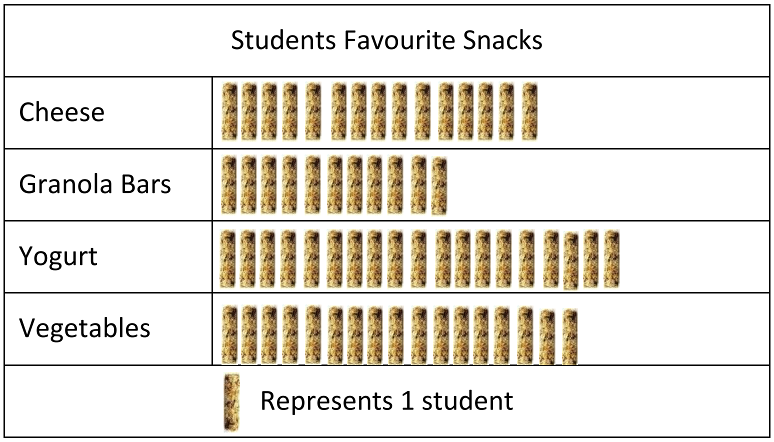
Favourite Snacks of Students

Snacks	**Tally Number**
Cheese	𝍸 𝍸 𝍸
Granola Bars	𝍸 𝍸 |
Yogurt	𝍸 𝍸 𝍸 ||||
Vegetables	𝍸 𝍸 𝍸 ||

How many more students prefer vegetables than cheese?

○ 2
○ 4
○ 6
○ 13

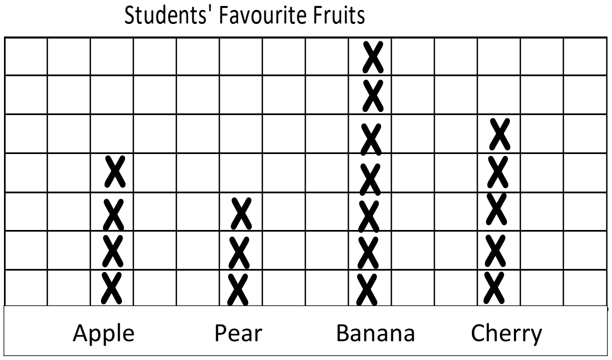
9. Rubina uses the data above to construct the pictograph below to show the results of students favourite snacks.



Which statement is true?

○ There are 30 students who prefer granola bars and yogurt.
○ There are 23 students who prefer cheese and yogurt.
○ There are 27 students who prefer granola bars and vegetables.
○ There are 35 students who prefer yogurt and vegetables.

10. Lee surveyed the Grade 2 students about their favourite fruits. The line plot below shows the results of this survey.



Which statement is **true**?

○ The students prefer apples to bananas.
○ The students prefer pears to cherries.
○ The apple is the most popular fruit.
○ The pear is the least popular fruit.

Mathematics in Grade 3 Lesson Learned 8
Problem Solving

Students need more exposure to application and analysis items in order to apply these higher order thinking skills when problem solving. Students need to be encouraged to understand that problems may have more than one entry point and there are many strategies to solve a problem. Students need to continue to work on translating between and among representations when problem solving. They do not seem to realize that they can use varied representations when asked to solve a word problem. These other representations may support their problem solving and their reasoning.

Learning through problem solving should be the focus of mathematics at all grade levels. Problem solving is one of the critical mathematical processes that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics. When students encounter new situations, and respond to questions such as, How would you …? or How could you …? the problem-solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.

In order for an activity to be problem-solving based, it must appropriately challenge students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but is simply practice. A true problem-solving activity requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement, perseverance, and collaboration. Problem solving is also a powerful teaching tool that fosters multiple, creative, and innovative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident mathematical risk takers.

Students need to be able to explore a wide variety of methods for solving and verifying problems in all areas of mathematics. They must be challenged to find multiple solutions for problems and be given opportunities to create and solve their own problems.

A. What conclusions can be drawn from the Nova Scotia Assessment: Mathematics in Grade 3?

We found that students have a good understanding of basic facts and procedures, and are successful when explicitly given all the information needed to do a knowledge question. But when given application and analysis items, they are not able to apply higher order thinking skills when problem solving. For example, students were not sure whether they should add or subtract when questions were presented in the context of a story problem. Overall, our students are experiencing challenges with problem solving across all mathematical strands. This appeared to be a theme throughout the assessment data when students were asked to “solve a word problem” or “solve a multi-step problem”. The data also showed that, across all mathematics strands, students struggle with items that require translating among representations in problem-solving contexts.

B. Do students have any misconceptions or errors in their thinking?

It is important to distinguish between misconceptions and errors. Misconceptions are, in effect, the misunderstandings about mathematical ideas which students entertain, and which usually lead to errors occurring. These are the most serious kind of errors which require urgent action on the part of the teacher. Errors may occur for a variety of underlying reasons, ranging from careless mistakes to errors resulting from misconceptions. - John SanGiovanni – (*Mine The Gap For Mathematical Understanding Grades K-2*)

It is also important to mention that when students encounter a problem-solving situation, they become confused by mathematical expressions which are not familiar to them or which are difficult to understand. In addition, when students consider that a problem is a mathematical problem, they believe, wrongly, that they should associate it simply to routine calculations with few concerns to the meaning of the context and to the credibility of the answers.

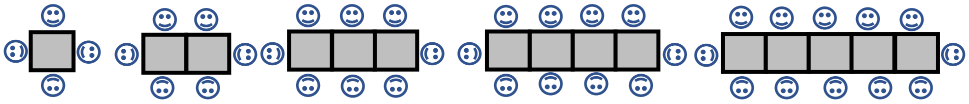
Many students, when given a problem-solving task, especially a “word problem” (solving a problem in a context) have the misconception that is it always too hard for them to attempt. Putting words around the numbers seems to obstruct their ability to think about the question. Another misconception students have is that there is only one way to solve a word problem.

When asked to solve a problem in a context, they struggle to identify a possible strategy and often fail to even attempt to solve the problem. Students appeared to have a limited repertoire of problem-solving strategies to help them attempt a problem. For example, when given an analysis problem in the context of an increasing pattern, students struggle with answering “what strategy could I use to solve this problem”? There are many strategies and ways to solve the problem. Some students find it difficult to find an entry point to begin to solve the problem.

Students also tend to forget that they may translate between and among representations to help them solve a problem. If asked to solve a word problem using words, symbols or pictures, most students only provide symbols. They do not seem to realize that they can use all representations when asked to solve a word problem. These other representations may support their problem solving.

Problem: Some friends are coming to your birthday party. A square table with 4 chairs can seat 4 of your friends. If 2 square tables are put together, you can seat 6 of your friends. How many friends can you seat with 5 square tables put together?

Solution 1: Make a drawing (pictorial representation)

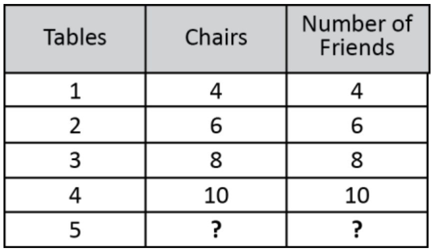


- I start with a drawing.
- There are 4 people seated at the first table.
- There are 6 people seated at two tables when put together.
- There are 8 people seated at three tables when put together.
- There are 10 people seated at four tables when put together.
- There are 12 people seated at five tables when put together.

Twelve friends can sit at five square tables.

Solution 2: Draw a chart

The table below represents the number of tables in the 1st column, the number of chairs in the 2nd column and the number of friends you can have seated at each table combination in the 3rd column.



Tables	Chairs	Number of Friends
1	4	4
2	6	6
3	8	8
4	10	10
5	**?**	**?**

12 friends can be seated at 5 square tables when they come to my birthday

- I see that each time I add a table, I must add 2 chairs. So, at 5 tables, there are 12 chairs, so 12 friends.
- 12 friends can be seated at 5 square tables when they come to my birthday party

Students could have used concrete materials/manipulatives such as coloured tiles, two-sided counters, cube-a-link blocks, and pictures or numbers to solve this question. Showing how the problem could be represented more abstractly with tiles would help make the connection to increasing patterns.

The student results of the **2018–2019 Nova Scotia Assessment Mathematics in Grade 3** shows the following:

- 53% of the students had difficulty using place-value strategies, to represent a number written in words symbolically
- 54% of the students had difficulty estimating the cost of objects in a context to the nearest hundred ($)

C. What are the next steps in instruction for the class and for individual students?

A significant part of learning to solve problems is learning about the problem-solving process. It is generally accepted that the problem-solving process consists of four steps – understand the problem, devise a plan, carry out the plan, and look back to determine the reasonableness of an answer. Teachers need to teach their lessons through a problem-solving approach. Students learn mathematics as a result of solving problems. It is important to point out that not all lessons students encounter must be taught through problem solving. If the purpose of the lesson being taught is to develop a certain skill for conceptual understanding, then some practice is required.

