

Nova Scotia Assessment: Mathematics in Grade 4 *Lessons Learned*

“For learners to succeed, teachers must assess students’ individual abilities and characteristics and choose appropriate and effective instructional strategies accordingly.”

– Helene J. Sherman

Contents

Purpose of this document.....	1
Overview of the Nova Scotia Assessment: Mathematics in Grade 4.....	2
Performance Levels	4
Assessment Results	4
Mathematics in Grade 4 Lessons Learned	5
Key Messages.....	7
Mathematics in Grade 4 Lesson Learned 1 – Translating Between and Among Representations.....	9
Mathematics in Grade 4 Lesson Learned 2 – Representing and Partitioning Whole Numbers	13
Mathematics in Grade 4 Lesson Learned 3 – Whole Number Operations	17
Mathematics in Grade 4 Lesson Learned 4 – Patterns and Relations	26
Mathematics in Grade 4 Lesson Learned 5 – Problem Solving.....	36
Mathematics in Grade 4 Lesson Learned 6 – Measurement	43
Mathematics in Grade 4 Lesson Learned 7 – Geometry	49
Mathematics in Grade 4 Lesson Learned 8 – Statistics and Probability	57
References	65
Appendix A: Cognitive Levels of Questioning	66
Appendix B: From Reading Strategies to Mathematics Strategies	68
Appendix C: Cognitive Levels of Sample Questions	77
Appendix D: Responses to the Sample Questions	78

Purpose of this document

This *Lessons Learned* document was developed based on an analysis of the Item Description Reports for the Nova Scotia Assessment: Mathematics in Grade 4 (2013–2014, 2014–2015, 2015–2016 and 2016–2017). It is intended to support all elementary classroom teachers (in particular grades Primary–3) and administrators at the school, board, and provincial levels, in using the information gained from this assessment to inform next steps for numeracy focus. The analysis of these items form the basis of this document, which was developed to support teachers as they further explore these areas through classroom-based instruction and assessment across a variety of mathematical concepts.

After the results for each mathematics assessment become available, an Item Description Report is developed to describe each item of the mathematics assessment in relation to the curriculum outcomes and cognitive processes involved with each mathematical strand. The percentage of students across the province who answered each item correctly is also connected to each item. Item description reports for mathematics are made available to school boards for distribution to schools, and they include provincial, board, and school data. Schools and boards should examine their own data in relation to the provincial data for continued discussions, explorations, and support for mathematics focus at the classroom, school, board, and provincial levels.

This document specifically addresses areas that students across the province found challenging based on provincial assessment evidence. It is essential that teachers consider assessment evidence from a variety of sources to inform the next steps most appropriate for their students. Effective classroom instruction and assessment strategies are responsive to the individual learners within a classroom.

The M4 assessment generates information that is useful in guiding classroom-based instruction and assessment in mathematics. This document provides an overview of the mathematics tasks included in the assessment, information about this year's mathematics assessment results, and a series of Lessons Learned for mathematics. Suggested next steps for classroom instruction and assessment are included for each Lesson Learned.

Overview of the Nova Scotia Assessment: Mathematics in Grade 4

Nova Scotia Assessments are large-scale assessments that provide reliable data about how well all students in the province are learning the mathematics curricula. It is different from many standardized tests in that all questions are written by Nova Scotia teachers to align with curriculum outcomes and the results reflect a snapshot of how well students are learning these outcomes. These results can be counted on to provide a good picture of how well students are learning curriculum outcomes within schools, boards and in the province as a whole. Since the assessments are based on the Nova Scotia curriculum, and are developed by Nova Scotia teachers, results can be used to determine whether the curriculum, approaches to teaching and allocation of resources are effective. Furthermore, because individual student results are available, these, in conjunction with other classroom assessment evidence, help classroom teachers understand what each student has under control and identify next steps to inform instruction.

The assessment provides information about mathematics for each student and complements assessment data collected in the classroom. This assessment is administered at the beginning of Grade 4. It is designed to provide detailed information for every student in the province regarding his or her progress in achieving a selection of mathematics curriculum outcomes at the end of Grade 3. Information from this assessment can be used by teachers to inform their instruction and next steps in providing support and intervention for their students.

The design of the assessment includes the following:

- mathematical tasks that reflect a selection of outcomes aligned with the curriculum from the end of grade 1 to the end of grade 3 from across all strands of the mathematics curriculum
- all items are in selected response format
- all items are designed to provide a broad range of challenge, thereby providing information about a range of individual student performance

Table 1: Specific Curriculum Outcomes Assessed in 2016–2017

Strand	Specific Curriculum Outcomes
Number (N)	1N04, 2N02, 2N03, 2N05, 2N09, 3N02, 3N03, 3N04, 3N05, 3N06, 3N08, 3N011, 3N012, 3N013
Patterns and Relations (PR)	2RR01, 3RR01, 3RR02, 3RR03
Measurement (M)	3M01, 3M02, 3M03, 3M04, 3M05
Geometry (G)	2G02, 3G01, 3G02
Statistics and Probability (SP)	2SP02, 3SP01, 3SP02

Cognitive levels of questions in mathematics are defined as:

- *Knowledge questions* may require students to recall or recognize information, names, definitions, or steps in a procedure.
- *Application/comprehension questions* may require students to make connections, represent a situation in more than one way (translating between representations), or solve contextual problems.
- *Analysis questions* may require students to go beyond comprehension and application to higher order thinking skills, such as generalizations and non-routine problem-solving.

These are the percentages of the questions on the Nova Scotia provincial assessments for M4:

- Knowledge 20–30%
- Application 50–60%
- Analysis 10–20%

These percentages are also recommended for classroom-based assessments.

Please refer to [Appendix A](#) for further information about cognitive levels of questioning.

The Nova Scotia Assessment: Mathematics 4 includes 96 items distributed over two days for a duration of 90 minutes each day; 48 items on day one and 48 items on day two. The chart below shows the distribution, by mathematical strand and cognitive level, of items each day.

Table 2: Number of Items by Strand and Cognitive Level

Number of Items Day 1				
	Knowledge	Application	Analysis	Total
Number	5	17	3	25
Patterns and Relations	2	3	1	6
Measurement	2	3	1	6
Geometry	2	3	1	6
Statistics and Probability	1	2	2	5

Number of Items Day 2				
	Knowledge	Application	Analysis	Total
Number	5	17	3	25
Patterns and Relations	2	3	1	6
Measurement	2	3	1	6
Geometry	2	3	1	6
Statistics and Probability	1	2	2	5

Performance Levels

Below are the Nova Scotia Assessment: Mathematics in Grade 4 Performance Levels

- Level 1:** Students at Level 1 can generally solve problems when they are simple and clearly stated or where the method to solve the problem is suggested to them. They can do addition and subtraction of whole numbers, but may not understand when each operation should be used. They can recognize some mathematical terms and symbols, mainly from earlier grades.
- Level 2:** Students at Level 2 can generally solve problems similar to problems they have seen before. They depend on a few familiar methods to solve problems. They can do addition and subtraction of whole numbers and usually understand when each operation should be used. They can understand and use some mathematical terms and symbols, especially those from earlier grades.
- Level 3:** Students at Level 3 can generally solve problems that involve several steps and may solve problems they have not seen before. They can apply number operations (+, −, as well as \times , \div to 5×5) correctly and can judge whether an answer makes sense. They can understand and use many mathematical terms and symbols, including those at grade level.
- Level 4:** Students at Level 4 can solve new and complex problems. They can apply number operations (+, −, \times , \div) with ease. They can think carefully about whether an answer makes sense. They find mathematical terms and symbols easy to use and to understand.

Assessment Results

The Nova Scotia Assessment: Mathematics in Grade 4 has been administered since the 2013–2014 school year. The following percentage of students performed at the expectation of level 3 or above on the assessment: 74% (2013–2014), 74% (2014–2015), 76% (2015–2016) and 77% (2016–2017).

The following is a breakdown of the 2016–2017 M4 results for each performance level (8177 grade 4 students participated in the M4 assessment):

- 77% of grade 4 students in the province have a performance level of 3 or above
- Performance Level 1: 7% of students in the province are below the expectations of this assessment
- Performance Level 2: 16% of students in the province are approaching the expectations of this assessment
- Performance Level 3: 62% of students in the province are at the expectations of this assessment
- Performance Level 4: 15% of students in the province are above the expectations of this assessment

Mathematics in Grade 4 Lessons Learned

The assessment information gathered from the Nova Scotia Assessment: Mathematics in Grade 4 data has been organized into 8 **Lessons Learned**:

- Translating Between and Among Representations
- Representing and Partitioning Whole Numbers
- Whole Number Operations
- Patterns and Relations
- Problem Solving
- Measurement
- Geometry
- Statistics and Probability.

Each Lesson Learned is divided into four sections that address the following questions

- A. What conclusions can be drawn from the Nova Scotia Assessment: Mathematics in Grade 4?
- B. Do students have any misconceptions or errors in their thinking?
- C. What are the next steps in instruction for the class and for individual students?
- D. What are the most appropriate methods and activities for assessing student learning?

Lessons Learned

1. **Translating Between and Among Representations**: Students were challenged when translating and moving flexibly between and among all representations of a concept. Students need to be encouraged to translate between words, pictures and symbols. If asked to represent a question using representations, (words, symbols and/or pictures), students usually only provide symbols. They do not seem to realize that they can use varied representations (words, symbols and/or pictures) when solving a question.
2. **Representing and Partitioning Whole Numbers**: Students were challenged when asked to apply their knowledge of basic facts and skills and to represent a situation or the steps in a procedure when given application questions. They struggled with translating between representations of a concept (contextually, concretely, pictorially, symbolically, and verbally). When students are working to partition whole numbers and to perform operations, it is very important for students to understand that numbers can be broken down into two or more parts in many different ways.
3. **Whole Number Operations**: Students were challenged when asked to apply basic skills, knowledge, and computational procedures to application and analysis questions. Students need to be able to apply the higher order thinking skills of problem solving, creativity, and reasoning to do application and analysis items. Students should have experiences with all of the story structures for addition and subtraction. Students also need to be encouraged to estimate before calculating an answer to a question.
4. **Patterns and Relations**: Students were challenged when asked to transfer their visual representation of patterns to numerical patterns. Students should be able to describe either an increasing pattern or a decreasing pattern, and need to recognize that each term has a numeric value. Students seemed to forget that a pattern rule must have a starting point. Without the starting point, the pattern rule is incomplete. They need to be encouraged to provide all the information for a pattern rule. Students need to continue to work with representations of patterns, concretely, contextually, pictorially, symbolically, and verbally.

