Mathematics in Grade 4 Lesson Learned 4 Patterns and Relations

Students were challenged when asked to transfer their visual representation of patterns to numerical patterns. Students should be able to describe either an increasing pattern or a decreasing pattern, and need to recognize that each term has a numeric value. Students seemed to forget that a pattern rule must have a starting point. Without the starting point, the pattern rule is incomplete. They need to be encouraged to provide all the information for a pattern rule. Students need to continue to work with representations of patterns, concretely, contextually, pictorially, symbolically, and verbally.

Patterns are the foundation for many mathematical concepts. Patterns should be taught throughout the year in situations that are meaningful to students. Patterns are explored in all the strands and are also developed through connections with other disciplines, such as science, social studies, English language arts, physical education, and music. Providing students with the opportunity to discover and create patterns, and then describe and extend those patterns, will result in more flexible thinking across strands and across subjects. Students should initially describe non-numerical patterns, such as shape, action, sound, and then incorporate numerical patterns by connecting them to the non-numerical patterns.

A large focus in Mathematics 3 is the introduction and development of decreasing patterns. Students use their knowledge of increasing patterns to make connections to the concept of decreasing patterns, since similar understandings are developed.

A. What conclusions can be drawn from the Nova Scotia Assessment: Mathematics in Grade 4? We noticed that our students did very well recognizing simple errors in increasing number patterns, identifying a pattern rule used to create a given increasing pattern, and identifying the next term in an increasing pictorial pattern. These types of items were either knowledge questions or application questions. Analysis questions related to patterns challenged our students. For example, students experience difficulty when asked to create an increasing pattern in which a specific element is identified (e.g., the 7th element is 56).

Students did extremely well when the patterns that they were working with were a visual representation of a pattern. See below for examples:



"The pattern for my bead is red (R), red (R), blue (B), yellow (Y)"

Students were challenged when they were transferring their knowledge of visual patterns to numerical patterns. Students should be able to describe an increasing pattern made of shapes, but need to recognize that each term in the pattern also has a numeric value. For example,



This expectation also applies to decreasing patterns. For example,



B. Do students have any misconceptions or errors in their thinking?

One of the most fundamental concepts in pattern work, but also one not clear to all students, is that, although the part of the pattern that they see is finite, when mathematicians talk about a pattern, they are talking about something that continues beyond what the student sees. (Small, 2009, p.568)

In Mathematics 2, students describe, reproduce, extend, and create repeating, increasing and, decreasing patterns. They use ordinal numbers (to tenth) to describe elements of repeating patterns. Students in Mathematics 3 explore increasing and decreasing patterns, both numerical patterns with numbers to 1000 and non-numerical patterns with concrete materials, pictures, sounds, and actions. They use ordinal numbers (to 100th) to refer to or to predict terms within a pattern.

Students have difficulty extending an increasing number pattern or a decreasing number pattern. A suggested strategy is to have students locate the numbers in the pattern on a hundred chart and place a transparent counter over each number. Have students use the visual pattern in the counters to extend the pattern. Help students relate the visual pattern to the starting point and the number added each time in the number pattern. (Pearson, 2009b, p. 14)

For example, when skip counting by 3, use only starting points that are multiples of 3 (3, 6, 9, 12, ...). This will result in many diagonal representations on a hundred chart. Skip counting by 5, results in a pattern that is two vertical columns with numbers ending in the digits 5 (5, 15, 25, ...) and 0 (10, 20, 30, ...). Students should also explore hundred charts to 1000 (1–100, 101–200, 201–300, ...) and look for patterns when counting by 2s, 5s, 10s, 25s, and 100s.

Another common error with decreasing patterns is that students do not extend a decreasing pattern correctly. A suggested strategy is to help students identify how each figure in the pattern differs from each previous figure.

For example, in the pattern below, the pattern rule is to start with 10 squares and decrease by 2 squares each time.