“The teacher provides a context or reason for the learning by beginning the lesson with a problem to be solved. This approach contrasts with the more traditional approach of, for example, presenting a new procedure and then adding a couple of word problems at the end for students to solve. Instead, the teacher gives students the opportunity to think about the problem and work through the solution in a variety of ways, and only then draws the procedures out of their work.” (Small, 2005, p. 154)

An important aspect of problem solving in grades 1–3 is addition and subtraction problems which can be categorized based on the kinds of relationships they represent. It is important that all the story problem structures are presented and developed from students’ experiences. Initial work with the story structures will focus on join and separate types of problems because students associate the actions in these problems with the operations. However, they must also experience addition and subtraction in part-part-whole and comparison situations.

Manipulatives can and should be used to model the strategies and the story structures. Examples of manipulatives that can be used for this purpose include

- two-sided counters
- linking cubes
- number cubes
- ten-frames
- walk-on number line
- base-ten blocks
- Rekenrek

A Problem-Solving Approach

A problem-solving approach is embedded and expected throughout our curriculum, grades Primary to 12, in all strands.

As noted in Pearson (2009a), problem-solving is a key strategy: Problem-solving is a key instructional strategy that enables students to take risks, secure in the knowledge that their thoughts, queries, and ideas are valued. As students share their solutions and findings, the teacher can provide direct instruction on problem-solving strategies. After students share their solutions and justifications, teachers can elaborate on their methods and encourage students to comment or ask questions of their peers. Using student findings and solution methods to guide instruction also allows students to see the value in their work, and encourages peers to share their strategies. While some strategies may be more efficient than others, several strategies may work and often a combination of strategies is required to solve a problem. Students must use strategies that are meaningful to them and make sense to them. (p. 13)

Problem-Solving Strategies

Students are already drawing on personal strategies for problem solving, in many of the activities they undertake. *Strategies Toolkit* lessons found in the *Pearson Math Makes Sense Series,* allows teachers to expand their students' personal repertoires through explicit instruction on a specific strategy. "When students develop a name for the strategy, they develop a stronger self-awareness of the personal strategies they are starting to use on their own" (Pearson, 2009a, p. 13).

The *Strategies Toolkit* lessons highlight these problem-solving strategies (Pearson, 2009a, p. 13):

- Make a chart or table
- Draw a picture
- Work backward
- Make an organized list
- Use a model
- Solve a simpler problem
- Guess and test
- Use a pattern

Van de Walle and Lovin (2006b), in their resource, *Teaching Student-Centered Mathematics Grades 3–5*, suggest a three-part lesson format for teaching through problem-solving. This same approach is used in our core resource, *Math Makes Sense* (Pearson, 2009b, pp. 13–14):

Before

Before students begin:

- Prepare meaningful problem scenarios for students. These should be sufficiently challenging.
- Ensure the problem is understood by all.
- Explain the expectations for the process and the product.

During

As students work through the problem:

- Let students approach the problem in a way that makes sense to them.
- Listen to the conversations to observe thinking.
- Assess student understanding of her/his solution.
- Provide hints or suggestions if students are on the wrong path.
- Encourage students to test their ideas.
- Ask questions to stimulate ideas.

After

After students, have solved their problem:

- Gather for a group meeting to reflect and share.
- Make the mathematics explicit through discussion.
- Highlight the variety of answers and methods.
- Encourage students to justify their solutions.
- Encourage students to comment positively or ask questions regarding their peer's solutions.

For more details on using a problem-solving approach to teach mathematics, see Van de Walle and Lovin (2006b), *Teaching Student-Centered Mathematics Grades 3–5,* from which these ideas are drawn (pp. 11–28).

Assessing Problem Solving

For information using a rubric to score problem-solving sample questions in either Mathematics in Grade 3 (M3) or Mathematics in Grade 6 (M6), please go to the Program of Learning Assessment for Nova Scotia (PLANS) website. On the Documents tab of each assessment page, you will find a Problem-Solving document which includes the provincial rubric and sample questions (plans.ednet.ns.ca/grade3/documents).

From Reading Strategies to Mathematics Strategies

With a problem-solving approach embedded and expected throughout our curriculum grades Primary to 12 in all strands, there are definite implications for teaching reading strategies in mathematics. Concepts and skills should also be connected to everyday situations and other curricular areas. For example, when developing literacy skills students learn to make text-to-world, text-to-text, and text-to-self connections. Students can also make connections to make mathematics come alive through math-to-world, math-to-math, and math-to-self connections.

There are classroom strategies with suggestions of how to use the strategy in a mathematical context that support teachers as they develop students' mathematical vocabulary, initiate effective ways to navigate informational text, and encourage students to reflect on what they have learned. For example, the Frayer Model, Concept Circles, Three-Read Strategy, Exit Cards, etc. These are only a few of the strategies that are found in Appendix B at the end of this document.

When teachers use these strategies in the instructional process or embed them in assessment tasks, the expectations for students must be made explicit. The students' understanding of the mathematics involved and maintaining the integrity of the curriculum are still the foremost concerns.

Please refer to the Appendix B found at the end of this document for strategies with illustrative examples.

D. What are the most appropriate methods and activities for assessing student learning?

Below are some sample questions related to problem-solving which will be used to represent some of the appropriate methods and activities for assessing student learning.

Encourage students to share their thinking, their strategies, and their solutions. Rich discussion fosters the development of alternative problem-solving strategies.

Examples:

For more information related to the sample questions, please refer to Appendix E.

1. Natasha has 4 T-shirts and 2 pairs of pants.
How many different outfits can Natasha make?

Show how you solved the problem and explain your strategy.
***Possible Strategy*:** Make a chart, table or draw a picture; Strand: Number/Stats and Probability; Application Question

2. Megan and Danielle each ordered the same size pizza.
Megan asked to have her pizza cut into fourths. Danielle asked to have her pizza cut into sixths.
Who has the larger pieces of pizza?

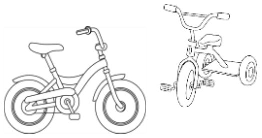
Show how you solved the problem and explain your strategy.
***Possible Strategy*:** Use a model, draw a picture; Strand: Number; Application Question

3. Peter has 123 marbles. He gives some marbles to his friend Paul. Now Peter has 86 marbles.
How many marbles did Peter give to Paul?

Show how you solved the problem and explain your strategy.
***Possible Strategy*:** Use a model; Strand: Number; Application Question

4. Sebastian and his sister have bicycles and tricycles.
The bicycles and tricycles have 21 wheels altogether.
If they have 3 tricycles, then how many bicycles do they have?
Show how you solved the problem and explain your strategy.

Show how you solved the problem and explain your strategy.
***Possible Strategy*:** *Work backward*; Strand: Number; Analysis Question



5. Susan has 23 apples.
She eats three apples every day.
How many apples did Susan have left after seven days?

Show how you solved the problem and explain your strategy.
***Possible Strategy*:** Use a pattern, *make a list, use a model*; Strand: Patterns and Relations; Analysis Question

6. Marbles come in packages of 10, 25, and 50.
You need 160 marbles.
Find 5 ways you could buy the marbles.

Show how you solved the problem and explain your strategy.
***Possible Strategy*:** *Make an organized list*; Strand: Number; Application Question

7. For James birthday, his mother wants to cover each table using a 4 m long piece of paper tablecloth. How many tables can she cover using a roll of paper tablecloth which is 23 m long?

   Show how you solved the problem and explain your strategy.
   Possible Strategy: *Make an organized list, work backward*; Strand: Number; Analysis Question

8. Marie's height is 3 cm more than Norman's height.
   Norman's height is 2 cm more than Jessica's height.
   If Marie's height is 126 cm, what is Jessica's height?

   Show how you solved the problem and explain your strategy.
   Possible Strategy: *Draw a picture*, work backwards; Strand: Measurement; Analysis Question

9. Lucy has some money. She has some 5 cent coins, some 10 cent coins and 25 cent coins.
   She buys a used book for 45 cents.
   She used all her money to buy this book and now has no money left.
   How many different ways could Lucy pay for this book using all of her money?

   Show how you solved the problem and explain your strategy.
   Possible Strategy: *Draw a picture, draw a chart*; Strand: Number; Analysis Question

Appendix E: Problem-Solving Scenarios

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Appendix A: Cognitive Levels of Questioning

Cognitive Levels of Questioning

Knowledge

Knowledge questions may require students to recall or recognize information, names, definitions, or steps in a procedure.

Key verbs: *identify, calculate, recall, recognize, find, evaluate, use and measure*

- Knowledge items focus on recall and recognition.
- Typically, the items specify what the student should do.
- The student must perform a procedure that can be performed mechanically.
- The student does not need to apply an original method to find the solution

Knowledge questions, items, and/or tasks:

- rely heavily on recall and recognition of facts, terms, concepts, or properties
- recognize an equivalent representation within the same form, for example, from symbolic to symbolic
- perform a specified procedure; for example, calculate a sum, difference, product, or quotient
- evaluate an expression in an equation or formula for a given variable
- draw or measure simple geometric figures
- read information from a graph, table, or figure

Application

Application questions may require students to make connections, represent a situation in more than one way (translating between representations), or solve contextual problems.