5. **Problem Solving:** Students were challenged with and require more exposure to application and analysis items in order to apply these higher order thinking skills when problem solving. Students need to be encouraged to understand that problems may have more than one entry point and there are many strategies to solve a problem. Students need to continue to work on translating between and among representations when problem solving. They do not seem to realize that they can use varied representations when asked to solve a word problem. These other representations may support their problem solving and their reasoning.
6. **Measurement:** Students were challenged with building conceptual understanding of what it means to measure with a ruler. Emphasis should be placed on counting the intervals between the numbers, rather than looking at the number on the ruler that is aligned with the end of the object. Students also need to use a ruler to measure the length of a pencil or other objects with and without using zero as the starting point. Students need to recognize which mass unit (gram or kilogram) is appropriate for measuring and comparing the mass of a specific item. Students need to be able to explain that perimeter is the distance around a shape or the length of the boundary of an enclosed region. Students need to find the perimeter of many different regular, irregular, and composite shapes, before being introduced to questions in pictorial form. Students need to work with perimeter in application and analysis questions.
7. **Geometry:** Students were challenged with developing their knowledge of 2-D shapes and 3-D objects by describing and sorting them according to their geometric attributes. Students need more experiences to identify the faces, edges, and vertices of 3-D objects including spheres, cones, cylinders, pyramids, cubes and other prisms. Students need to be provided with opportunities to explore these attributes through sorting and constructing activities. Students need to extend their knowledge of both regular and irregular polygons. They need to focus on comparing the number of sides as the key attribute for classifying polygons. Students need more experiences with regular and irregular polygons, so that they begin to realize that a polygon, regardless of its dimensions, remains the same shape.
8. **Statistics and Probability:** Students were challenged using tally marks, lists, charts, line plots, and bar graphs to organize data relevant to their everyday life. Students need opportunities and experiences to interpret information collected, organized, and displayed in tally charts, charts, line plots and bar graphs. Students need to develop the skill of interpreting graphs, and answering questions and to draw conclusions from those tally charts, line plots and bar graphs. They need to be encouraged to ask or write questions that go beyond simplistic reading of a graph. Both literal and inferential comprehension questions need to be asked.

Key Messages

The following key messages should be considered when using this document to inform classroom instruction and assessment.

The Nova Scotia mathematics curriculum is based upon several key assumptions or beliefs about mathematics learning that have grown out of research and practice. These beliefs include the following:

- Mathematics learning is an active and constructive process.
- Learning is most effective when standards of expectation are made clear with ongoing assessment and feedback.
- Learners are individuals who bring a wide range of prior knowledge and experiences and who learn via various styles and at different rates.
- Learning is most likely to occur when placed in meaningful contexts and in an environment, that supports exploration, risk taking, and critical thinking and that nurtures positive attitudes and sustained effort.
- Students are curious, active learners with individual interests, abilities, and needs. They come to classrooms with varying knowledge, life experiences, and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.
- Students learn by attaching meaning to what they do and need to construct their own meaning of mathematics. This meaning is best constructed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of models and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students and enhance the formation of sound, transferable, mathematical concepts. At all levels, students benefit from working with and translating through a variety of materials, tools, and contexts when constructing meaning about new mathematical ideas. Meaningful discussions can provide essential links among concrete, pictorial, contextual, and symbolic representations of mathematics.
(EECD, 2013b, p. 23)

Research has shown that assessment for learning (formative assessment) practices produce significant and often substantial learning gains, close achievement gaps, and build students' ability to learn new skills (Black & Wiliam 1998; OECD 2006). Student involvement in assessment promotes learning. Timely and effective teacher feedback and student self-assessment allow students to reflect on and articulate their understanding of mathematical concepts and ideas.

- Provincial assessment results form part of the larger picture of assessment for each student and complements assessment data collected in the classroom. Ongoing assessment for learning (formative assessment) is essential to effective teaching and learning. Assessment for learning can and should happen every day as part of classroom instruction. Assessment of learning (summative assessment) should also occur regularly and at the end of a cycle of learning. Both forms of assessment are necessary to guide teaching, stimulate learning, and produce achievement gains.
- It is important to construct assessment activities that require students to complete tasks across the cognitive levels. While it is important for students to be able to answer factual and procedural type questions, it is also important to embed activities that require strategic reasoning and problem-solving.

- Ongoing assessment for learning involves the teacher focusing on how learning is progressing during the lesson and the unit, determining where improvements can be made and identifying the next steps. *“Such assessment becomes ‘formative assessment’ when the evidence is actually used to adapt the teaching to meet learning needs,”* (Black, Harrison, Lee, Marshall & Wiliam, 2003, p.2). Effective strategies of assessment for learning during a lesson include: strategic questioning, observing, conversing (conferring with students to “hear their thinking”), analyzing student’s work (product), engaging students in reviewing their progress, as well as, providing opportunities for peer and self-assessment.
- Assessment of learning involves the process of collecting and interpreting evidence for the purpose of summarizing learning at a given point in time, and making judgments about the quality of student learning on the basis of established criteria. The information gathered may be used to communicate the student’s achievement to students, parents, and others. It occurs at or near the end of a learning cycle.
- All forms of assessment should be planned with the end in mind, thinking about the following questions:
 - What do I want students to learn? (identifying clear learning targets)
 - What does the learning look like? (identifying clear criteria for success)
 - How will I know they are learning?
 - How will I design the learning so that all will learn?
- Before planning for instruction using the suggestions for instruction and assessment, it is important that teachers review individual student results in conjunction with current mathematics assessment information. A variety of current classroom assessments should be analyzed to determine specific strengths and areas for continued instructional focus or support.

Balanced Assessment in Mathematics: Effective ways to gather information about a student’s mathematical understanding

- Conversations/Conferences/Interviews: Individual, Group, Teacher-initiated, Child-initiated
- Products/Work Samples: Mathematics journals, Portfolios, Drawings, Charts, Tables, Graphs, Individual and classroom assessment, Pencil-and-paper tests, Surveys, Self-assessment
- Observations: Planned (formal), Unplanned (informal), Read-aloud (literature with mathematics focus), Shared and guided mathematics activities, Performance tasks, Individual conferences, Anecdotal records, Checklists, Interactive activities

(EECD, 2013a, p. 4)

Mathematics in Grade 4 Lesson Learned 1

Translating Between and Among Representations

Students were challenged when translating and moving flexibly between and among all representations of a concept. Students need to be encouraged to translate between words, pictures and symbols. If asked to represent a question using representations, (words, symbols and/or pictures), students usually only provide symbols. They do not seem to realize that they can use varied representations (words, symbols and/or pictures) when solving a question.

Mathematics should be viewed as an integrated whole rather than as the study of separate strands or units. Connections must also be made between and among the different representational modes – contextual, concrete, pictorial, linguistic/verbal, and symbolic. The process of making connections, in turn, facilitates a child’s learning.

“Children who have difficulty translating a concept from one representation to another are the same children who have difficulty solving problems and understanding computations. Strengthening the ability to move between and among these representations improves the growth of children’s concepts” (Van De Walle, John A. 2001, *Elementary and Middle School Mathematics*, Fourth Edition, p. 34).

One way to encourage children to use multiple representations is to explicitly ask for them.

Ask questions such as

- How many ways can you show the number 20 using words, pictures, models, and numbers?
- How many ways can you write 75?
- Can you represent a rectangle as a combination of other shapes?
- Can you represent this line plot as a bar graph?
- Can you use an equation to represent how you thought about this story problem?

A. What conclusions can be drawn from the Nova Scotia Assessment: Mathematics in Grade 4?

Historically, our students were challenged when asked to translate between and among representations of a concept. However, the student results of the 2016–2017 Nova Scotia Assessment Mathematics in Grade 4 reveals that there has been improvement over time. Students now have translating between and among representation well under control. We found that students have a good understanding of basic facts and procedures, but when given application items, they appear to want to rush to symbolic. For example, when problem-solving, students are able to understand the context of the question but many are not able to translate between the representations (translating from words to pictures or symbolic to pictures, etc.).

B. Do students have any misconceptions or errors in their thinking?

If asked to represent a question using words, symbols and/or pictures, students usually only provided symbols. They do not seem to realize that they can use various representations (words, symbols and/or pictures) when solving a question. For example, when given an analysis question concerning the perimeter (distance around) a named shape, students struggle with knowing how many sides the named shape would have, unless the actual shape is shown in the question as a picture/diagram. Students should be encouraged to translate between the name of the shape (words) and a picture. Rather than relying on the question itself, students could draw the picture and check how many sides the named shape has.

The student results of the 2016–2017 Nova Scotia Assessment Mathematics in Grade 4 reveals that the students have translating between and among representation well under control.

C. What are the next steps in instruction for the class and for individual students?

Next steps in instruction should provide opportunities for students to use representations to communicate mathematical ideas. They should have experiences selecting, applying, modeling, and translating among mathematical representations to solve problems. The five representations of a concept are contextual, concrete, pictorial, symbolic, and verbal (written/oral). All five representations should remain a focus for students.

“Representational competence (Novick, 2004) for students is knowing how and when to use particular mathematical representations. A key aspect of understanding mathematics means not only knowing how to use a representation during problem-solving situations but also being able to make connections between representations” (*Teaching Children Mathematics*. NCTM, August 2010, p. 40). When students are able to move between and among the different representations of a concept, we say that they have concept attainment.

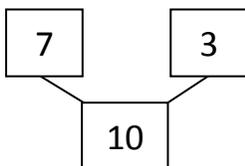
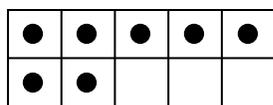
There are three specific strategies that provide opportunities to support students’ development of representational competency

- “Engaging in dialogue about the explicit connections between representations
- Alternating directionality of the connections made among representations
- Encouraging purposeful selection of representation”

(*Teaching Children Mathematics*. NCTM, August 2010, p. 40)

For example, during instruction students focus on specific parts of one representation and think about the correspondence with parts in another. Teachers should ask questions that require students to translate between parts. Alternating directionality supports students’ thinking with various representations. For example, consider the activity below, *All About Ten* (*Teaching Children Mathematics*. NCTM, August 2010, p. 44).

All About Ten: Fill in the box diagram to show the two parts of ten shown in the ten-frame. Write number sentences to match. [Below is the completed activity.]



$7 + 3 = 10$	$10 = 7 + 3$
$3 + 7 = 10$	$10 = 3 + 7$
$10 - 7 = 3$	$3 = 10 - 7$
$10 - 3 = 7$	$7 = 10 - 3$

Instructional Strategies

“Three specific instructional strategies create opportunities that may support students’ development of representational competence ...” (*Teaching Children Mathematics*. NCTM, August 2010, p. 40).

1. Students need to **engage** in dialogue about the explicit connections between representations. Pose questions to students such as
 - How is the number 10 represented in each diagram?
 - How are the three representations of the number the same? How are they different?
 - Can you show the number 10 in a different way?

2. Students need to be encouraged to **alternate directionality** in order to make connections among representations. When discussing multiple representations, ask focused questions such as
 - Can you describe the 3 in the ten-frame, in the box diagram, and in each of the equations?
 - What meaning does the 3 have in each of the diagrams?