As students describe decreasing shape patterns, help them recognize that each term has a numeric value. The above pattern can be expressed as 10, 8, 6, ... by counting the number of squares in each term. Students may also find it useful to record the change from one term to the next as shown below:



Remind students that a pattern rule must have a starting point or the pattern rule is incomplete. For example, if a student describes the pattern 10, 8, 6, ... as a decrease by 2 without indicating that the pattern starts at 10, the pattern rule is incomplete.

The student result of the 2016–2017 Nova Scotia Assessment Mathematics in Grade 4 shows the following data:

- 47% of the students were not able to determine the 23rd element of a given repeating pattern.
- 56% of the students were not able to determine a missing number in a given counting pattern.
- C. What are the next steps in instruction for the class and for individual students? As students identify the core of a pattern, they should use appropriate patterning vocabulary, such as core (the repeating part of the pattern) and elements (the actual objects used in the pattern). It is important to create patterns that have the core repeated at least three times. To help students identify the pattern core, it is suggested students highlight, or isolate, the core each time it repeats. Remind students that repeating patterns can be extended in both directions. Encourage students to reference the position of the elements of the pattern using ordinal numbers. The core of the shape pattern below is: circle, square, triangle. There are three elements in this pattern, namely a circle, a square, and a triangle.



The pattern above is also a three-element pattern. The core of this three-element pattern is circle (1st element), square (2nd element), and triangle (3rd element).

Increasing Numeric Patterns

Students should be able to describe an increasing pattern. An increasing pattern is a growing pattern where the size of the term increases in a predictable way. The terms in an increasing pattern grow by either a constant amount or by an increasing amount each time. Students need sufficient time to explore increasing patterns using various manipulatives, such as cube-a-links, tiles, toothpicks, counters, pattern blocks, base-ten blocks, ten-frames, bread tags, stickers, or buttons, to realize they increase in a predictable way. As students describe increasing shape patterns, help them recognize that each term has a numeric value. For example,



A counting sequence is an increasing pattern where each number represents a term in the pattern. For example, in the counting sequence 1, 2, 3, 4, ..., 1 represents the first term, 2 the second term, 3 the third term ... This counting sequence can be connected to ordinal numbers where students should be able to recognize that the 34th term is 34 and that 57 is the 57th term in the sequence. These ordinal number patterns should be investigated for numbers up to 100.

Students should be able to describe a given increasing pattern by stating the pattern rule. A pattern rule tells how to make the pattern and can be used to extend an increasing pattern. Give students the first three or four terms of an increasing pattern. Ask them to state the pattern rule by identifying the term that represents the starting point and describing how the pattern continues. For example, in the pattern below, the pattern rule is, start with 2 counters and add 3 counters each time.



As students describe concrete or pictorial patterns, help them recognize that each term has a numeric value. For example, the above pattern can be expressed as, 2, 5, 8, 11, ... by counting the number of counters in each term. Students may also find it useful to record the change from one number to the next as shown below.



Students should be able to extend a pattern by identifying the rule, and then use the rule to build and draw the next three terms. Initially, students should replicate the first three terms with concrete materials and then extend the pattern. The use of the concrete materials allows them to make changes if necessary and to build onto one term to make the next term. Students should be able to explain why their extension follows the

pattern. It is important to note that for some patterns, there may be more than one way to extend the pattern. For example, if only one term is given, such as the third term 12, some possible solutions could be

4, 8, 12, 16, ... 3, 7, 12, 18, ... 2, 6, 12, 20, ... 6, 9, 12, 15, ...

Students need opportunities to compare numeric patterns, discussing how they are the same and how they are different. When comparing increasing patterns, compare the starting points and how each term increases. For example, one way students may address this is by using a page with four small hundred charts. Ask them to skip count starting a 0 and shade one chart by 2s, one chart by 5s, one chart by 10s, and one chart by 25s. Then, discuss the pattern rule in each chart comparing the starting points and the amount of increases.

Students should be able to create various representations of an increasing pattern that follow a given pattern rule. Initially students will create patterns with concrete materials, then pictures, and then numbers. Some students may create a simple pattern, while other students may create a more complex pattern.