Key verbs: *sort, organize, estimate, interpret, compare and explain*

- Items are more flexible in terms of mathematical thinking and response choice.
- Questions require more than the usual response.
- Resolution method is not indicated.
- Students should make their own decisions about what to use as informal methods or problem-solving strategies.
- Students need to know a variety of skills and knowledge from a variety of fields to be able to solve problems

Application questions, items, and/or tasks:

- select and use different representations, depending on situation and purpose
- involve more flexibility of thinking
- solve a word problem
- use reasoning and problem-solving strategies
- may bring together skills and knowledge from various concepts or strands
- make connections between facts, terms, properties, or operations
- represent a situation mathematically in more than one way
- compare figures or statements
- explain and provide justification for steps in a solution process
- translate between representations
- extend a pattern
- use information from a graph, table, or figure to solve a problem
- create a routine problem, given data, and conditions
- interpret a simple argument

Analysis

Analysis questions may require students to go beyond comprehension and application to higher order thinking skills, such as generalizations and non-routine problem-solving.

Key verbs: *analyze, investigate, formulate, explain, describe and prove*

The following illustrates some of the requirements of an “Analysis” item:

- The items are very demanding when it comes to mathematical thinking.
- The items encourage the student to think, plan, analyze, synthesize, judge and think creatively.
- Items require the student to think in an abstract and sophisticated way.

Analysis questions, items, and/or tasks:

- require problem solving, reasoning, planning, analysis, judgment, and creative thought
- thinking in abstract and sophisticated ways
- explain relationships among facts, terms, properties, or operations
- describe how different representations can be used for different purposes
- analyze similarities and differences between procedures and concepts
- generalize a pattern
- solve a novel problem, a multi-step, and/or multiple decision point problem
- solve a problem in more than one way
- justify a solution to a problem and/or assumption made in a mathematical model
- describe, compare, and contrast solution methods
- formulate a mathematical model for a complex situation, such as probability experiments
- provide a mathematical justification and/or analyze or produce a deductive argument

Below are the percentages of knowledge, application, and analysis questions in the Nova Scotia provincial assessments for Mathematics in Grade 3:

• Knowledge	20–30%
• Application	50–60%
• Analysis	10–20%

These percentages are also recommended for classroom-based assessments.

Appendix B: From Reading Strategies to Mathematics Strategies

The following table illustrates when strategies are to be used, and during what part of the three-part lesson format (as referenced in Lessons Learned 1).

Name of Strategy	Before	During	After	Assessment
1. Concept Circles	X	X	X	X
2. Frayer Model	X	X	X	X
3. Concept Definition Map	X	X	X	X
4. Word Wall	X	X	X	
5. Three-Read		X	X	
6. Graphic Organizer	X	X	X	X
7. K-W-L	X		X	X
8. Think-Pair-Share	X	X	X	X
9. Think-Aloud	X	X		X
10. Academic Journal-Mathematics	X	X	X	X
11. Exit Cards			X	X

1. **Concept Circle**

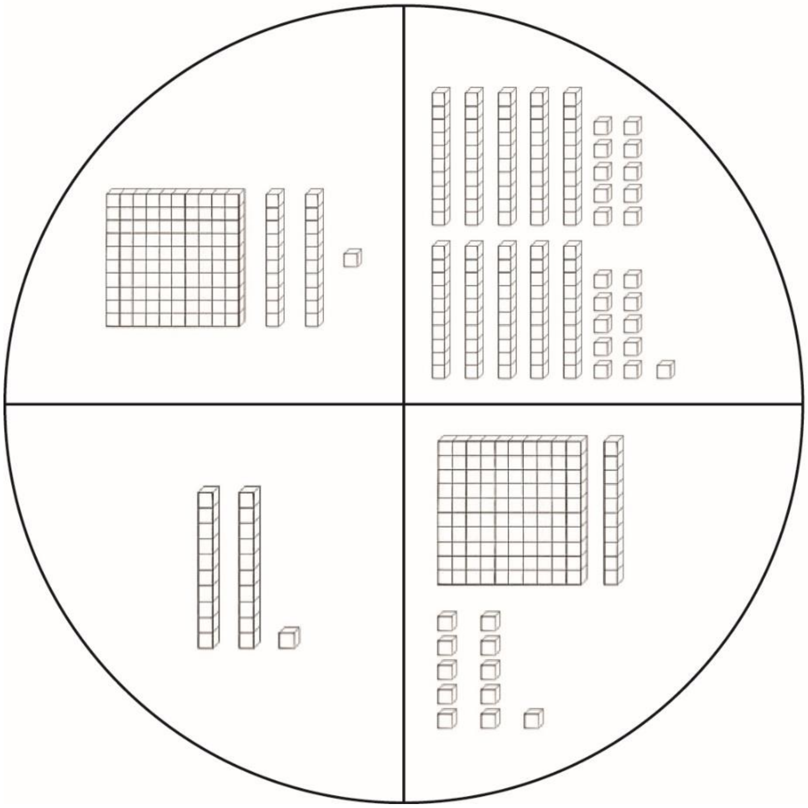
   A concept circle is a way for students to conceptually relate words, terms, expressions, etc. As a “before” activity, it allows students to predict or discover relationships. As a “during” or “after” activity, students can determine the missing concept or attribute or identify an attribute that does not belong.

   The following steps illustrate how the organizer can be used:
   - Draw a circle with the number of sections needed.
   - Choose the common attributes and place them in the sections of the circle.
   - Have students identify the common concepts to the attributes.

   This activity can be approached in other ways.
   - Supply the concept and some of the attributes and have students apply the missing attributes.
   - Insert an attribute that is not an example of the concept and have students find the one that does not belong and justify their reasoning.

Example of a Concept Circle

Concept: Which pictures of base-ten blocks represent 121?

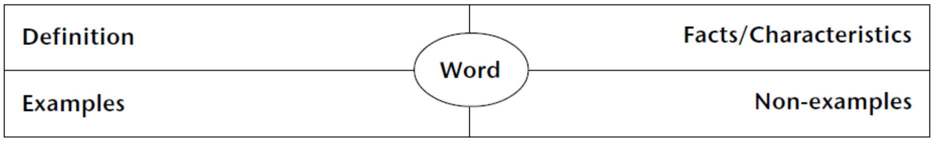


2. Frayer Model

The Frayer Model is a graphic organizer used to categorize a word and build vocabulary. It prompts students to think about and describe the meaning of a word by

- giving a definition
- describing main characteristics
- providing examples and non-examples of the word or concept

It is especially helpful to use with a concept that might be confusing because of its close connections to another concept.

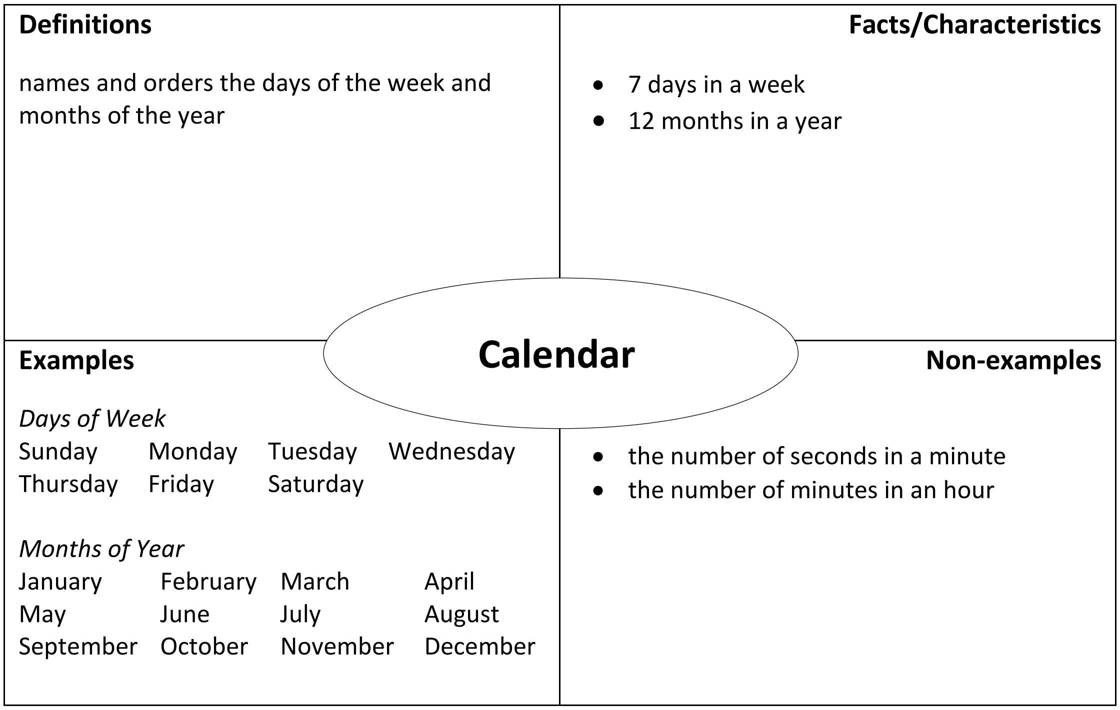


The following steps illustrate how the organizer can be used.