“The **directionality** of the connections made between the representations and the problem situation is another important feature of representational competence. For example, translating from a box diagram to a ten-frame, and vice versa, promotes the use of different mathematical thought processes. An important aspect of developing understanding of mathematics means not only knowing how to use a representation during problem solving situations but also being able to move flexibly between different representations, making connections from one representation to the other, and vice versa.”

(Teaching Children Mathematics. NCTM, August 2010, p. 44)

3. Students need to be **encouraged** to purposefully select the most appropriate representation. “Encourage students to consider the suitability of a representation. Discuss a variety of reasons to use particular representations, including but not limited to the following: efficiency, accuracy, ease of use, appropriateness with respect to the problem context, and student preference. By comparing the use of multiple representations for the same problem, students can more easily see the suitability of one representation over another” *(Teaching Children Mathematics. NCTM, August 2010, p. 46).*

D. What are the most appropriate methods and activities for assessing student learning?

The strands Measurement (M) and Geometry (G) will be used to represent some of the appropriate methods and activities for assessing student learning.

Encourage students to share their thinking, their strategies, and their solutions. Rich discussion fosters the development of alternative problem-solving strategies.

Examples:

- 1.** Peter found a stick that was more than a 1 m long.
How many centimetres long could the stick be?

- 80 cm
- 90 cm
- 100 cm
- 115 cm

Show how you know using words, pictures, and/or symbols.

- 2.** The perimeter (distance around) of a regular pentagon is 20 cm.
How long is each side?

- 4 cm
- 5 cm
- 20 cm
- 25 cm

Show how you know using words, pictures, and/or symbols.

- 3.** I am a 3-D object.
I have 5 faces.
I have 5 vertices.
I have 8 edges.
Which shape am I?

- cube
- sphere
- square-based pyramid
- triangular-based pyramid

Show how you know using words, pictures, and/or symbols.

Mathematics in Grade 4 Lesson Learned 2

Representing and Partitioning Whole Numbers

Students were challenged when asked to apply their knowledge of basic facts and skills and to represent a situation or the steps in a procedure when given application questions. They struggled with translating between representations of a concept (contextually, concretely, pictorially, symbolically, and verbally). When students are working to partition whole numbers and to perform operations, it is very important for students to understand that numbers can be broken down into two or more parts in many different ways.

A. What conclusions can be drawn from the Nova Scotia Assessment: Mathematics in Grade 4?

We noticed that students were able to do well when explicitly given all the information needed to do the question. Students performed well on knowledge questions that required them to use basic skills, symbolic procedures, and factual knowledge. For example, students were successful when asked to calculate $61 + 35$. Students were successful problem solvers and performed well on questions that required analysis and non-routine problem-solving.

However, our assessment information also shows that many students experienced challenges with application questions. Our students were challenged when asked to apply their knowledge of basic facts and skills to a context. They also struggled with translating between representations of a concept (contextually, concretely, pictorially, symbolically, and verbally).

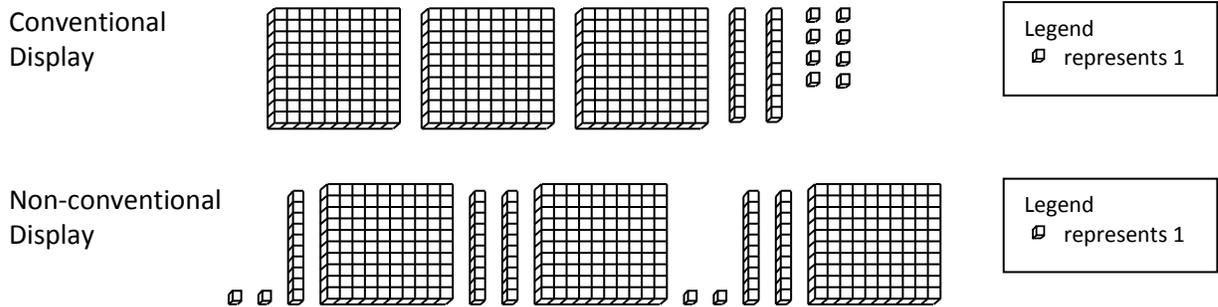
B. Do students have any misconceptions or errors in their thinking?

Students were very capable of performing well with partitioning numbers when given knowledge questions. For example, students represented a number using base-ten blocks in a conventional display. However, the difficulty appeared when students were asked to apply their knowledge or to represent a situation or the steps in a procedure when given application questions. For example, students could correctly represent a number, such as 75 using a conventional display of base-ten blocks but were challenged when asked to partition a number in a variety of ways, such as $70 + 5$, $50 + 25$, or $60 + 12 + 3$, $25 + 25 + 20$. Many students have the misconception that these are expressions that have an answer of 75, and do not understand that these also represent four ways of writing 75. An expression names a number. Sometimes an expression is a number such as 150. Sometimes an expression shows an arithmetic expression, such as $125 + 25$. 150 may also be represented by its partitions, such as $80 + 70$, $100 + 50$, and $50 + 50 + 50$. Numbers can also be represented by a difference expression, such $175 - 25$.

The student result of the 2016–2017 Nova Scotia Assessment Mathematics in Grade 4 reveals that the students have representing and partitioning of numbers well under control.

C. What are the next steps in instruction for the class and for individual students?

When students translate between and among the five representations of a concept (contextual, concrete, pictorial, symbolic and verbal), we say that they have concept attainment. Students need numerous experiences representing numbers to 1000 and translating between and among these representations of a concept to strengthen their knowledge. They need many experiences with base-ten materials, pictures such as number lines and tallies, ten-frames, words, and contexts to conceptualize a number being made up of two or more parts. It is extremely important that students have opportunities to view and create numbers using conventional and non-conventional displays of base-ten blocks.



Partitioning numbers using the models above supports students' ability to recognize that any number can be partitioned into two or more parts. It also helps students develop part-part-whole thinking. Although it is important for students to experience a variety of partitions including traditional expanded notation ($425 = 400 + 20 + 5$), they should also continue to experience partitions such as $424 + 1$, $325 + 100$, $200 + 200 + 10 + 10 + 5$. This is the most important understanding that can be developed about number relationships.

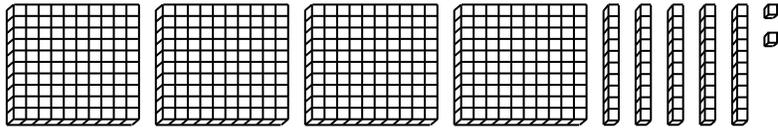
D. What are the most appropriate methods and activities for assessing student learning?

Below are some sample questions related to representing and partitioning whole numbers which will be used to represent some of the appropriate methods and activities for assessing student learning.

Encourage students to share their thinking, their strategies, and their solutions. Rich discussion fosters the development of alternative problem-solving strategies.

Examples:

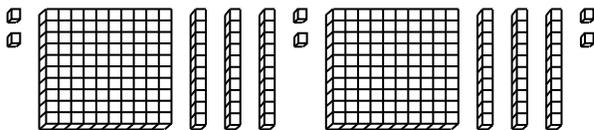
1. What number do these base-ten blocks represent?



Legend
 represents 1

Write the number, _____.

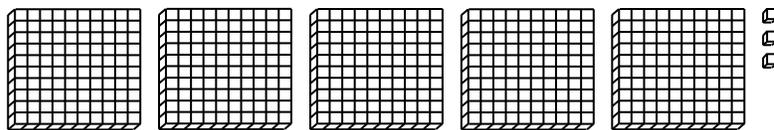
2. What number do these base-ten blocks represent?



Legend
 represents 1

Write the number, _____.

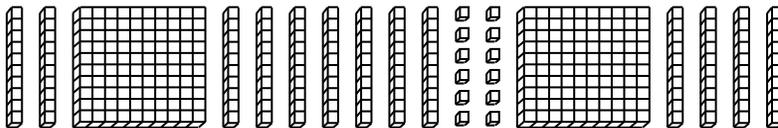
3. What number do these base-ten blocks represent?



Legend
 represents 1

Write the number, _____.

4. What number do these base-ten blocks represent?



Legend
 represents 1

Write the number, _____.

5. Draw a picture of base-ten blocks to show 236 in 3 different ways.

Legend
□ represents 1

--	--	--

6. The number 358 is the same as:

- $100 + 100 + 50 + 8 + 100$
- $300 + 5 + 8$
- $400 - 58$
- $3 + 5 + 8$

7. Choose the number that is equal to thirty-one tens.

- 31
- 301
- 310
- 3010

8. The number 642 is the same as:

- 5 hundreds, 2 tens, and 14 ones
- 64 tens and 2 ones
- 6 tens and 42 ones
- 6 hundreds, 20 tens and 4 ones

9. Write three expressions that can be used to represent 53.

53 is the same as _____

53 is the same as _____

53 is the same as _____

10. Write the following number in words:

263 _____

373 _____

487 _____

597 _____

Mathematics in Grade 4 Lesson Learned 3

Whole Number Operations

Students were challenged when asked to apply basic skills, knowledge, and computational procedures to application and analysis questions. Students need to be able to apply the higher order thinking skills of problem solving, creativity, and reasoning to do application and analysis items. Students should have experiences with all of the story structures for addition and subtraction. Students also need to be encouraged to estimate before calculating an answer to a question.

The Mathematics curriculum documents for grades Primary to 3 note that a true sense of number goes well beyond the skills of simply counting, memorizing facts, and the situational rote use of algorithms. Number sense develops when students connect numbers to real-life experiences and use benchmarks and referents. This results in students who are computationally fluent, flexible with numbers, and have intuition about numbers. The evolution of number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing rich mathematical tasks that allow students to make connections. Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects, and concepts.

A. What conclusions can be drawn from the Nova Scotia Assessment: Mathematics in Grade 4?

We noticed that students were able to do well when explicitly given all the information needed to do the question. Students performed well on knowledge questions that required them to use basic facts and skills, symbolic procedures, and factual knowledge. For example, students were able to solve $487 - 37$ when the problem was presented symbolically.

However, when students were asked to apply basic skills, knowledge, and computational procedures to application and analysis questions, they were challenged. This appeared to be a theme throughout the assessment data. At times, students were not sure whether they should add, subtract, multiply, or divide when questions were presented in the context of a story problem. The assessment analysis also showed that our students did not understand the relationship between addition and subtraction. Many of the students were doing addition and subtraction questions as procedures and were not making any connection between these two operations.

B. Do students have any misconceptions or errors in their thinking?

Addition and Subtraction

When solving computation questions, many students performed better when solving the computation using an alternative algorithm, such as presenting the digits to be added or subtracted horizontally. Students did less well, when the traditional algorithm was used. This may result because the traditional algorithm focuses on single digits within the computation rather than thinking about the number as a whole.