Students should be able to create increasing patterns, concretely, pictorially, and symbolically, and be able to describe the pattern rule they used to create their representations. When creating increasing patterns, initially students need to choose a starting point and then decide on the amount of increase. The amount of increase may be either a constant amount or an increasing amount. Students should be able to describe their pattern by clearly explaining how it changes from one term to the next. Students may share their patterns and the strategies they used to create their pattern.

Students should have frequent experiences using increasing patterns to solve real-world problems that interest and challenge them. They should use concrete materials or pictures to model the problem before determining the pattern rule. A variety of strategies may be used to solve the problem such as using a number line, a hundred chart, a picture, concrete materials, or skip counting.

Students should be able to identify and describe the strategy used to determine a missing term in a given increasing pattern. Since patterns increase in a predictable way, to determine a missing term, students should first look at the term that comes before and after. One strategy may be to identify and use the pattern rule. Other possible strategies may include using a number line, a hundred chart, a picture, concrete materials, or skip counting.

Decreasing Numeric Patterns

Students should be able to describe a decreasing pattern. A decreasing pattern is a shrinking pattern that decreases by a constant amount each time. Students need sufficient time to explore decreasing patterns using various manipulatives, such as cube-a -links, tiles, toothpicks, counters, pattern blocks, base-ten blocks, tenframes, bread tags, stickers, and buttons. Sometimes students are more comfortable during the exploration stage if they can experiment first, using manipulatives, then pictures, and eventually numbers.

Students should be able to identify and describe various decreasing patterns such as horizontal, vertical, and diagonal patterns found on a hundred chart. Working with decreasing patterns can be connected to skip counting in outcome N01. Provide copies of hundred charts. Ask students to begin at 100 and skip count backward by a given number, shading in the number for each count all the way to 1. Then they write a description of the pattern. For example, if they chose to skip count by 10s, the pattern results in one vertical column, regardless of the starting point.

As students begin to investigate patterns, they sometimes confuse repeating patterns with decreasing patterns. Remind them to look for a core first. If they cannot find a core, then the pattern is not a repeating pattern.

Earlier, students became familiar with assigning a numeric value to each element in an increasing pattern. This expectation also applies to decreasing patterns.



Students should be able to describe a given decreasing pattern by stating the pattern rule. A pattern rule includes a term representing a starting point and a description of how the pattern continues. A pattern rule tells how to make the pattern and can be used to extend a pattern. For example, in the pattern above, the pattern rule is to start with 12 squares and decrease by 4 squares each time.

As students describe decreasing shape patterns, help them recognize that each term has a numeric value. The above pattern can be expressed as 12, 8, 4, ... by counting the number of circles in each term. Students may also find it useful to record the change from one term to the next as shown below.



Remind students that a pattern rule must have a starting point or the pattern rule is incomplete. For example, if a student describes the pattern 12, 8, 4, ... as a decrease by 4 pattern without indicating that it starts at 12, the pattern rule is incomplete.

Students need opportunities to compare numeric patterns and to discuss how they are the same and how they are different. When comparing decreasing patterns, compare the starting points and how each term decreases using a variety of representations such as shape patterns, hundred charts, and number patterns. For example, give students a page with four small hundred charts. Ask them to skip count backward starting at 100 and shade one chart by 2s, one chart by 5s, one chart by 10s, and one chart by 25s. Then discuss the pattern rule in each chart indicating the starting point and the amount of decrease.

Students should be able to create various representations of a decreasing pattern that follow a given pattern rule. Initially students will create patterns with concrete materials, then pictures, and then numbers. Some students may create a simple pattern, while other students may create a more complex pattern.

Students should be able to create decreasing patterns, concretely, pictorially, and symbolically, and be able to describe the pattern rule they used to create their representations. When creating decreasing patterns, initially students need to choose a starting point and then decide on the amount of the decrease. The amount of decrease may be either a constant amount or an amount that increases each time. Students should be able to describe their pattern by clearly explaining how it changes from one term to the next. Students may share their patterns and the strategies they used to create their pattern.

Students should have frequent experiences using decreasing patterns to solve real-world problems that interest and challenge them. They should use concrete materials or pictures to model the problem before determining the pattern rule. A variety of strategies may be used to solve the problem, such as using a number line, a hundred chart, a picture, concrete materials, or skip counting.