1. Display the template for the Frayer model and discuss the various headings and what is being sought.
2. Model how to use this example by using a common word or concept. Give students explicit instructions on the quality of work that is expected.
3. Establish the groupings (e.g. pairs) to be used and assign the concept(s) or word(s).
4. Have students share their work with the entire class.

This is an excellent activity to do in poster form to display in class. Each group might do the same word or concept, or different words or concepts could be assigned.

Example of a Frayer Model



Definitions

names and orders the days of the week and months of the year

Facts/Characteristics

- 7 days in a week
- 12 months in a year

Calendar

Examples

Days of Week

Sunday	Monday	Tuesday	Wednesday
Thursday	Friday	Saturday	

Months of Year

January	February	March	April
May	June	July	August
September	October	November	December

Non-examples

- the number of seconds in a minute
- the number of minutes in an hour

3. Concept Definition Map

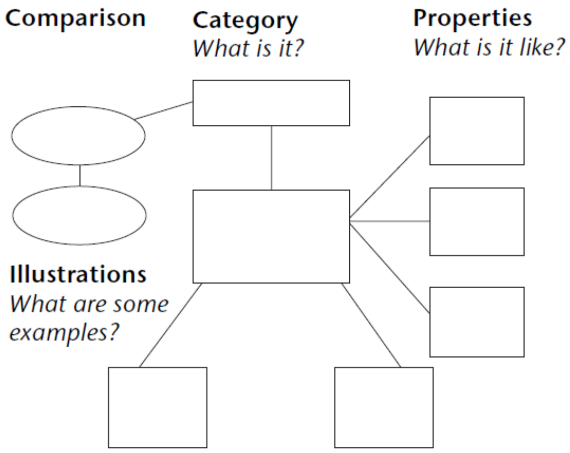
The purpose of a concept definition map is to prompt students to identify the main components of a concept, show the interrelatedness, and build vocabulary. Information is placed into logical categories, allowing students to identify properties, characteristics, and examples of the concept.

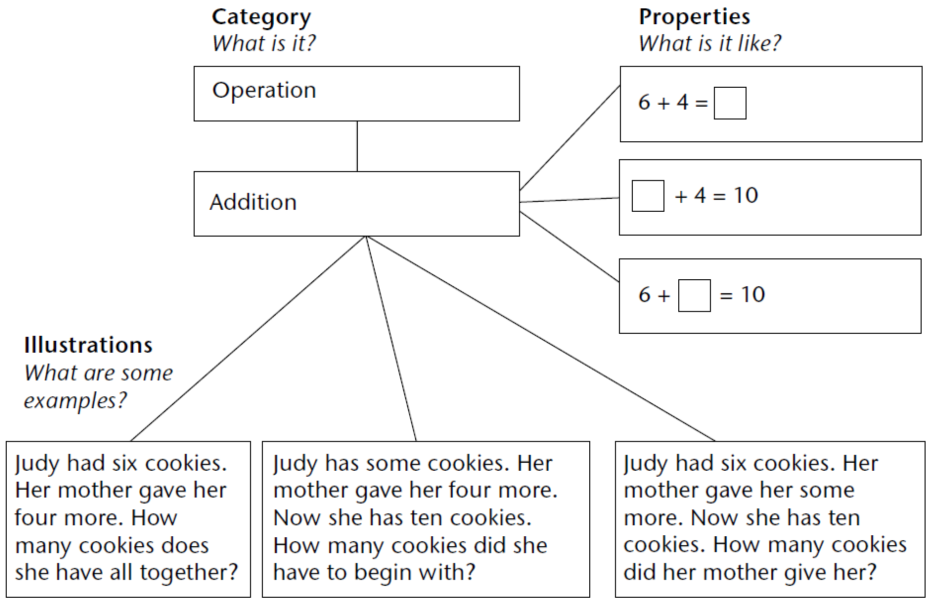
The following steps illustrate how this organizer can be used.

1. Display the template for the concept definition map.
2. Discuss the different headings, what is being sought, and the quality of work that is expected.
3. Model how to use this map by using a common concept.
4. Establish the concept(s) to be developed.
5. Establish the groupings (e.g. pairs) and materials to be used to complete the task.
6. Complete the activity by having the students write a complete definition of the concept.

Encourage students to refine their map as more information becomes available.

Example of a Concept Definition Map





Judy had six cookies. Her mother gave her four more. How many cookies does she have all together?

Judy has some cookies. Her mother gave her four more. Now she has ten cookies. How many cookies did she have to begin with?

Judy had six cookies. Her mother gave her some more. Now she has ten cookies. How many cookies did her mother give her?

4. **Word Wall**

   A mathematics word wall is based upon the same principle as a reading word wall, found in many classrooms. It is an organized collection of words that is prominently displayed in the classroom and helps students learn the language of mathematics. A word wall can be dedicated to a concept, big idea, or unit in the mathematics curriculum. Words are printed in bold block letters on cards and then posted on the wall or bulletin region.

   Illustrations placed next to the word on the word wall can add to the students' understanding. Students may also elaborate on the word in their journals by illustrating, showing an example, and using the word in a meaningful sentence or short paragraph. Students can be assigned a word and its illustration to display on the word wall. Room should be left to add more words and diagrams as the unit or term progresses.

   As a new mathematical term is introduced to the class, students can define and categorize the word in their mathematics journal under an appropriate unit of study. Then the word can be added to the mathematics word wall so students may refer to it as needed. Students will be surprised at how many words fall under each category and how many new words they learn to use in mathematics.
   Note: The word wall is developed one word at a time as new terminology is encountered.

   Use the following steps to set up a word wall.
   1. Determine the key words that students need to know or will encounter in the topic or unit.
   2. Print each word in large block letters and add the appropriate illustrations.
   3. Display cards when appropriate.
   4. Regularly review the words as a warm-up or refresher activity.

5. **Three-Read Strategy**

   Using this strategy, found in *Toward a Coherent Mathematics Program: A Study Document for Educators* (Nova Scotia Department of Education 2002), the teacher encourages students to read a problem three times before they attempt to solve it. There are specific purposes for each reading.

   First Read
   The students try to visualize the problem to get an impression of its overall context. They do not need specific details at this stage, only a general idea so they can describe the problem in broad terms.

   Second Read
   The students begin to gather facts about the problem to make a more complete mental image of it. As they listen for more detail, they focus on the information to determine and clarify the question.

   Third Read
   The students check each fact and detail in the problem to verify the accuracy of their mental image and to complete their understanding of the question.

   During the Three-Read strategy, the students discuss the problem, including any information needed to solve it. Reading becomes an active process that involves oral communication among students and teachers; it also involves written communication as teachers encourage students to record information and details from their reading and to represent what they read in other ways with pictures, symbols, or charts. The teacher facilitates the process by posing questions that ask students to justify their reasoning, support their thinking, and clarify their solutions.

To teach the Three-Read Strategy, teachers should exaggerate each step as they model it. When they have students practice the strategy, teachers should ask questions that stimulate the kinds of questions that students should be asking themselves in their internal conversations. In every classroom, an ongoing discussion of this Three-Read strategy must be conducted, and many students will need to be reminded to use this strategy.

6. **Graphic Organizer**

A graphic organizer can be of many forms: web, chart, diagram, etc. Graphic organizers use visual representations as effective tools to do such things as

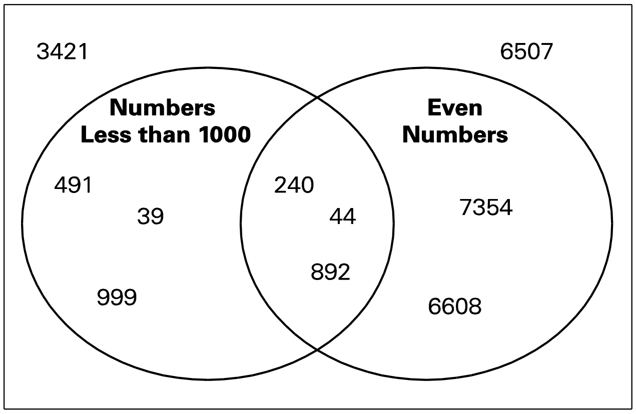
- activate prior knowledge
- analyze
- compare and contrast
- make connections
- organize
- summarize

The following steps illustrate how the organizer can be used.

1. Present a template of the organizer and explain its features.
2. Model how to use the organizer, being explicit about the quality of work that is expected.
3. Present various opportunities for students to use graphic organizers in the classroom.