Examples:

$$67 - 32 = 35$$

$$\begin{array}{r} 650 \\ - 421 \\ \hline 229 \end{array}$$

Many students have the misconception that they always subtract the smaller number from the larger number. They apply this thinking regardless of whether the position of the number is in the subtrahend (a number which is to be subtracted from another number) or the minuend (a number from which another number is to be subtracted) in the question. Below are two examples.

$$\begin{array}{r} 451 \text{ (minuend)} \\ - 231 \text{ (subtrahend)} \\ \hline 220 \text{ (difference)} \end{array} \qquad \begin{array}{r} 509 \text{ (minuend)} \\ - 389 \text{ (subtrahend)} \\ \hline 280 \text{ (difference)} \end{array}$$

In the first question, when subtracting the tens, 5 tens – 3 tens = 2 tens, (50 – 30 = 20) the student completes the question correctly using his/her understanding of subtracting the smaller number from the larger number. But in the second question, the student tries to use the same method (smaller number 0 subtracted from larger number 8) and they get an answer of 8 tens, which is incorrect.

At times, students forgot to regroup when adding. They often wrote a two-digit number where there should have only been one digit. Below is an example.

$$\begin{array}{r} 145 \\ + 247 \\ \hline 3812 \end{array}$$

Some students misaligned the digits when recording their calculations and computed incorrectly. Ways to address these errors or misconceptions is to focus on developing personal strategies and alternative algorithms which tend to focus on the meaning of the number, rather than on individual digits.

Multiplication

It is important when developing multiplication and division concepts that children are able to explain the connection between their models and the story problems using verbal expressions such as “groups of,” “rows of” and “jumps of”.

Early in the development of the division concept, students should encounter situations involving remainders. Also, the connection between words and symbols must be carefully developed for division. For example, when writing $12 \div 4$, we should say aloud, “How many groups of 4 are in 12?” As well, students should be familiar with both forms of the symbolic expression, and below are two examples.

$$4 \overline{)12} \qquad 12 \div 4$$

A misconception that students have about multiplication is that the product of two numbers is always greater than the sum of those two numbers. So, when they encountered expressions like 8×1 , 8×0 , and 2×2 where this did not apply, they were puzzled.

The student result of the **2016–2017 Nova Scotia Assessment Mathematics in Grade 4** shows the following data:

- 52% of the students were not able to round a 3-digit number to the nearest hundred.
- 53% of the students were not able to identify an array represented by a given number of rows and a given number of columns.
- 55% of the students were not able to round 2-digit and 3-digit numbers to the nearest hundred in a context.
- 56% of the students were not able to identify a multiplication fact represented by an array.
- 61% of the students had difficulty recognizing the place value of a number and being able to rename the same number in a variety of ways.
- 61% of the students were not able to subtract two 3-digit numbers correctly when the largest number in the subtraction question contained a zero in the tens place.

C. What are the next steps in instruction for the class and for individual students?

Students were challenged when applying estimation strategies. They knew how to round numbers in isolation but could not estimate sums and differences in context. One of the first steps in instruction is making sure that children are exposed to and understand how to estimate sums, differences, products, and quotients. Estimating allows students to predict answers, check their calculations, and ask themselves if their actual answer is reasonable. The factors that may influence estimating sums, differences, products, and quotients is the context and the numbers and operations involved.

Addition and Subtraction

Students need to learn that addition and subtraction are related. They are inverse operations; they undo each other. The basic facts of addition and subtraction do not have to be learned as separate facts. Below are some examples.

$$\begin{array}{ll} 6 + 5 = 11, \text{ so } 5 + 6 = 11 & 11 - 5 = 6, \text{ so } 11 - 6 = 5 \\ 11 = 6 + 5, \text{ so } 11 = 5 + 6 & 6 = 11 - 5, \text{ so } 5 = 11 - 6 \end{array}$$

Addition and subtraction problems can be categorized based on the kinds of relationships they represent. It is important that all the story problem structures are presented and developed from students' experiences. Initial work with the story structures will focus on join and separate types of problems because students associate the actions in these problems with the operations. However, they must also experience addition and subtraction in part-part-whole and comparison situations. Please refer to the grade level appropriate curriculum documents (*Mathematics 1*, p. 64; *Mathematics 2*, p. 68; and *Mathematics 3*, p. 71) for more information about the story structures and instructional strategies.

Students will be expected to use and describe strategies to determine sums and differences using manipulatives and visual aids. Initial strategies include

- counting on or counting backwards
- one more or one less
- making ten
- doubles
- near doubles

Other strategies are described in the curriculum documents, *Mathematics 1*, *Mathematics 2*, and *Mathematics 3*.

Manipulatives can and should be used to model not only the above strategies but also model the story structures. Examples of manipulatives that can be used for this purpose include

- two-sided counters
- linking cubes
- game materials (number cubes)
- ten-frames
- walk-on number line



Multiplication and Division

When introducing multiplication, students should be introduced to multiplication through situations (equal-group story problems) that lend themselves to modelling with sets, arrays, and linear measurement models, such as number lines. For example, 3×4 can be represented with the

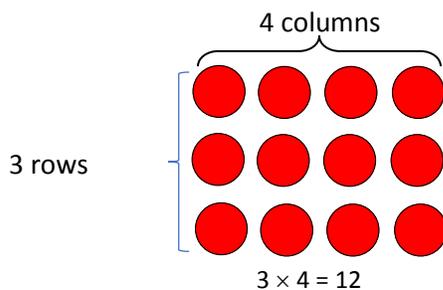
- Set Model – represent 3 groups of 4 birds (equal groups of birds) which equals 12 birds



$4 + 4 + 4 = 12$ is connected to $3 \times 4 = 12$

This example shows a repeated addition of equal groups.

- Array Model – represent 3 rows of 4 groups of counters by making 3 rows of 4 columns



The first factor 3 represents the number of rows. The second factor 4 represents the number of columns.

Note: The array on the right, of 4 rows and 3 columns, represents the multiplication for 4×3 .

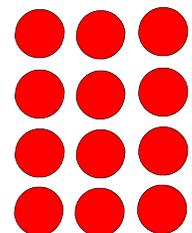
The first array represents $3 \times 4 = 12$ counters.

The second array represents $4 \times 3 = 12$ counters.

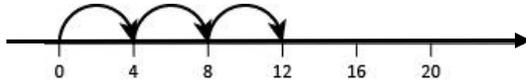
Then, $3 \times 4 = 12$ and $4 \times 3 = 12$ which is the commutativity of the multiplication.

The students thinking is incorrect if they believe that the arrays are equal

$4 \times 3 = 12$ because $3 \times 4 = 4 \times 3$.



- Linear Model – represent 3 jumps of 4 on a number line



This representation is another way to show the multiplication 3×4 as a repeated addition ($4 + 4 + 4$). This can then be connected to the multiplication equation $3 \times 4 = 12$.

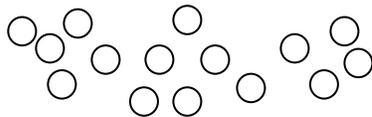
Division

Students should be introduced to division through story problems. For this instruction, there are two types of situations, equal-sharing and equal-grouping, which need to be considered. Equal-sharing problems are those in which the number of groups is known and the number in each group needs to be found. Equal-grouping problems are those in which the number in each group is known and the number of groups needs to be found. Students should also see that dividing the class into groups of 5, giving each student 4 pencils, and placing books into stacks of 4 are all examples of equal-grouping situations.

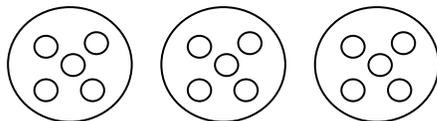
Students should see that dividing the class into two groups, sharing 12 pieces of paper with 4 students, and sharing a large bag of candy into 3 small bags are all examples of equal-sharing situations.

Below are some sample questions that represent division through story problems.

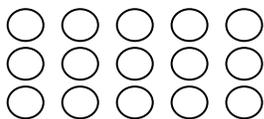
1. Three friends want to share 15 candies. How many candies will each friend get?



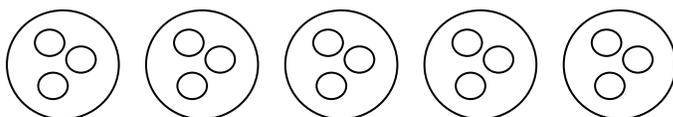
Each friend will get 5 candies. This may be described that when 15 is divided into 3 groups, there are 5 in each group.



2. Friends want to share 15 candies by each taking 3 candies. How many friends will get candies?



Five friends will get candies. This may be described that when 15 is divided into groups of 3, there are 5 groups.



Students should solve many examples of both types of division problems by modelling them concretely, recording them pictorially, and describing the division in words before they are introduced to division sentences.

D. What are the most appropriate methods and activities for assessing student learning?

Below are some sample questions related to operations with whole numbers which will be used to represent some of the appropriate methods and activities for assessing student learning.

Encourage students to share their thinking, their strategies, and their solutions. Rich discussion fosters the development of alternative problem-solving strategies.

Examples:

1. Choose the correct answer for the following addition $363 + 25$

- 308
- 388
- 618
- 5113

2. Choose the correct answer for the following subtraction $809 - 489$

- 320
- 420
- 480
- 1298

3. Forty-two students were in the gym. Twenty-six of them were girls. How many were boys?
Choose the equation that shows how to work out this problem.

26	\triangle
42	

- $42 = 26 + \triangle$
 - $26 + 42 = \triangle$
 - $\triangle - 26 = 42$
 - $26 - \triangle = 42$
4. Which number is missing in this equation? $\square = 7 + 9$
- 19
 - 16
 - 13
 - 6

5. Which number is missing in this equation $3 + \square = 12 - 6$

- 1
- 3
- 6
- 9

6. Luke has 72 stickers. Lilly has 29 stickers.
About how many more stickers does Luke have than Lilly?
Choose the best estimate.

- 100
- 90
- 40
- 30

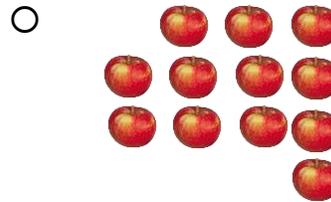
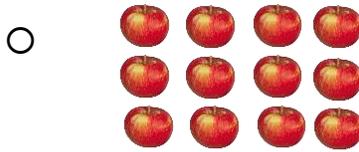
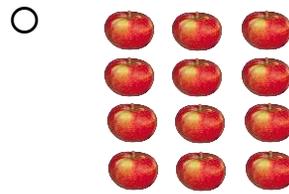
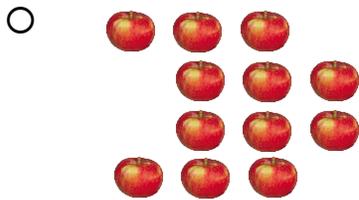
7. Marley did this subtraction.
 $675 - 346 = 329$
Which expression could help her check her work?