Students should be able to identify and describe the strategy used to determine a missing term in a given decreasing pattern. Since patterns decrease in a predictable way, to determine a missing term the students should first look at the term that comes before and after. One strategy may be to identify and use the pattern rule. Other possible strategies include using a number line, a hundred chart, a picture, concrete materials, or skip counting.

D. What are the most appropriate methods and activities for assessing student learning? Below are some sample questions related to pattern and relations which will be used to represent some of the appropriate methods and activities for assessing student learning.

Encourage students to share their thinking, their strategies, and their solutions. Rich discussion fosters the development of alternative problem-solving strategies.

Examples:

- 1. What are the two missing numbers in the number pattern Below?
 - 66, 61, 56, 51, ____, 41, 36, ____, ...
 - O 45 and 35
 - O 46 and 31
 - O 52 and 31
 - O 52 and 37
- Natalie created the following decreasing pattern: 546, 536, 526, 516, 506, 496, ... What is the rule for this pattern?
 - O Subtract 10.
 - O Start at 546 subtract 5 each time.
 - O Start at 496 and subtract 10 each time.
 - O Start at 546 and subtract 10 each time.
- **3.** Monique created the following decreasing number pattern. Two numbers are missing in this pattern. 55, 50, 45, 35, 30, 25, 20, 10, 5, ...

What are the two missing numbers?

- O 40 and 30
- O 15 and 25
- O 40 and 15
- O 45 and 15

4. Simon created the following pattern.



How many small circles are there in the fifth figure?

- O 15
- O 12
- O 10
- O 5
- 5. Examine the following pattern:



How many squares are there in fourth figure?

- O 14 squares
- O 13 squares
- O 12 squares
- O 11 squares
- 6. Which statement about the two following patterns is true?

62, 74, 86, 98, ... and 62, 50, 38, 26, ...

- O They have the same starting point and increase in the same way.
- O They have the same starting point and they are increasing patterns.
- O They have the same starting point and they are decreasing patterns.
- O They have the same starting point and they do not increase in the same way.

7. Examine the following pattern:



How many circles are there in the fifth figure?

- O 2
- О з
- O 4
- O 5
- **8.** Examine the following pattern:



How many squares are there in the first figure?

- O 1 square
- O 4 squares
- O 12 squares
- O 14 squares
- 9. Marthe created the following pattern using yellow and red counters:



What is the rule for this pattern?

- O From figure 1, add one yellow counter to the left and one red counter to the right each time.
- O From figure 1, add two red counters to the left and two yellow counters to the right each time.
- O From figure 1, add one yellow counter and one red counter to the left, and one yellow counter and one red counter to the right each time.
- O From figure 1, add one red counter to the left and one yellow counter to the right each time.

10. This part of a hundred chart has some numbers missing.



What number belongs in the shaded box?

O 42

O 43

O 52

O 63

Mathematics in Grade 4 Lesson Learned 5 Problem Solving

Students need more exposure to application and analysis items in order to apply these higher order thinking skills when problem solving. Students need to be encouraged to understand that problems may have more than one entry point and there are many strategies to solve a problem. Students need to continue to work on translating between and among representations when problem solving. They do not seem to realize that they can use varied representations when asked to solve a word problem. These other representations may support their problem solving and their reasoning.

Learning through problem solving should be the focus of mathematics at all grade levels. Problem solving is one of the critical mathematical processes that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics. When students encounter new situations, and respond to questions such as, How would you ...? or How could you ...? the problem-solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.

In order for an activity to be problem-solving based, it must appropriately challenge students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but is simply practice. A true problem-solving activity requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement, perseverance, and collaboration. Problem solving is also a powerful teaching tool that fosters multiple, creative, and innovative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident mathematical risk takers.

Students need to be able to explore a wide variety of methods for solving and verifying problems in all areas of mathematics. They must be challenged to find multiple solutions for problems and be given opportunities to create and solve their own problems.