Students should be encouraged to use graphic organizers on their own as ways of organizing their ideas and work. If the graphic organizer being used is a Venn diagram, it is important to draw a rectangle around Venn diagrams to represent the entire group that is being sorted. This will show the items that do not fit the attributes of the circle(s) outside of them, but within the rectangle. Therefore, elements of the set that do belong to Attribute A or Attribute B are shown within the rectangle but not within the circles in the rectangles.

Example of a Venn diagram



7. K-W-L (Know/Want to Know/Learned)

K-W-L is an instructional strategy that guides students through a text or mathematics word problem and uses a three-column organizer to consolidate the important ideas. Students brainstorm what they know about the topic and record it in the K column. They then record what they want to know in the W column. During and after the reading, students' record what they have learned in the L column.

The K-W-L strategy has several purposes:

- to illustrate a student's prior knowledge of a topic
- to give a purpose to the reading
- to help a student monitor their comprehension

Example: 3-D Objects

Know	Want to Know	Learned
A prism has vertices, faces and edges. *The faces of the prism are rectangles.* *A milk box is a prism.* *A sphere is round.*	*Are there any other 3-D objects besides the prism?*	*A prism is named after its base.* *A prism has two bases.* *Other than the bases, faces of a prism are rectangles.* *There are the pyramids.* *A pyramid has only one base.* *We name a pyramid after its base.* *Other than the base, the faces of a pyramid are triangles.* *There is the cylinder.* *A cylinder has 2 bases which are circles.*

The following steps illustrate how the K-W-L can be used.

1. Present a template of the organizer to students, explain its features, and be explicit about the quality of work that is expected.
2. Ask them to fill out the first two sections (what they know and what they want to know before proceeding).
3. Check the first section for any misconceptions in thinking or weakness in vocabulary.
4. Have the students read the text, and taking notes as they look for answers to the questions they posed.
5. Have students complete the last column to include the answers to their questions and other pertinent information.
6. Discuss this new information with the class, and address any questions that were not answered.

8. **Think-Pair-Share**

   Think-Pair-Share is a learning strategy designed to encourage students to participate in class and keep them on task. It focusses students' thinking on specific topics and provides them with an opportunity to collaborate and have meaningful discussion about mathematics.

   - First, teachers ask students to think individually about a newly introduced topic, concept, or problem. This provides essential time for each student to collect his or her thoughts and focus on his or her thinking.
   - Second, each student pairs with another student, and together the partners discuss each other's ideas and points of view. Students are more willing to participate because they do not feel the peer pressure that is involved when responding in front of the class. Teachers ensure that sufficient time is allowed for each student to voice his or her views and opinions. Students use this time to talk about personal strategies, compare solutions, or test ideas with their partners. This helps students to make sense of the problem in terms of their prior knowledge.
   - Third, each pair of students shares with the other pairs of students in large-group discussion. In this way, each student has the opportunity to listen to all of the ideas and concerns discussed by the other pairs of students. Teachers point out similarities, overlapping ideas, or discrepancies among the pairs of students and facilitate an open discussion to expand upon any key points or arguments they wish to pursue.

9. **Think-Aloud**

   Think-aloud is a self-analysis strategy that allows students to gain insight into the thinking process of a skilled reader as he or she works through a piece of text. Thoughts are verbalized, and meaning is constructed around vocabulary and comprehension. It is a useful tool for such things as brainstorming, exploring text features, and constructing meaning when solving problems. When used in mathematics, it can reveal to teachers the strategies that are part of a student's experience and those that are not. This is helpful in identifying where a student's understanding may break down and may need additional support.

   The think-aloud process will encourage students to use the following strategies as they approach a piece of text.

   - Connect new information to prior knowledge.
   - Develop a mental image.
   - Make predictions and analogies.
   - Self-question.
   - Revise and fix up as comprehension increases.

   The following steps illustrate how to use the think-aloud strategy.

   1. Explain that reading in mathematics is important and requires students to be thinking and trying to make sense of what they are reading.
   2. Identify a comprehension problem or piece of text that may be challenging to students; then read it aloud and have students read it quietly.
   3. While reading, model the process verbalizing what you are thinking, what questions you have, and how you would approach a problem.
   4. Then model this process a second time, but have a student read the problem and do the verbalizing.
   5. Once students are comfortable with this process, a student should take a leadership role.

10. **Academic Mathematics Journal**

An academic journal in mathematics is an excellent way for students to keep personal work and other materials that they have identified as being important for their personal achievement in mathematics. The types of materials that students would put in their journals would include:

- strategic lessons – lessons that they would identify as being pivotal as they attempt to understand mathematics
- examples of problem-solving strategies
- important vocabulary

Teachers are encouraged to allow students to use these journals as a form of assessment. This will emphasize to the student that the material that is to be placed in his or her journal has a purpose. Mark these journals only based on how students are using them and whether or not they have appropriate entries.

The goal of writing in mathematics is to provide students with opportunities to explain their thinking about mathematical ideas and then to re-examine their thoughts by reviewing their writing. Writing will enhance students' understanding of math as they learn to articulate their thought processes in solving math problems and learning mathematics concepts.

11. **Exit Card**

Exit cards are quick tools for teachers to become better aware of a students' mathematics understanding. They are written student responses to questions that have been posed in class or solutions to problem-solving situations. They can be used at the end of a day, week, lesson, or unit. An index card is given to each student (with a question that promotes understanding on it), and the student must complete the assignment before they can "exit" the classroom. The time limit should not exceed 5 to 10 minutes, and the student drops the card into some sort of container on the way out. The teacher now has a quick assessment of a concept that will help in planning instruction.

Appendix C: Cognitive Levels of Sample Questions

Translating Between and Among Representations		Representing and Partitioning Whole Numbers		Whole Number Operations		Patterns and Relations	
Question	**What type of question?**	**Question**	**What type of question?**	**Question**	**What type of question?**	**Question**	**What type of question?**
1	Application	1	Application	1	Application	1	Application
2	Application	2	Application	2	Application	2	Application
3	Analysis	3	Application	3	Application	3	Application
4	Analysis	4	Application	4	Application	4	Analysis
5	Application	5	Analysis	5	Application	5	Analysis
6	Application	6	Application	6	Application	6	Application
		7	Application	7	Application	7	Analysis
		8	Application	8	Application	8	Analysis
		9	Application	9	Application	9	Application
		10	Application	10	Application	10	Application
				11	Application		
				12	Application		
				13	Application		
				14	Application		

Measurement		Geometry		Statistics and Probability		Problem Solving	
Question	**What type of question?**	**Question**	**What type of question?**	**Question**	**What type of question?**	**Question**	**What type of question?**
1	Knowledge	1	Application	1	Knowledge	1	Application
2	Application	2	Knowledge	2	Application	2	Application
3	Knowledge	3	Knowledge	3	Application	3	Application
4	Knowledge	4	Application	4	Analysis	4	Application
5	Knowledge	5	Application	5	Application	5	Analysis
6	Knowledge	6	Analysis	6	Application	6	Analysis
7	Application	7	Knowledge	7	Analysis	7	Analysis
8	Application	8	Analysis	8	Analysis	8	Analysis
9	Analysis	9	Analysis	9	Application	9	Analysis
Examples Page 37		10	Knowledge	10	Application		
1	Application						
2	Application						
3	Analysis						

Appendix D: Answers to the Sample Questions

Translating Between and Among Representations		Representing and Partitioning Whole Numbers		Whole Number Operations		Patterns and Relations	
Question	**Answer**	**Question**	**Answer**	**Question**	**Answer**	**Question**	**Answer**
1	D	1	452	1	C	1	B
2	A	2	266	2	D	2	D
3	C	3	503	3	B	3	C
4	30 + 7 ≠ 30 + 5	4	342	4	A	4	A
5	15 cubes	5	Answers will vary	5	A	5	B
6	a) 17	6	A	6	B	6	D
	b) 11 cubes + 6 cubes (many combinations that add up to 17)	7	C	7	B	7	A
	c) 12 cubes + 5 cubes (many combinations that add up to 17)	8	B	8	C	8	D
		9	Answers will vary	9	C	9	D
		10	263, 373, 487 and 597 written in words	10	C	10	A
				11	C		
				12	A		
				13	D		
				14	C		

Measurement		Geometry		Statistics and Probability		Problem Solving	
Question	**Answer**	**Question**	**Answer**	**Question**	**Answer**	**Question**	**Answer**
1	C	1	D	1	C	1	8 outfits
2	B	2	B	2	A	2	Megan
3	A	3	B	3	B	3	37 marbles
4	C	4	A	4	C	4	6 bicycles
5	D	5	D	5	D	5	2 apples
6	B	6	D	6	D	6	10 combinations
7	A	7	D	7	B	7	5 tables
8	B	8	C	8	D	8	121 cm
9	A	9	B	9	A	9	8 combinations
Examples Page 37			C	10	D		
1	D						
2	B						
3	C						

Appendix E: Problem-Solving Sample Question Strategies (Pages 60-61)

1. Natasha has 4 T-shirts and 2 pairs of pants.
How many different outfits can Natasha make?

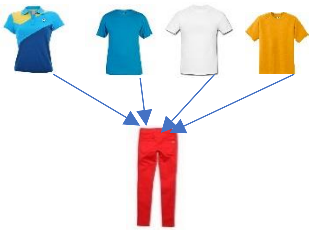
Show how you solved the problem and explain your strategy.