- $675 + 329$
- $675 + 346$
- $329 + 346$
- $346 - 329$

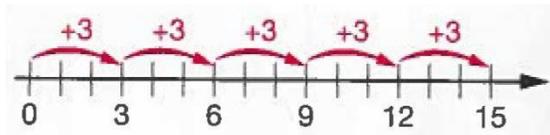
8. This array represents which multiplication fact?

- 1×8
- 2×4
- 2×8
- 4×2

9. Look at the following pictures of apples.
Which picture represents the multiplication array 4×3 ?



10. Look at the following number line:



Which operation is represented by this number line?

- 3×5
- 5×3
- $15 + 3 + 3 + 3 + 3 + 3$
- 15×3

11. Which statement about the array below is **not true**?



- The array has 2 rows and 4 columns.
- The array represents the multiplication $4 \times 2 = 8$.
- The array represents $4 + 4 = 8$ items.
- The array represents the multiplication $2 \times 4 = 8$.

12. Mila made 2 rectangular cakes. Each cake can be cut into 37 pieces.

About how many pieces of cake did Mila make?

- 40
- 70
- 80
- 90

13. Amir has 12 tomatoes. He puts the tomatoes into 3 boxes in equal groups.

Which equation below would help you determine the number of tomatoes in each box?

- $12 + 3 = 15$
- $12 - 3 = 9$
- $12 \times 3 = 36$
- $12 \div 3 = 4$

14. An apple was cut into equal pieces.

Choose the fraction that would represent the smallest piece of apple.

- $\frac{1}{4}$
- $\frac{1}{2}$
- $\frac{1}{8}$
- $\frac{1}{10}$

Mathematics in Grade 4 Lesson Learned 4 Patterns and Relations

Students were challenged when asked to transfer their visual representation of patterns to numerical patterns. Students should be able to describe either an increasing pattern or a decreasing pattern, and need to recognize that each term has a numeric value. Students seemed to forget that a pattern rule must have a starting point. Without the starting point, the pattern rule is incomplete. They need to be encouraged to provide all the information for a pattern rule. Students need to continue to work with representations of patterns, concretely, contextually, pictorially, symbolically, and verbally.

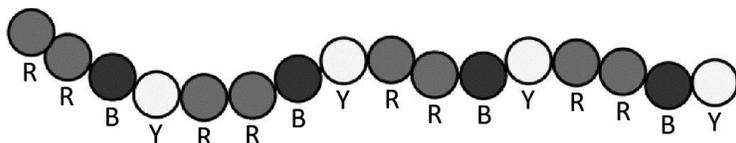
Patterns are the foundation for many mathematical concepts. Patterns should be taught throughout the year in situations that are meaningful to students. Patterns are explored in all the strands and are also developed through connections with other disciplines, such as science, social studies, English language arts, physical education, and music. Providing students with the opportunity to discover and create patterns, and then describe and extend those patterns, will result in more flexible thinking across strands and across subjects. Students should initially describe non-numerical patterns, such as shape, action, sound, and then incorporate numerical patterns by connecting them to the non-numerical patterns.

A large focus in Mathematics 3 is the introduction and development of decreasing patterns. Students use their knowledge of increasing patterns to make connections to the concept of decreasing patterns, since similar understandings are developed.

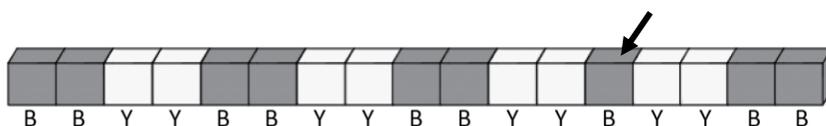
- A. What conclusions can be drawn from the Nova Scotia Assessment: Mathematics in Grade 4? We noticed that our students did very well recognizing simple errors in increasing number patterns, identifying a pattern rule used to create a given increasing pattern, and identifying the next term in an increasing pictorial pattern. These types of items were either knowledge questions or application questions. Analysis questions related to patterns challenged our students. For example, students experience difficulty when asked to create an increasing pattern in which a specific element is identified (e.g., the 7th element is 56).

Students did extremely well when the patterns that they were working with were a visual representation of a pattern. See below for examples:

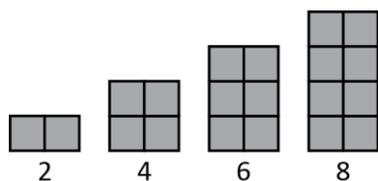
“The pattern for my bead is red (R), red (R), blue (B), yellow (Y)”



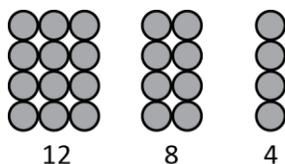
“I see a mistake in this block pattern. It needs another blue block here.”



Students were challenged when they were transferring their knowledge of visual patterns to numerical patterns. Students should be able to describe an increasing pattern made of shapes, but need to recognize that each term in the pattern also has a numeric value. For example,



This expectation also applies to decreasing patterns. For example,



B. Do students have any misconceptions or errors in their thinking?

One of the most fundamental concepts in pattern work, but also one not clear to all students, is that, although the part of the pattern that they see is finite, when mathematicians talk about a pattern, they are talking about something that continues beyond what the student sees. (Small, 2009, p.568)

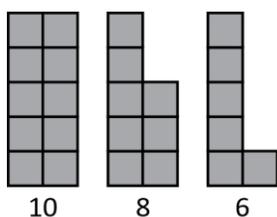
In Mathematics 2, students describe, reproduce, extend, and create repeating, increasing and, decreasing patterns. They use ordinal numbers (to tenth) to describe elements of repeating patterns. Students in Mathematics 3 explore increasing and decreasing patterns, both numerical patterns with numbers to 1000 and non-numerical patterns with concrete materials, pictures, sounds, and actions. They use ordinal numbers (to 100th) to refer to or to predict terms within a pattern.

Students have difficulty extending an increasing number pattern or a decreasing number pattern. A suggested strategy is to have students locate the numbers in the pattern on a hundred chart and place a transparent counter over each number. Have students use the visual pattern in the counters to extend the pattern. Help students relate the visual pattern to the starting point and the number added each time in the number pattern. (Pearson, 2009b, p. 14)

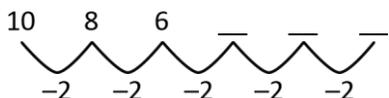
For example, when skip counting by 3, use only starting points that are multiples of 3 (3, 6, 9, 12, ...). This will result in many diagonal representations on a hundred chart. Skip counting by 5, results in a pattern that is two vertical columns with numbers ending in the digits 5 (5, 15, 25, ...) and 0 (10, 20, 30, ...). Students should also explore hundred charts to 1000 (1–100, 101–200, 201–300, ...) and look for patterns when counting by 2s, 5s, 10s, 25s, and 100s.

Another common error with decreasing patterns is that students do not extend a decreasing pattern correctly. A suggested strategy is to help students identify how each figure in the pattern differs from each previous figure.

For example, in the pattern below, the pattern rule is to start with 10 squares and decrease by 2 squares each time.



As students describe decreasing shape patterns, help them recognize that each term has a numeric value. The above pattern can be expressed as 10, 8, 6, ... by counting the number of squares in each term. Students may also find it useful to record the change from one term to the next as shown below:



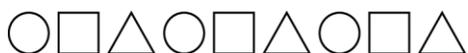
Remind students that a pattern rule must have a starting point or the pattern rule is incomplete. For example, if a student describes the pattern 10, 8, 6, ... as a decrease by 2 without indicating that the pattern starts at 10, the pattern rule is incomplete.

The student result of the 2016–2017 Nova Scotia Assessment Mathematics in Grade 4 shows the following data:

- 47% of the students were not able to determine the 23rd element of a given repeating pattern.
- 56% of the students were not able to determine a missing number in a given counting pattern.

C. What are the next steps in instruction for the class and for individual students?

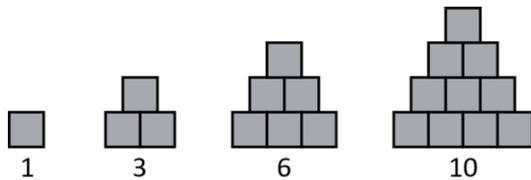
As students identify the core of a pattern, they should use appropriate patterning vocabulary, such as **core** (the repeating part of the pattern) and **elements** (the actual objects used in the pattern). It is important to create patterns that have the core repeated at least three times. To help students identify the pattern core, it is suggested students highlight, or isolate, the core each time it repeats. Remind students that repeating patterns can be extended in both directions. Encourage students to reference the position of the elements of the pattern using ordinal numbers. The core of the shape pattern below is: circle, square, triangle. There are three elements in this pattern, namely a circle, a square, and a triangle.



The pattern above is also a three-element pattern. The core of this three-element pattern is circle (1st element), square (2nd element), and triangle (3rd element).

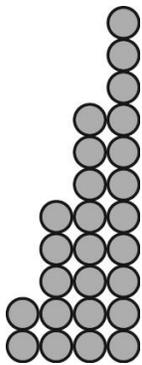
Increasing Numeric Patterns

Students should be able to describe an increasing pattern. An increasing pattern is a growing pattern where the size of the term increases in a predictable way. The terms in an increasing pattern grow by either a constant amount or by an increasing amount each time. Students need sufficient time to explore increasing patterns using various manipulatives, such as cube-a-links, tiles, toothpicks, counters, pattern blocks, base-ten blocks, ten-frames, bread tags, stickers, or buttons, to realize they increase in a predictable way. As students describe increasing shape patterns, help them recognize that each term has a numeric value. For example,

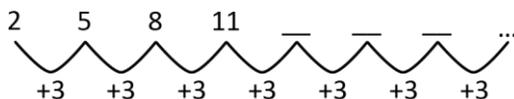


A counting sequence is an increasing pattern where each number represents a term in the pattern. For example, in the counting sequence 1, 2, 3, 4, ..., 1 represents the first term, 2 the second term, 3 the third term ... This counting sequence can be connected to ordinal numbers where students should be able to recognize that the 34th term is 34 and that 57 is the 57th term in the sequence. These ordinal number patterns should be investigated for numbers up to 100.

Students should be able to describe a given increasing pattern by stating the pattern rule. A pattern rule tells how to make the pattern and can be used to extend an increasing pattern. Give students the first three or four terms of an increasing pattern. Ask them to state the pattern rule by identifying the term that represents the starting point and describing how the pattern continues. For example, in the pattern below, the pattern rule is, start with 2 counters and add 3 counters each time.



As students describe concrete or pictorial patterns, help them recognize that each term has a numeric value. For example, the above pattern can be expressed as 2, 5, 8, 11, ... by counting the number of counters in each term. Students may also find it useful to record the change from one number to the next as shown below.