A. What conclusions can be drawn from the Nova Scotia Assessment: Mathematics in Grade 4? We found that students have a good understanding of basic facts and procedures, and are successful when explicitly given all the information needed to do a knowledge question. But when given application and analysis items, they are not able to apply higher order thinking skills when problem solving. For example, students were not sure whether they should add or subtract when questions were presented in the context of a story problem. Overall, our students are experiencing challenges with problem solving across all mathematical strands. This appeared to be a theme throughout the assessment data when students were asked to "solve a word problem" or "solve a multi-step problem". The data also showed that, across all mathematics strands, students struggle with items that require translating among representations in problem-solving contexts.

B. Do students have any misconceptions or errors in their thinking?

It is important to distinguish between misconceptions and errors. Misconceptions are, in effect, the misunderstandings about mathematical ideas which students entertain and which usually lead to errors occurring. These are the most serious kind of errors which require urgent action on the part of the teacher. Errors may occur for a variety of underlying reasons, ranging from careless mistakes to errors resulting from misconceptions.

It is also important to mention that when students encounter a problem-solving situation, they become confused by mathematical expressions which are not familiar to them or which are difficult to understand. In addition, when students consider that a problem is a mathematical problem, they believe, wrongly, that they should associate it simply to routine calculations with few concerns to the meaning of the context and to the credibility of the answers.

Many students, when given a problem-solving task, especially a "word problem" (solving a problem in a context) have the misconception that is it always too hard for them to attempt. Putting words around the numbers seems to obstruct their ability to think about the question. Another misconception students have is that there is only one way to solve a word problem.

When asked to solve a problem in a context, they struggle to identify a possible strategy and often fail to even attempt to solve the problem. Students appeared to have a limited repertoire of problem-solving strategies to help them attempt a problem. For example, when given an analysis problem in the context of an increasing pattern, students struggle with answering "what strategy could I use to solve this problem"? There are many strategies and ways to solve the problem. Some students find it difficult to find an entry point to begin to solve the problem.

Students also tend to forget that they may translate between and among representations to help them solve a problem. If asked to solve a word problem using words, symbols and/or pictures, most students only provide symbols. They do not seem to realize that they can use all representations when asked to solve a word problem. These other representations may support their problem solving.

Problem: Some friends are coming to your birthday party. A square table with 4 chairs can seat 4 of your friends. If 2 square tables are put together, you can seat 6 of your friends. How many friends can you seat with 5 square tables?

Solution 1: What do I know and how can I use it to help me solve this problem?

- I know that one square table can seat 4 of my friends.
- I know that if 2 square tables are put together, 6 of my friends can have seats.
- So, if I continue this pattern, 3 square tables put together, can seat 8 of my friends.
- 4 square tables put together, can seat 10 of my friends, and 5 square tables put together, can seat 12 of my friends.
- So, 5 square tables can seat 12 of my friends

Solution 2: Draw a picture and a chart



Tables	Chairs	Number of Friends
1	4	4
2	6	6
3	8	8
4	10	10
5	?	?

So, 12 friends can be seated at 5 square tables when they come to my birthday

Students could have used concrete materials/manipulatives such as coloured tiles, two-sided counters, cube-a-link blocks, and pictures or numbers to solve this question. . It would be nice to add that graphic here. Showing how the problem could be represented more abstractly with tiles would help make the connection to increasing patterns.

The student results of the 2016–2017 Nova Scotia Assessment Mathematics in Grade 4 shows the following data:

- 56% of the students were not able to solve a word problem involving subtraction or division.
- 58% of the students were not able to solve a word problem involving estimations.
- 59% of the students had difficulty to solve correctly a word problem involving division.

C. What are the next steps in instruction for the class and for individual students?

A significant part of learning to solve problems is learning about the problem-solving process. It is generally accepted that the problem-solving process consists of four steps – understand the problem, devise a plan, carry out the plan, and look back to determine the reasonableness of an answer. Teachers need to teach their lessons through a problem-solving approach. Students learn mathematics as a result of solving problems. It is important to point out that not all lessons students encounter must be taught through problem solving. If the purpose of the lesson being taught is to develop a certain skill for conceptual understanding, then some practice is required.