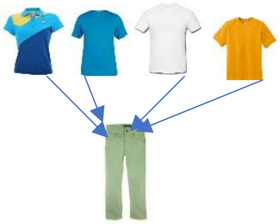
Make a chart

	Jeans (J1)	Jeans (J2)
T-shirt (T1)	T1J1	T1J2
T-Shirt (T2)	T2J1	T2J2
T-shirt (T3)	T3J1	T3J2
T-shirt (T4)	T4J1	T4J2

Natasha can make eight outfits.

Draw a picture





Natasha can make eight outfits.

2. Megan and Danielle each ordered the same size pizza.
Megan asked to have her pizza cut into fourths. Danielle asked to have her pizza cut into sixths.
Who has the larger pieces of pizza?

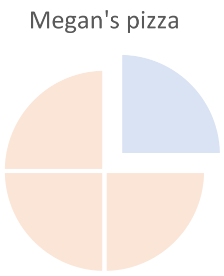
Show how you solved the problem and explain your strategy.

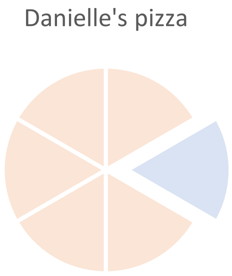
Use a model

The model can be: two fractional circles, one with 4 sections and one with 6 sections
- Cuisenaire rods
- fraction strips

Draw a picture

The drawing shows that Michelle has the larger pieces of pizza.





3. Peter has 123 marbles. He gives some marbles to his friend Paul. Now Peter has 86 marbles. How many marbles did Peter give to Paul?

Show how you solved the problem and explain your strategy.

Use a model

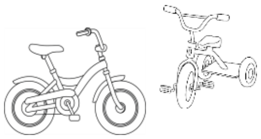
Students can use coloured tokens, tiles or other concrete material to represent this subtraction situation. Students can use an equation such as:

123 – ☐ = 86, where ☐ represents the number of marbles that Peter gave Paul.

Draw a picture

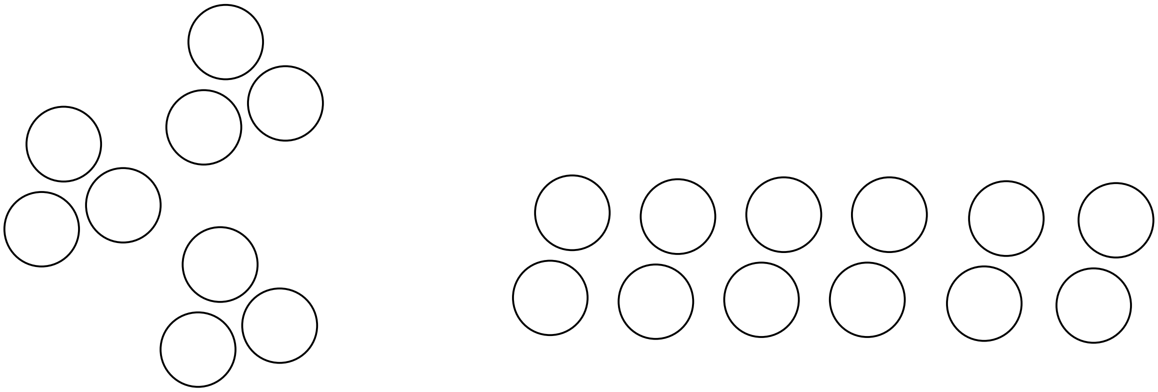
Students can draw a set of 123 small circles and then separate a set of 86 small circles by drawing arrows to show the action of separating and count the number of circles that remain in the starting set.

4. Sebastian and his sister have bicycles and tricycles.
The bicycles and tricycles have 21 wheels altogether.
If they have 3 tricycles, then how many bicycles do they have?
Show how you solved the problem and explain your strategy.



Use a model

Students can use coloured tokens, tiles or other concrete materials to represent this situation.



The 3 tricycles have 9 wheels. There are 6 bicycles with 12 wheels.

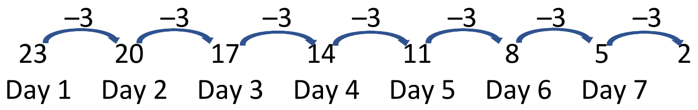
Draw a picture

There are 3 tricycles and 6 bicycles.

5. Monette has 23 apples.
She eats three apples every day.
How many apples will Monette have left after seven days?

Show your reasoning and explain your strategy.

Make a pattern



The number of apples decreases according to a pattern whose rule is: Start at 23, subtract 3 each day. After 7 days, there are 2 apples left.

Make a chart

Number of Apples Eaten	
Number of Days	Number of Apples
Day 1	23 – 3 = 20
Day 2	20 – 3 = 17
Day 3	17 – 3 = 14
Day 4	14 – 3 = 11
Day 5	11 – 3 = 8
Day 6	8 – 3 = 5
Day 7	5 – 3 = 2 apples remain

After seven days, there are two apples left.

Use a model

Students can use a set of 23 two-coloured counters, 23 coloured tiles, or any other appropriate manipulative.

Day 1: Remove 3 chips from the set of 23 chips. There are 20 chips left.
Day 2: Remove 3 chips from these 20 chips. There are 17 chips left.
Day 3: Remove 3 chips from these 17 chips. There are 14 chips left.
Day 4: Remove 3 chips from these 14 chips. There are 11 chips left.
Day 5: Remove 3 chips from these 11 chips. There are 8 chips left.
Day 6: Remove 3 chips from these 8 chips. There are 5 chips left.
Day 7: Removes 3 chips from these 5 chips. There are 2 chips left.

After seven days, there are two apples left.

6. Marbles come in packages of 10, 25, and 50.
You need 160 marbles.
Find 5 ways you could buy the marbles.

Show how you solved the problem and explain your strategy.

Make a table

The possible combinations are:

Package of 10 marbles	Package of 25 marbles	Package of 50 marbles	Total
16 packages	0 packages	0 packages	16 × 10 = 160 marbles
11 packages	2 packages	0 packages	11 × 10 + 2 × 25 = 160 marbles
6 packages	4 packages	0 packages	6 × 10 + 4 × 25 = 160 marbles
1 package	6 packages	0 packages	1 × 10 + 6 × 25 = 160 marbles
1 package	0 packages	3 packages	1 × 10 + 3 × 50 = 160 marbles
6 packages	0 packages	2 packages	6 × 10 + 2 × 50 = 160 marbles
11 packages	0 packages	1 packages	11 × 10 + 1 × 50 = 160 marbles
6 packages	2 packages	1 package	6 × 10 + 2 × 25 + 1 × 50 = 160 marbles
1 packages	2 packages	2 packages	1 × 10 + 2 × 25 + 2 × 50 = 160 marbles
1 package	4 packages	1 package	1 × 10 + 4 × 25 + 1 × 50 = 160 marbles

There are 10 possible combinations.

Use a model

Students may use coloured tokens, tiles or any other available manipulative.

7. For James birthday, his mother wants to cover each table using paper tablecloth which is 4 m long. How many tables can she cover using a roll of tablecloth which is 23 m long?

Show how you solved the problem and explain your strategy.

Make a table

Number of Tables to be Covered		
Strips of Paper Tablecloth	**Required Length (m)**	**Number of Tables**
1	4	1
2	8	2
3	12	3
4	16	4
5	20	5

The table shows that the remaining 3 meters of the roll of paper tablecloth are not enough to cover a 6th table.

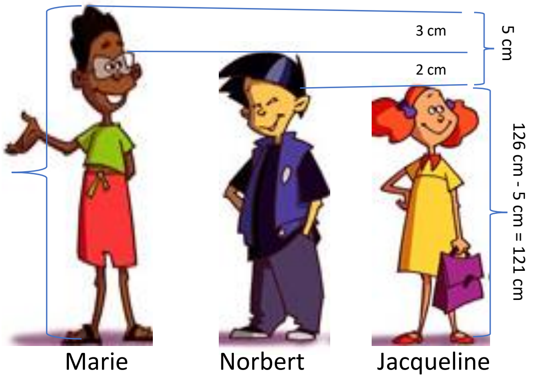
Jacques' mother can cover five tables.

Draw a picture

There are 3 m of paper tablecloth left. The paper tablecloth can cover 5 tables.

8. Marie's height is 3 cm more than Norman's height.
Norman's height is 2 cm more than Jacqueline's height.
If Marie's height is 126 cm, what is Jacqueline's height?

Show how you solved the problem and explain your strategy.
Draw a picture



Jacqueline's height is 121 cm.