Students should be able to extend a pattern by identifying the rule, and then use the rule to build and draw the next three terms. Initially, students should replicate the first three terms with concrete materials and then extend the pattern. The use of the concrete materials allows them to make changes if necessary and to build onto one term to make the next term. Students should be able to explain why their extension follows the

pattern. It is important to note that for some patterns, there may be more than one way to extend the pattern. For example, if only one term is given, such as the third term 12, some possible solutions could be

4, 8, 12, 16, ...
3, 7, 12, 18, ...
2, 6, 12, 20, ...
6, 9, 12, 15, ...

Students need opportunities to compare numeric patterns, discussing how they are the same and how they are different. When comparing increasing patterns, compare the starting points and how each term increases. For example, one way students may address this is by using a page with four small hundred charts. Ask them to skip count starting a 0 and shade one chart by 2s, one chart by 5s, one chart by 10s, and one chart by 25s. Then, discuss the pattern rule in each chart comparing the starting points and the amount of increases.

Students should be able to create various representations of an increasing pattern that follow a given pattern rule. Initially students will create patterns with concrete materials, then pictures, and then numbers. Some students may create a simple pattern, while other students may create a more complex pattern.

Students should be able to create increasing patterns, concretely, pictorially, and symbolically, and be able to describe the pattern rule they used to create their representations. When creating increasing patterns, initially students need to choose a starting point and then decide on the amount of increase. The amount of increase may be either a constant amount or an increasing amount. Students should be able to describe their pattern by clearly explaining how it changes from one term to the next. Students may share their patterns and the strategies they used to create their pattern.

Students should have frequent experiences using increasing patterns to solve real-world problems that interest and challenge them. They should use concrete materials or pictures to model the problem before determining the pattern rule. A variety of strategies may be used to solve the problem such as using a number line, a hundred chart, a picture, concrete materials, or skip counting.

Students should be able to identify and describe the strategy used to determine a missing term in a given increasing pattern. Since patterns increase in a predictable way, to determine a missing term, students should first look at the term that comes before and after. One strategy may be to identify and use the pattern rule. Other possible strategies may include using a number line, a hundred chart, a picture, concrete materials, or skip counting.

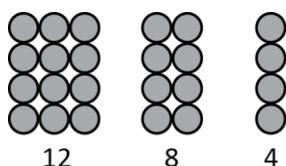
Decreasing Numeric Patterns

Students should be able to describe a decreasing pattern. A decreasing pattern is a shrinking pattern that decreases by a constant amount each time. Students need sufficient time to explore decreasing patterns using various manipulatives, such as cube-a -links, tiles, toothpicks, counters, pattern blocks, base-ten blocks, ten-frames, bread tags, stickers, and buttons. Sometimes students are more comfortable during the exploration stage if they can experiment first, using manipulatives, then pictures, and eventually numbers.

Students should be able to identify and describe various decreasing patterns such as horizontal, vertical, and diagonal patterns found on a hundred chart. Working with decreasing patterns can be connected to skip counting in outcome N01. Provide copies of hundred charts. Ask students to begin at 100 and skip count backward by a given number, shading in the number for each count all the way to 1. Then they write a description of the pattern. For example, if they chose to skip count by 10s, the pattern results in one vertical column, regardless of the starting point.

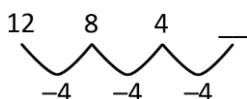
As students begin to investigate patterns, they sometimes confuse repeating patterns with decreasing patterns. Remind them to look for a core first. If they cannot find a core, then the pattern is not a repeating pattern.

Earlier, students became familiar with assigning a numeric value to each element in an increasing pattern. This expectation also applies to decreasing patterns.



Students should be able to describe a given decreasing pattern by stating the pattern rule. A pattern rule includes a term representing a starting point and a description of how the pattern continues. A pattern rule tells how to make the pattern and can be used to extend a pattern. For example, in the pattern above, the pattern rule is to start with 12 squares and decrease by 4 squares each time.

As students describe decreasing shape patterns, help them recognize that each term has a numeric value. The above pattern can be expressed as 12, 8, 4, ... by counting the number of circles in each term. Students may also find it useful to record the change from one term to the next as shown below.



Remind students that a pattern rule must have a starting point or the pattern rule is incomplete. For example, if a student describes the pattern 12, 8, 4, ... as a decrease by 4 pattern without indicating that it starts at 12, the pattern rule is incomplete.

Students need opportunities to compare numeric patterns and to discuss how they are the same and how they are different. When comparing decreasing patterns, compare the starting points and how each term decreases using a variety of representations such as shape patterns, hundred charts, and number patterns. For example, give students a page with four small hundred charts. Ask them to skip count backward starting at 100 and shade one chart by 2s, one chart by 5s, one chart by 10s, and one chart by 25s. Then discuss the pattern rule in each chart indicating the starting point and the amount of decrease.

Students should be able to create various representations of a decreasing pattern that follow a given pattern rule. Initially students will create patterns with concrete materials, then pictures, and then numbers. Some students may create a simple pattern, while other students may create a more complex pattern.

Students should be able to create decreasing patterns, concretely, pictorially, and symbolically, and be able to describe the pattern rule they used to create their representations. When creating decreasing patterns, initially students need to choose a starting point and then decide on the amount of the decrease. The amount of decrease may be either a constant amount or an amount that increases each time. Students should be able to describe their pattern by clearly explaining how it changes from one term to the next. Students may share their patterns and the strategies they used to create their pattern.

Students should have frequent experiences using decreasing patterns to solve real-world problems that interest and challenge them. They should use concrete materials or pictures to model the problem before determining the pattern rule. A variety of strategies may be used to solve the problem, such as using a number line, a hundred chart, a picture, concrete materials, or skip counting.

Students should be able to identify and describe the strategy used to determine a missing term in a given decreasing pattern. Since patterns decrease in a predictable way, to determine a missing term the students should first look at the term that comes before and after. One strategy may be to identify and use the pattern rule. Other possible strategies include using a number line, a hundred chart, a picture, concrete materials, or skip counting.

D. What are the most appropriate methods and activities for assessing student learning?

Below are some sample questions related to pattern and relations which will be used to represent some of the appropriate methods and activities for assessing student learning.

Encourage students to share their thinking, their strategies, and their solutions. Rich discussion fosters the development of alternative problem-solving strategies.

Examples:

1. What are the two missing numbers in the number pattern Below?

66, 61, 56, 51, _____, 41, 36, _____, ...

- 45 and 35
- 46 and 31
- 52 and 31
- 52 and 37

2. Natalie created the following decreasing pattern:

546, 536, 526, 516, 506, 496, ...

What is the rule for this pattern?

- Subtract 10.
- Start at 546 subtract 5 each time.
- Start at 496 and subtract 10 each time.
- Start at 546 and subtract 10 each time.

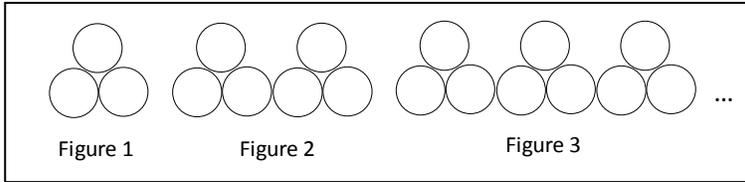
3. Monique created the following decreasing number pattern. Two numbers are missing in this pattern.

55, 50, 45, 35, 30, 25, 20, 10, 5, ...

What are the two missing numbers?

- 40 and 30
- 15 and 25
- 40 and 15
- 45 and 15

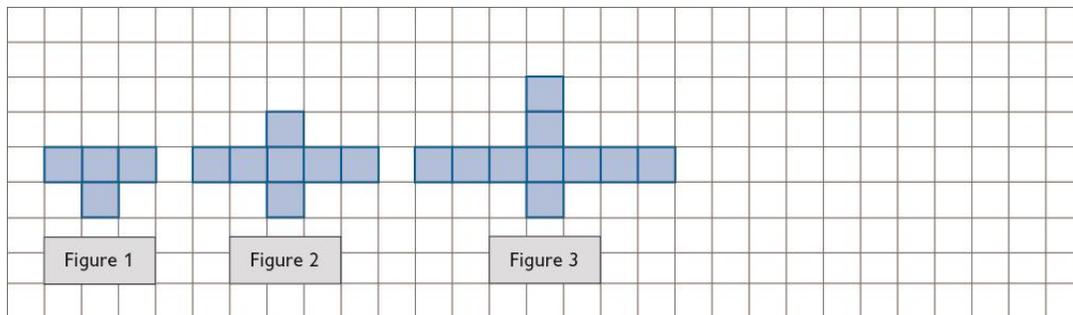
4. Simon created the following pattern.



How many small circles are there in the fifth figure?

- 15
- 12
- 10
- 5

5. Examine the following pattern:



How many squares are there in fourth figure?

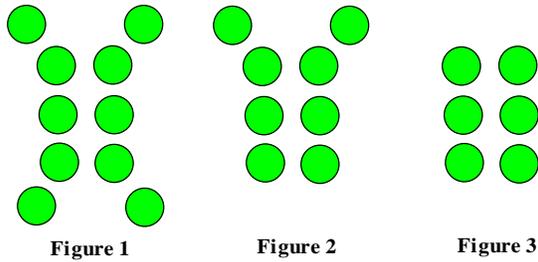
- 14 squares
- 13 squares
- 12 squares
- 11 squares

6. Which statement about the two following patterns is true?

62, 74, 86, 98, ... and 62, 50, 38, 26, ...

- They have the same starting point and increase in the same way.
- They have the same starting point and they are increasing patterns.
- They have the same starting point and they are decreasing patterns.
- They have the same starting point and they do not increase in the same way.

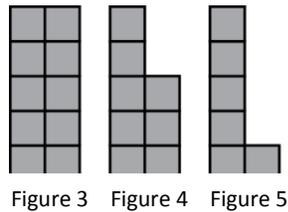
7. Examine the following pattern:



How many circles are there in the fifth figure?

- 2
- 3
- 4
- 5

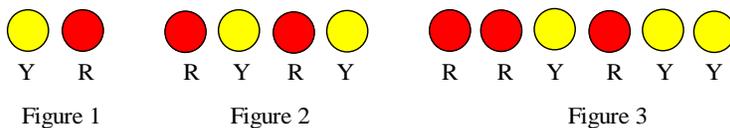
8. Examine the following pattern:



How many squares are there in the first figure?

- 1 square
- 4 squares
- 12 squares
- 14 squares

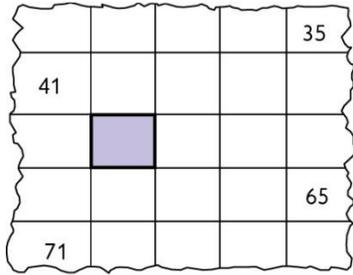
9. Marthe created the following pattern using yellow and red counters:



What is the rule for this pattern?