"The teacher provides a context or reason for the learning by beginning the lesson with a problem to be solved. This approach contrasts with the more traditional approach of, for example, presenting a new procedure and then adding a couple of word problems at the end for students to solve. Instead, the teacher gives students the opportunity to think about the problem and work through the solution in a variety of ways, and only then draws the procedures out of their work." (Small, 2005, p. 154)

An important aspect of problem solving in grades 1–3 is addition and subtraction problems which can be categorized based on the kinds of relationships they represent. It is important that all of the story problem structures are presented and developed from students' experiences. Initial work with the story structures will focus on join and separate types of problems because students associate the actions in these problems with the operations. However, they must also experience addition and subtraction in part-part-whole and comparison situations. Please refer to the grade level appropriate curriculum documents (Mathematics 1,

p. 64; Mathematics 2, p. 68; and Mathematics 3, p. 71) for more information about the story structures and instructional strategies.

Manipulatives can and should be used to model the strategies and the story structures. Examples of manipulatives that can be used for this purpose include

- two-sided counters
- linking cubes
- number cubes
- ten-frames
- walk-on number line
- base-ten blocks

A Problem-Solving Approach

A problem-solving approach is embedded and expected throughout our curriculum, grades Primary to 12, in all strands.

As noted in Pearson (2009a), problem-solving is a key strategy: Problem-solving is a key instructional strategy that enables students to take risks, secure in the knowledge that their thoughts, queries, and ideas are valued. As students share their solutions and findings, the teacher can provide direct instruction on problem-solving strategies. After students share their solutions and justifications, teachers can elaborate on their methods and encourage students to comment or ask questions of their peers. Using student findings and solution methods to guide instruction also allows students to see the value in their work, and encourages peers to share their strategies. While some strategies may be more efficient than others, several strategies may work and often a combination of strategies is required to solve a problem. Students must use strategies that are meaningful to them and make sense to them. (p. 13)

Problem-Solving Strategies

Students are already drawing on personal strategies for problem solving, in many of the activities they undertake. *Strategies Toolkit* lessons found in the *Pearson Math Makes Sense Series*, allows teachers to expand their students' personal repertoires through explicit instruction on a specific strategy. "When students develop a name for the strategy, they develop a stronger self-awareness of the personal strategies they are starting to use on their own" (Pearson, 2009a, p. 13).

The *Strategies Toolkit* lessons highlight these problem-solving strategies (Pearson, 2009a, p. 13):

- Make a chart or table
- Draw a picture
- Work backward
- Make an organized list
- Use a model
- Solve a simpler problem
- Guess and test
- Use a pattern

Van de Walle and Lovin (2006b), in their resource, *Teaching Student-Centered Mathematics Grades 3–5*, suggest a three-part lesson format for teaching through problem-solving. This same approach is used in our core resource, *Math Makes Sense* (Pearson, 2009b, pp. 13–14):

Before

Before students begin:

- Prepare meaningful problem scenarios for students. These should be sufficiently challenging.
- Ensure the problem is understood by all.
- Explain the expectations for the process and the product.

During

As students work through the problem:

- Let students approach the problem in a way that makes sense to them.
- Listen to the conversations to observe thinking.
- Assess student understanding of her/his solution.
- Provide hints or suggestions if students are on the wrong path.
- Encourage students to test their ideas.
- Ask questions to stimulate ideas.

After

After students, have solved their problem:

- Gather for a group meeting to reflect and share.
- Make the mathematics explicit through discussion.
- Highlight the variety of answers and methods.
- Encourage students to justify their solutions.
- Encourage students to comment positively or ask questions regarding their peer's solutions.

For more details on using a problem-solving approach to teach mathematics, see Van de Walle and Lovin (2006b), *Teaching Student-Centered Mathematics Grades 3–5*, from which these ideas are drawn (pp. 11–28).