Work Backwards
Norman is 2 cm taller than Jacqueline. Marie is 3 cm taller than Norman.
Marie is three inches taller than Jacqueline.
Jacqueline measures 126 cm – 5 cm = 121 cm.

9. Lucy has some money. Lucy has some 5 cent coins, some 10 cent coins and 25 cent coins.
Lucy buys a used book for 45 cents.
Lucy used all of the money to buy this book and now has no money left.
How many different ways could Lucy pay for this book using all of her money?

Show how you solved the problem and explain your strategy.

Make a Table

5 cent coins	10 cent coins	25 cent coins	Possible Combinations
9 coins	0 coins	0 coins	9 × 5 = 45 cents
7 coins	1 coins	0 coins	7 × 5 + 1 × 10 = 45 cents
5 coins	2 coins	0 coins	5 × 5 + 2 × 10 = 45 cents
3 coins	3 coins	0 coins	3 × 5 + 3 × 10 = 45 cents
1 coins	4 coins	0 coins	1 × 5 + 4 × 10 = 45 cents
4 coins	0 coins	1 coins	4 × 5 + 1 × 25 = 45 cents
2 coins	1 coins	1 coins	2 × 5 + 1 × 10 + 1 × 25 = 45 cents
0 coins	2 coins	1 coins	2 × 10 + 1 × 25 = 45 cents

Lucy could have paid for the novel in eight different ways.

Use a model
Note: Students can use play money to determine possible combinations.

Appendix F: Problem-Solving Scenarios

In mathematics, problem solving is seen as a complex process of mathematical modelling. A scripted contextual problem is a research exercise that is a challenge to the individual, one that engages their faculties and comprehension skills. The idea is to present students with real problems, authentic problem scenarios, which they will be motivated to solve. This approach enables them to acquire a methodical approach to exploration aimed at finding a solution that clearly describes their strategy using appropriate terminology.

Example 1: Fall Festival

The exciting evening of the Fall Festival is celebrated on October 31st. Children and their parents will go to the houses in search of the best treats. On the street, where John lives and goes trick or treating, there are 12 houses beautifully decorated with pumpkins, haystacks, giant spiders and tombstones.

When John was trick or treating on the street, John counted the following decorative objects:

- 1 haystack in front of four houses
- 1 pumpkin in front of each house
- 1 giant spider in front of six houses
- 1 bunch of dried wheat in front of the second and the tenth house

1. The houses on the left side of the street are even numbers, starting at 612 and adding 2 each time.

   What is the number of the seventh house on this street?

   ○ 7
   ○ 24
   ○ 622
   ○ 624

Work Space

2. Construct a line plot to represent the number of objects that John counted.

3. The mass of the little pumpkin, placed in front of John's house, was 935 g. After John carved and emptied this pumpkin, its mass became 458 g.
How many grams did John's pumpkin lose after it was carved and emptied?

○ 427 g
○ 477 g
○ 523 g
○ 1 393 g

Solve the problem.
Show all your work.
You may use words, pictures, or symbols.

4. A haystack has the shape of a rectangular prism.

How many vertices, faces and edges does the haystack have?

- ○ 6 vertices, 6 faces and 6 edges
- ○ 6 vertices, 8 faces and 12 edges
- ○ 8 vertices, 6 faces and 8 edges
- ○ 8 vertices, 6 faces and 12 edges

Please Note: Students will use informal geometric language at this stage rather than specific mathematical language. They can say “corners” rather than “vertices” and “sides” rather than “faces for three-dimensional objects”.

Example 1: Fall Festival – Answers and Justifications

The exciting evening of the Fall Festival is celebrated on October 31st. Children and their parents will go to the houses in search of the best treats. On the street, where John lives and goes trick or treating, there are 12 houses beautifully decorated with pumpkins, haystacks, giant spiders and tombstones.

When John was trick or treating on the street, John counted the following decorative objects:

- 1 haystack in front of four houses
- 1 pumpkin in front of each house
- 1 giant spider in front of six houses
- 1 bunch of dried wheat in front of the second and the tenth house

1. The houses on the left side of the street are even numbers, starting at 612 and adding 2 each time.

   What is the number of the seventh house on this street?

   ○ 7
   ○ 24
   ○ 622
   ● 624

Work Space

SCO: 3PR01
Item Description: Extend a pattern to determine the number of a house on a street.

Big Idea: The increasing numerical patterns describe real situations.

Cognitive Level: Application
Difficulty Level: Medium

Attention: A common error made by students is due to a misunderstanding of the difference between the placement and the value of a term of a given numerical pattern.

Selected Answers/Next Steps

The student chose A (7) The student does not understand the difference between the placement and the value of a term of a given pattern. ***The Next Step*** Using simple examples, verify whether the student is familiar with increasing numeric patterns. Provide the student with the following pattern: 12, 14, 16, -----, 18 ... Ask them to indicate the first term, to state the pattern rule, to locate the fourth missing term and its value, to extend this pattern until the twelfth term and to find the value of the seventh term. It is often easier for students to show that they understand an increasing numeric pattern by extending it much more than describing it.	***The student chose B (24)*** The student's work reveals that they can skip count by 2. The error they have made is because they have forgotten the number of hundreds, 6. ***The Next Step*** Ask the student to look closely at the house numbers. Once he notices that the first number (the first term of the numerical pattern) is 612, he could quickly recognize that his mistake is due to forgetting 6.
The student chose C (622) The error made by the student is because they considered the number of the sixth house. ***The Next Step*** Check if the student has correctly extended the given pattern. Ask them to tell you the placement of each term of this pattern. Once, they reach the seventh term, they could discover the cause of the error.	***The student chose D (624)*** *D is the correct answer.* The student's work shows that they have increasing numeric patterns under control the. ***The Next Step*** A short conversation with the student can show you whether they are capable of easily determining the starting point, the number added each time, the placement and value of any term of a given numeric regularity.

2. Construct a line plot to represent the number of objects that John counted.

SCO: 3SP01
Item Description: Construct a line plot to represent the number of decorative objects counted on a street.

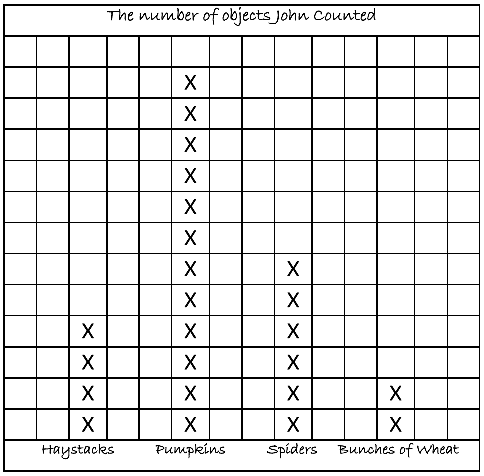
Big Idea: The use of tally marks, lists, charts, line plots, and bar graphs to organize data relevant to everyday life situations.

Cognitive Level: Application
Difficulty Level: Medium

Attention: The students forget to label the titles and may not put enough x's in the vertical line of the line plot.

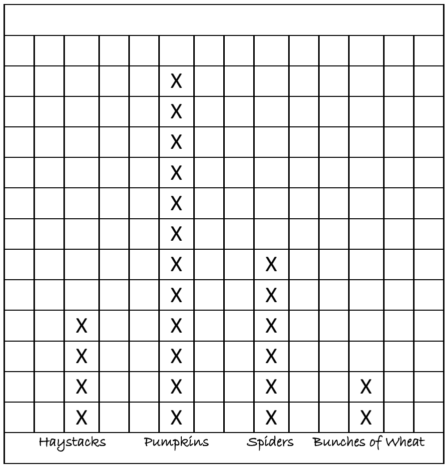
Selected Answers/Next Steps

The correct Answer
The student is aware of the common attributes of a line plot such as the title, the labels, the horizontal axis, and the use of crosses in a vertical line.



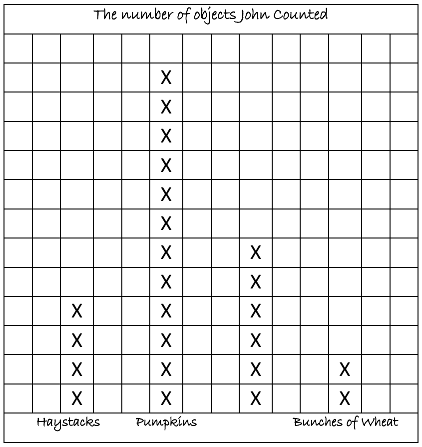
The Next Step
Have a conversation with the student that the common attributes of a line plot can differ.

First possibility of incorrect answer
The title is missing.
It is hard to make sense of a line plot if it does not have a title.



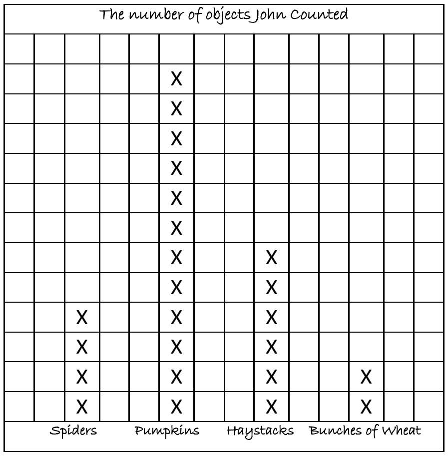
The Next Step
Have a conversation with the student about the common attributes of a line plot (refer to the correct answer).