- From figure 1, add one yellow counter to the left and one red counter to the right each time.
- From figure 1, add two red counters to the left and two yellow counters to the right each time.
- From figure 1, add one yellow counter and one red counter to the left, and one yellow counter and one red counter to the right each time.
- From figure 1, add one red counter to the left and one yellow counter to the right each time.

10. This part of a **hundred chart** has some numbers missing.



What number belongs in the shaded box?

- 42
- 43
- 52
- 63

Mathematics in Grade 4 Lesson Learned 5 Problem Solving

Students need more exposure to application and analysis items in order to apply these higher order thinking skills when problem solving. Students need to be encouraged to understand that problems may have more than one entry point and there are many strategies to solve a problem. Students need to continue to work on translating between and among representations when problem solving. They do not seem to realize that they can use varied representations when asked to solve a word problem. These other representations may support their problem solving and their reasoning.

Learning through problem solving should be the focus of mathematics at all grade levels. Problem solving is one of the critical mathematical processes that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics. When students encounter new situations, and respond to questions such as, How would you ...? or How could you ...? the problem-solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.

In order for an activity to be problem-solving based, it must appropriately challenge students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but is simply practice. A true problem-solving activity requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement, perseverance, and collaboration. Problem solving is also a powerful teaching tool that fosters multiple, creative, and innovative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident mathematical risk takers.

Students need to be able to explore a wide variety of methods for solving and verifying problems in all areas of mathematics. They must be challenged to find multiple solutions for problems and be given opportunities to create and solve their own problems.

- A. What conclusions can be drawn from the Nova Scotia Assessment: Mathematics in Grade 4?
We found that students have a good understanding of basic facts and procedures, and are successful when explicitly given all the information needed to do a knowledge question. But when given application and analysis items, they are not able to apply higher order thinking skills when problem solving. For example, students were not sure whether they should add or subtract when questions were presented in the context of a story problem. Overall, our students are experiencing challenges with problem solving across all mathematical strands. This appeared to be a theme throughout the assessment data when students were asked to “solve a word problem” or “solve a multi-step problem”. The data also showed that, across all mathematics strands, students struggle with items that require translating among representations in problem-solving contexts.

B. Do students have any misconceptions or errors in their thinking?

It is important to distinguish between misconceptions and errors. Misconceptions are, in effect, the misunderstandings about mathematical ideas which students entertain and which usually lead to errors occurring. These are the most serious kind of errors which require urgent action on the part of the teacher. Errors may occur for a variety of underlying reasons, ranging from careless mistakes to errors resulting from misconceptions.

It is also important to mention that when students encounter a problem-solving situation, they become confused by mathematical expressions which are not familiar to them or which are difficult to understand. In addition, when students consider that a problem is a mathematical problem, they believe, wrongly, that they should associate it simply to routine calculations with few concerns to the meaning of the context and to the credibility of the answers.

Many students, when given a problem-solving task, especially a “word problem” (solving a problem in a context) have the misconception that it is always too hard for them to attempt. Putting words around the numbers seems to obstruct their ability to think about the question. Another misconception students have is that there is only one way to solve a word problem.

When asked to solve a problem in a context, they struggle to identify a possible strategy and often fail to even attempt to solve the problem. Students appeared to have a limited repertoire of problem-solving strategies to help them attempt a problem. For example, when given an analysis problem in the context of an increasing pattern, students struggle with answering “what strategy could I use to solve this problem”? There are many strategies and ways to solve the problem. Some students find it difficult to find an entry point to begin to solve the problem.

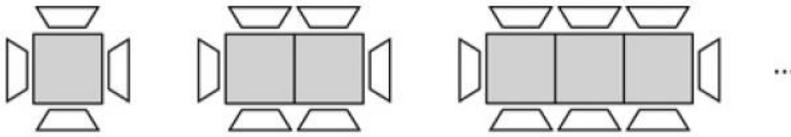
Students also tend to forget that they may translate between and among representations to help them solve a problem. If asked to solve a word problem using words, symbols and/or pictures, most students only provide symbols. They do not seem to realize that they can use all representations when asked to solve a word problem. These other representations may support their problem solving.

Problem: Some friends are coming to your birthday party. A square table with 4 chairs can seat 4 of your friends. If 2 square tables are put together, you can seat 6 of your friends. How many friends can you seat with 5 square tables?

Solution 1: What do I know and how can I use it to help me solve this problem?

- I know that one square table can seat 4 of my friends.
- I know that if 2 square tables are put together, 6 of my friends can have seats.
- So, if I continue this pattern, 3 square tables put together, can seat 8 of my friends.
- 4 square tables put together, can seat 10 of my friends, and 5 square tables put together, can seat 12 of my friends.
- So, 5 square tables can seat 12 of my friends

Solution 2: Draw a picture and a chart



Tables	Chairs	Number of Friends
1	4	4
2	6	6
3	8	8
4	10	10
5	?	?

So, 12 friends can be seated at 5 square tables when they come to my birthday

Students could have used concrete materials/manipulatives such as coloured tiles, two-sided counters, cube-a-link blocks, and pictures or numbers to solve this question. . It would be nice to add that graphic here. Showing how the problem could be represented more abstractly with tiles would help make the connection to increasing patterns.

The student results of the 2016–2017 Nova Scotia Assessment Mathematics in Grade 4 shows the following data:

- 56% of the students were not able to solve a word problem involving subtraction or division.
- 58% of the students were not able to solve a word problem involving estimations.
- 59% of the students had difficulty to solve correctly a word problem involving division.

C. What are the next steps in instruction for the class and for individual students?

A significant part of learning to solve problems is learning about the problem-solving process. It is generally accepted that the problem-solving process consists of four steps – understand the problem, devise a plan, carry out the plan, and look back to determine the reasonableness of an answer. Teachers need to teach their lessons through a problem-solving approach. Students learn mathematics as a result of solving problems. It is important to point out that not all lessons students encounter must be taught through problem solving. If the purpose of the lesson being taught is to develop a certain skill for conceptual understanding, then some practice is required.

“The teacher provides a context or reason for the learning by beginning the lesson with a problem to be solved. This approach contrasts with the more traditional approach of, for example, presenting a new procedure and then adding a couple of word problems at the end for students to solve. Instead, the teacher gives students the opportunity to think about the problem and work through the solution in a variety of ways, and only then draws the procedures out of their work.” (Small, 2005, p. 154)

An important aspect of problem solving in grades 1–3 is addition and subtraction problems which can be categorized based on the kinds of relationships they represent. It is important that all of the story problem structures are presented and developed from students’ experiences. Initial work with the story structures will focus on join and separate types of problems because students associate the actions in these problems with the operations. However, they must also experience addition and subtraction in part-part-whole and comparison situations. Please refer to the grade level appropriate curriculum documents (Mathematics 1,

p. 64; Mathematics 2, p. 68; and Mathematics 3, p. 71) for more information about the story structures and instructional strategies.

Manipulatives can and should be used to model the strategies and the story structures. Examples of manipulatives that can be used for this purpose include

- two-sided counters
- linking cubes
- number cubes
- ten-frames
- walk-on number line
- base-ten blocks

A Problem-Solving Approach

A problem-solving approach is embedded and expected throughout our curriculum, grades Primary to 12, in all strands.

As noted in Pearson (2009a), problem-solving is a key strategy: Problem-solving is a key instructional strategy that enables students to take risks, secure in the knowledge that their thoughts, queries, and ideas are valued. As students share their solutions and findings, the teacher can provide direct instruction on problem-solving strategies. After students share their solutions and justifications, teachers can elaborate on their methods and encourage students to comment or ask questions of their peers. Using student findings and solution methods to guide instruction also allows students to see the value in their work, and encourages peers to share their strategies. While some strategies may be more efficient than others, several strategies may work and often a combination of strategies is required to solve a problem. Students must use strategies that are meaningful to them and make sense to them. (p. 13)

Problem-Solving Strategies

Students are already drawing on personal strategies for problem solving, in many of the activities they undertake. *Strategies Toolkit* lessons found in the *Pearson Math Makes Sense Series*, allows teachers to expand their students' personal repertoires through explicit instruction on a specific strategy. "When students develop a name for the strategy, they develop a stronger self-awareness of the personal strategies they are starting to use on their own" (Pearson, 2009a, p. 13).

The *Strategies Toolkit* lessons highlight these problem-solving strategies (Pearson, 2009a, p. 13):

- Make a chart or table
- Draw a picture
- Work backward
- Make an organized list
- Use a model
- Solve a simpler problem
- Guess and test
- Use a pattern

Van de Walle and Lovin (2006b), in their resource, *Teaching Student-Centered Mathematics Grades 3–5*, suggest a three-part lesson format for teaching through problem-solving. This same approach is used in our core resource, *Math Makes Sense* (Pearson, 2009b, pp. 13–14):

Before

Before students begin:

- Prepare meaningful problem scenarios for students. These should be sufficiently challenging.
- Ensure the problem is understood by all.
- Explain the expectations for the process and the product.

During

As students work through the problem:

- Let students approach the problem in a way that makes sense to them.
- Listen to the conversations to observe thinking.
- Assess student understanding of her/his solution.
- Provide hints or suggestions if students are on the wrong path.
- Encourage students to test their ideas.
- Ask questions to stimulate ideas.

After

After students, have solved their problem:

- Gather for a group meeting to reflect and share.
- Make the mathematics explicit through discussion.
- Highlight the variety of answers and methods.
- Encourage students to justify their solutions.
- Encourage students to comment positively or ask questions regarding their peer's solutions.

For more details on using a problem-solving approach to teach mathematics, see Van de Walle and Lovin (2006b), *Teaching Student-Centered Mathematics Grades 3–5*, from which these ideas are drawn (pp. 11–28).

Assessing Problem Solving

For information using a rubric to score problem-solving sample questions in either Mathematics in Grade 4 (M4) or Mathematics in Grade 6 (M6), please go to the Program of Learning Assessment for Nova Scotia (PLANS) website. On the Documents tab of each assessment page, you will find a Problem-Solving document which includes the provincial rubric and sample questions (plans.ednet.ns.ca/grade4/documents).

From Reading Strategies to Mathematics Strategies

With a problem-solving approach embedded and expected throughout our curriculum grades Primary to 12 in all strands, there are definite implications for teaching reading strategies in mathematics. Concepts and skills should also be connected to everyday situations and other curricular areas. For example, when developing literacy skills students learn to make text-to-world, text-to-text, and text-to-self connections. Students can also make connections to make mathematics come alive through math-to-world, math-to-math, and math-to-self connections.

There are classroom strategies with suggestions of how to use the strategy in a mathematical context that support teachers as they develop students' mathematical vocabulary, initiate effective ways to navigate informational text, and encourage students to reflect on what they have learned. For example, the Frayer Model, Concept Circles, Three-Read Strategy, Exit Cards, etc. These are only a few of the strategies that are found in Appendix B at the end of this document.