Assessing Problem Solving

For information using a rubric to score problem-solving sample questions in either Mathematics in Grade 4 (M4) or Mathematics in Grade 6 (M6), please go to the Program of Learning Assessment for Nova Scotia (PLANS) website. On the Documents tab of each assessment page, you will find a Problem-Solving document which includes the provincial rubric and sample questions (<u>plans.ednet.ns.ca/grade4/documents</u>).

From Reading Strategies to Mathematics Strategies

With a problem-solving approach embedded and expected throughout our curriculum grades Primary to 12 in all strands, there are definite implications for teaching reading strategies in mathematics. Concepts and skills should also be connected to everyday situations and other curricular areas. For example, when developing literacy skills students learn to make text-to-world, text-to-text, and text-to-self connections. Students can also make connections to make mathematics come alive through math-to-world, math-to-math, and math-to-self connections.

There are classroom strategies with suggestions of how to use the strategy in a mathematical context that support teachers as they develop students' mathematical vocabulary, initiate effective ways to navigate informational text, and encourage students to reflect on what they have learned. For example, the Frayer Model, Concept Circles, Three-Read Strategy, Exit Cards, etc. These are only a few of the strategies that are found in Appendix B at the end of this document.

When teachers use these strategies in the instructional process or embed them in assessment tasks, the expectations for students must be made explicit. The students' understanding of the mathematics involved and maintaining the integrity of the curriculum are still the foremost concerns.

Please refer to the <u>Appendix B</u> found at the end of this document for strategies with illustrative examples.

D. What are the most appropriate methods and activities for assessing student learning? Below are some sample questions related to problem-solving which will be used to represent some of the appropriate methods and activities for assessing student learning.

Encourage students to share their thinking, their strategies, and their solutions. Rich discussion fosters the development of alternative problem-solving strategies.

Examples:

 Natasha has 4 T-shirts and 2 pairs of pants. How many different outfits can Natasha make? Show how you solved the problem and explain your strategy.

Possible Strategy: Make a chart, table or draw a picture; Strand: Number/Stats and Probability; Application Question

 Megan and Danielle each ordered the same size pizza. Megan asked to have her pizza cut into fourths. Danielle asked to have her pizza cut into sixths. Who has the larger pieces of pizza? Show how you solved the problem and explain your strategy.

Possible Strategy: Use a model, draw a picture; Strand: Number; Application Question

 Peter has 123 marbles. He gives some marbles to his friend Paul. Now Peter has 86 marbles. How many marbles did Peter give to Paul? Show how you solved the problem and explain your strategy.

Possible Strategy: Use a model; Strand: Number; Application Question

Sebastian was at a bicycle sale.
There were bicycles and tricycles.
Altogether, there are 21 wheels.
How many bicycles and tricycles are there?
Show how you solved the problem and explain your strategy.



Possible Strategy: Work backward; Strand: Number; Analysis Question

5. Susan has 23 apples.

She eats three apples every day. How many apples did Susan have left after seven days? Show how you solved the problem and explain your strategy.

Possible Strategy: Use a pattern, make a list, use a model; Strand: Patterns and Relations; Analysis Question

6. Marbles come in packages of 10, 25, and 50. You need 160 marbles.Find 5 ways you could buy the marbles.Show how you solved the problem and explain your strategy.

Possible Strategy: Make an organized list; Strand: Number; Application Question

 For James birthday, his mother wants to cover each table using paper tablecloth which is 4 m long. How many tables can she cover using a roll of table cloth which is 23 m long? Show how you solved the problem and explain your strategy.

Possible Strategy: Make an organized list, work backward; Strand: Number; Analysis Question

8. Marie's height is 3 cm more than Norman's height. Norman's height is 2 cm more than Jessica's height. If Marie's height is 126 cm, what is Jessica's height? Show how you solved the problem and explain your strategy.

Possible Strategy: Draw a picture; Strand: Measurement; Analysis Question

9. Lucy has some money. She has some 5 cent coins, some 10 cent coins and 25 cent coins. She buys a used book for 45 cents.
She used all her money to buy this book and now has no money left.
How many different ways could Lucy pay for this book using all of her money?
Show how you solved the problem and explain your strategy.

Possible Strategy: Draw a picture, draw a chart; Strand: Number; Analysis Question