Second possibility of incorrect answer
At least one label is missing.
It is hard to make sense of a line plot if one or more labels are missing.



The Next Step
Have a conversation with the student about the common attributes of a line plot (refer to the correct answer).

Third possibility of incorrect answer
At least one label is not matching the correct number of the objects counted by John.



The Next Step
Have a conversation with the student about the common attributes of a line plot (refer to the correct answer).

3. The mass of the little pumpkin, placed in front of John's house, was 935 g. After John carved and emptied this pumpkin, its mass became 458 g.
How many grams did John’s pumpkin lose after it was carved and emptied?

○ 427 g
● 477 g
○ 523 g
○ 1393 g

Solve the problem.
Show all your work.
You may use words, pictures, or symbols.

Small pumpkin = 935 g

Empty Pumpkin = 458 g

$$\begin{array}{r} 935\text{ g} \\ -\ 458\text{ g} \\ \hline 477\text{ g} \end{array}$$

or 935 g – 458 g = 477 g

John’s pumpkin after it was carved and emptied now had a mass of 477 g.

OR use a Part-Part-Whole Chart

Part 935 g	**Part** 458 g
Whole ?	

Part 935 g	**Part** 458 g
Whole 477 g	

A Strip Diagram can also be used. It is set up identical to the Part-Part-Whole Chart but without naming the parts.

SCO: 3N09
Item Description: Subtraction of two 3-digit numerals to determine the mass of the pumpkin after it has been carved and emptied.

Big Idea: Subtraction with answers to 1000 by using personal strategies (write an equation, use a part-part whole chart, a strip diagram, etc.) manipulatives (base-ten blocks, Rekenrek counting frame, number lines).

Cognitive Level: Application
Difficulty Level: Medium

Attention: Many students have difficulty working with groups of 10 and groups of 100; therefore, they experience many difficulties with subtraction when regrouping and trading.

Selected Answers/Next Steps

The student chose A (427 g)
The student understood that this was a subtraction story problem, but does not have place value, regrouping and trading of whole numbers under control.

The Next Step
Have a conversation/interview with this student. Ask the student if they can explain to you in their own words what strategy they used to solve the story problem. This student was able to compute the differences involving two 3-digit numbers with errors. Their work shows that this student does not have place value of whole numbers, trading and regrouping of whole numbers under control.

Example: $\begin{array}{r} 935 \\ -\,458 \\ \hline 427 \end{array}$ $\quad\begin{array}{r} \overset{8}{\cancel{9}}3\,15 \\ -\,45\;8 \\ \hline 427 \end{array}$ Or 935 – 458 = 427

The student knew they had to trade with a group of 10 and a group of 100 when performing subtraction on these numbers. As shown in the example, the student recorded an 8 above the 9 and added one ten to the number 5 in the one's column; subtracted 15 minus 8 equals 7. They realized they couldn't subtract a 3 from a 5; they subtracted 5 from 3 and got a 2; subtracted 4 from 8 and get 4. This symbolic recording is organized but mathematically it is not correct.

The student chose B (477 g)
B is the correct answer.
The student was able to solve the subtraction story problem, and subtract two 3-digit numerals efficiently.

The Next Step
This is a separate story problem, result unknown, that has the action of causing a decrease. This student was able to compute the differences involving two 3-digit numbers. They had the ability to use strategies that are reliable, accurate and efficient, and reflect their thinking.
This student's symbolic recording of this computation was to use a Part-Part Whole Chart. Whether they used the standard algorithm or a personal strategy, the symbolic recording must be mathematically correct, organized, and efficient.

Example: 935 – 458 = 477

Part	$\begin{array}{r} 935 \\ -\,400 \\ \hline 535 \end{array}$	**Part**	$\begin{array}{r} 535 \\ -\;\;58 \\ \hline 477 \end{array}$
Whole	477		

This student shows that they have place value of whole numbers, trading and regrouping of whole numbers under control.

The student chose C (523 g)
The student understood that this was a subtraction story problem, but does not have place value, regrouping and trading of whole numbers under control.

The Next Step
Have a conversation/interview with this student. Ask the student to explain their strategy that they used to solve this separate story structure problem.

Example: $\begin{array}{r} 935 \\ -\,458 \\ \hline 523 \end{array}$ $\quad\begin{array}{r} \mathbf{9}35 \\ -\,\mathbf{4}58 \\ \hline \mathbf{5}23 \end{array}$ Or **9**35 – **4**58 = **5**23

This student did not have regrouping and trading under control. They realized they couldn't subtract an 8 from a 5 in the one's column; so, they subtracted 5 from 8 and got a 3; they did the same thing in the ten's column; subtracted 5 from 3 and got 2. The hundreds column was easy, they could subtract **4** from **9** and get a **5**; this symbolic recording is organized but mathematically it is not correct.

The student chose D (1393 g)
The student was not able to solve the subtraction story problem; they thought it was an addition story problem.

The Next Step
Have a conversation/interview with this student. Ask the student if they can explain to you in their own words what strategy they used to solve the story problem. The computation of this story problem was done correctly if it had been an addition story problem. Unfortunately, it was a subtraction question. Because the student had to decide where to place the two given numbers in the strip diagram, they must carefully read the story problem to determine whether each given quantity is a part or a whole.

935	458
? (1393)	

The principal use of strip diagrams is as a strategy to help students interpret story problems.

4. A haystack has the shape of a rectangular prism.

How many vertices, faces and edges does the haystack have?

○ 6 vertices, 6 faces and 6 edges
○ 6 vertices, 8 faces and 12 edges
○ 8 vertices, 6 faces and 8 edges
● 8 vertices, 6 faces and 12 edges

SCO: 3G01
Item Description: Determine the number of vertices, faces and edges of a haystack (3-D Object).

Big Idea: Many of the properties and attributes that apply to 2-D shapes also apply to 3-D objects.

Cognitive Level: Application
Difficulty Level: Medium

Attention: Many students make errors when counting the number of vertices, faces and edges because they lose track of which components have already been counted; or they mix up the vertices and edges or the faces (the base).

Selected Answers/Next Steps

The student chose A (*6 vertices, 6 faces and 6 edges*) The error made by the student is that they may have lost count of the number of vertices, faces, and edges. ***The Next Step*** Have a conversation/interview with this student to discover what error they may have made. Ask the student to show you how they arrived at their answer. If they counted the number of each attribute, the vertices, the faces and the edges, suggest that they mark a starting point for counting and that they count in a systematic way. Perhaps starting from the base (face) up, then mark a starting point. Students will use informal geometric language at this stage rather than precise mathematical language. They may say “corners” rather than “vertices” and “sides” rather than “faces for 3-D objects.	***The student chose B (*6 vertices, 8 faces and 12 edges*)*** The error made by the student is that they may have mixed up the vertices, edges and the faces with each other. ***The Next Step*** Have a conversation/interview with this student to discover what error they may have made. Ask the student to discuss with you the attributes of the 3-D object, and to point to the picture of the haystack where they found vertices, faces and edges. If the student is having difficulty identifying the attributes that the question is asking them to find, then provide sorting and constructing activities for them. As the students become more familiar with identifying the attributes, they can determine the number of faces, edges and vertices. Students will use informal geometric language at this stage rather than precise mathematical language. They may say “corners” rather than “vertices” and “sides” rather than “faces for 3-D objects.
The student chose C (*8 vertices, 6 faces and 8 edges*) The error this student may have made is a combination of loosing the count of the number of vertices, faces and edges. As well as mixing up these attributes up with each other. ***The Next Step*** Have a conversation/interview with this student to discover what error they may have made. Ask the student to show you how they arrived at their answer. If they show you how they counted the number of attributes, suggest to the student that they mark a starting point for counting and they count in a systematic way. If the student is having difficulty identifying the attributes that the question is asking them to find, then provide sorting and constructing activities for them. As the students become more familiar with identifying the attributes, they can determine the number of faces, edges and vertices. Students will use informal geometric language at this stage rather than precise mathematical language.	***The student chose D (*8 vertices, 6 faces and 12 edges*)*** *D is the correct answer.* The student was able to identify the correct number of vertices, faces, and edges. ***The Next Step*** This student appears to have the knowledge to determine the number of vertices, faces and edges of the haystack. To confirm what the student knows and has under control, schedule a conversation/interview with the student. Ask the student to show you how they arrived at their answer. If they show you how they counted the number of attributes of the 3-D object, marking a starting point for counting the vertices, faces and edges, and that they counted in a systematic way, then they have the conceptual knowledge of identifying the attributes of a 3-D object under control. Students will use informal geometric language at this stage rather than precise mathematical language.