When teachers use these strategies in the instructional process or embed them in assessment tasks, the expectations for students must be made explicit. The students' understanding of the mathematics involved and maintaining the integrity of the curriculum are still the foremost concerns.

Please refer to the [Appendix B](#) found at the end of this document for strategies with illustrative examples.

D. What are the most appropriate methods and activities for assessing student learning?

Below are some sample questions related to problem-solving which will be used to represent some of the appropriate methods and activities for assessing student learning.

Encourage students to share their thinking, their strategies, and their solutions. Rich discussion fosters the development of alternative problem-solving strategies.

Examples:

1. Natasha has 4 T-shirts and 2 pairs of pants.
How many different outfits can Natasha make?
Show how you solved the problem and explain your strategy.

Possible Strategy: Make a chart, table or draw a picture; Strand: Number/Stats and Probability; Application Question

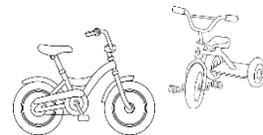
2. Megan and Danielle each ordered the same size pizza.
Megan asked to have her pizza cut into fourths. Danielle asked to have her pizza cut into sixths.
Who has the larger pieces of pizza?
Show how you solved the problem and explain your strategy.

Possible Strategy: Use a model, draw a picture; Strand: Number; Application Question

3. Peter has 123 marbles. He gives some marbles to his friend Paul. Now Peter has 86 marbles.
How many marbles did Peter give to Paul?
Show how you solved the problem and explain your strategy.

Possible Strategy: Use a model; Strand: Number; Application Question

4. Sebastian was at a bicycle sale.
There were bicycles and tricycles.
Altogether, there are 21 wheels.
How many bicycles and tricycles are there?
Show how you solved the problem and explain your strategy.



Possible Strategy: Work backward; Strand: Number; Analysis Question

5. Susan has 23 apples.
She eats three apples every day.
How many apples did Susan have left after seven days?
Show how you solved the problem and explain your strategy.

Possible Strategy: Use a pattern, *make a list*, *use a model*; Strand: Patterns and Relations; Analysis Question

6. Marbles come in packages of 10, 25, and 50.
You need 160 marbles.
Find 5 ways you could buy the marbles.
Show how you solved the problem and explain your strategy.

Possible Strategy: *Make an organized list*; Strand: Number; Application Question

7. For James birthday, his mother wants to cover each table using paper tablecloth which is 4 m long.
How many tables can she cover using a roll of table cloth which is 23 m long?
Show how you solved the problem and explain your strategy.

Possible Strategy: *Make an organized list*, *work backward*; Strand: Number; Analysis Question

8. Marie's height is 3 cm more than Norman's height.
Norman's height is 2 cm more than Jessica's height.
If Marie's height is 126 cm, what is Jessica's height?
Show how you solved the problem and explain your strategy.

Possible Strategy: *Draw a picture*; Strand: Measurement; Analysis Question

9. Lucy has some money. She has some 5 cent coins, some 10 cent coins and 25 cent coins.
She buys a used book for 45 cents.
She used all her money to buy this book and now has no money left.
How many different ways could Lucy pay for this book using all of her money?
Show how you solved the problem and explain your strategy.

Possible Strategy: *Draw a picture*, *draw a chart*; Strand: Number; Analysis Question

Mathematics in Grade 4 Lesson Learned 6

Measurement

Students are expected to build conceptual understanding of what it means to measure with a ruler. Emphasis should be placed on counting the intervals between the numbers, rather than looking at the number on the ruler that is aligned with the end of the object. Students also need to use a ruler to measure the length of a pencil or other objects with and without using zero as the starting point. Students need to recognize which mass unit (gram or kilogram) is appropriate for measuring and comparing the mass of a specific item. Students need to be able to explain that perimeter is the distance around a shape or the length of the boundary of an enclosed region. Students need to find the perimeter of many different regular, irregular, and composite shapes, before being introduced to questions in pictorial form. Students need to work with perimeter in application and analysis questions.

A. What conclusions can be drawn from the Nova Scotia Assessment: Mathematics in Grade 4?

We noticed that students did very well when presented with questions related to direct measure, time referents, and approximate measure when using personal referents. For example, students were able to identify how many centimetres are in one metre (Knowledge question). They were also able to solve a given problem involving the number of seconds in a minute, the number of minutes in an hour, and the number of hours in a day (Knowledge question).

Students were able to use their personal referents for 1 g and 1 kg to estimate the mass of common objects, such as a bag of sugar or a paper clip (Application question). Students also did well when estimating the length or height of an object using personal referents. For example, students used the height of a doorknob from the floor as a personal referent for 1 m (Application question).

Having these personal referents helps students visualize measurements and estimate more accurately. Personal referents also help students identify the units required for specific measurements.

A big idea developed in Grade 3 is perimeter. Students appeared to not understand the concept of perimeter even though a definition was given in parentheses in a question on the assessment. Students need to be able to explain that perimeter is the distance around a shape or the length of the boundary of an enclosed region.

B. Do students have any misconceptions or errors in their thinking?

A common error that many students make is the placement of the ruler when measuring an object. This may indicate that they do not understand that they are counting the intervals between the numbers to determine length. Some students fail to consider the gap between the end of the ruler and the zero mark. Other students may begin to measure at other points on a ruler, other than the 0 cm mark when they first start to use a standard measurement tool such as a centimetre ruler. Some students ignore the 0 cm mark and begin at the 1 cm mark. This may indicate that students do not realize that the scale of the ruler begins at 0 cm, especially if it is not labelled at the beginning of the ruler. Note however, that some students may start at 1 on the ruler but still use the ruler accurately by taking this into consideration as shown in the picture found below. The pencil being measured starting at the 0 cm mark measures 4 cm. The pencil being measured starting at 1 cm and ends at 5 cm, still measures 4 cm.



Students have misconceptions related to estimating mass and comparing the kilogram and the gram. Students need to recognize which mass unit (kilogram or gram) is appropriate for measuring and comparing the mass of a specific item. Students have difficulty matching an item with its estimated mass. When you measure how heavy an object is, you're measuring its mass. Students may not understand that mass and size are not necessarily related. Different sized objects can have the same mass, while objects of the same size can have different masses.

Another common misconception that students have is related to perimeter and area. Some students have the misconception that perimeter and area of an object are always the same measures. Students should recognize that area and perimeter are independent of one another. Students often do not make the distinction between area and perimeter and do not understand when to use each one in a problem. They may calculate the area instead of the perimeter or vice versa. Others think that perimeter and area are interchangeable. This is seen when students are finding the perimeter of many different regular, irregular, and composite shapes drawn on 1 cm paper. They find it difficult to count the units along the perimeter.

The student result of the 2016–2017 Nova Scotia Assessment Mathematics in Grade 4 shows the following data:

- 64% of the students were not able to determine the perimeter of a regular hexagon.
- 73% of the students were not able to determine the length of a pen using a ruler.

C. What are the next steps in instruction for the class and for individual students?

Standard Units: Working with standard units is integral to students' understanding of measurement. Students may start using standard units to measure length when they realize that non-standard units mean different things to different people. For example, if someone says a book is 15 cm long, everyone knows how long that is, but to say the book is 15 cards long would be more difficult to interpret. Students need to develop a familiarity with standard units and explore the relationship between them.

Students begin to use a standard tool to measure length in Mathematics 3. They learn about two basic standard units of length—centimetre and metre. It is valuable to initially use simpler rulers that are created by the students. Then, they should move on to tools that are easy for them to read. Students should use rulers (or the side of the ruler) that show only numbered centimetres and not millimetres. Students should identify objects from around the classroom that would be an appropriate referent for a centimetre or a metre; for example, a pencil, a garbage can, a teacher's desk, or a glue stick.

Measuring using a Ruler: When students begin to use rulers to measure, it is important for students to line up the 0 mark with one end of the shape being measured. Emphasis should be placed on counting the intervals between the numbers, rather than looking at the number on the ruler that is aligned with the end of the object. Lining up small cubes from base-ten materials along the ruler will demonstrate that the numbers on the ruler correspond to the number of small cubes, starting at 0. Observe how students use a ruler to measure a shape that is longer than the ruler. Show students how to measure something that is longer than a ruler by marking, recording, and starting again.

Using a centimetre ruler, students should measure the length, width, or height of a given 3-D object in the classroom, such as a lunch box, their desk, or a cereal box. Students can record their measurements using both the number and the measurement unit; for example, 3 cm or 3 centimetres. Ensure students are clear about the distance they should be measuring.

Instruction should continue to support students in learning to use rulers accurately. A strategy often used to help students is to break up plastic rulers. Give a piece of broken ruler to each student and ask them to measure items in the classroom. Observe how they attempt to measure items, especially if there is not a familiar starting point for them, such as 0 cm or 1 cm on their piece of ruler.

Another important strategy to help students learn to measure accurately is to encourage them to estimate measurements before actually verifying them using a measurement tool.

Mass and Weight: The terms mass and weight are similar, but they are not the same. Weight measures how heavy an object is and depends upon gravity, so it will vary with height above sea level. On the other hand, mass measures the amount of matter in an object and will be the same at all heights above sea level. As with all measurement units, it is important that students have a personal referent for a gram and a kilogram. Students should recognize which mass unit (gram or kilogram) is appropriate for measuring the mass of a specific item. It is helpful for students to investigate how everyday items, such as food items, are measured. Include items that are small and dense, such as a golf ball, as well as those that are large and hollow or porous, such as a beach ball. Students need to understand that grams are used to measure very light objects and kilograms are more appropriate units for heavier objects.

Using their understanding of a kilogram, ask students to brainstorm items that may have a mass of 1 gram. They may also use a small base-ten cube as a personal referent of a gram. You may wish to provide students with items such as a raisin, bean seed, jelly bean, or a paper clip, to conceptualize the sense of how a gram feels.

It would be beneficial for students to have an opportunity to make a kilogram mass of their own. Provide students with materials such as sand, flour, sugar, and small cubes from base-ten materials to fill a container until it exactly balances with a 1 kg mass on a balance scale. Using this kilogram container, they can now compare its mass to items in the classroom to help them find a personal referent for 1 kg.

Using objects from the classroom, for example a counter, a raisin, a paper clip, a textbook, a sneaker, or a lunch box, ask students to identify whether the object is an appropriate referent for a gram or a kilogram. Once students have established a personal referent for 1 g and 1 kg, they can now use their referents to estimate the mass of common objects such as an eraser, an apple, a juice box, or a textbook, or to estimate whether an object is heavier or lighter than 1 kg.

Estimating mass is more difficult than estimating other measures, as the object's size and shape is not directly related to its mass. A strategy often used to help students with this misconception is to encourage them to use the known mass of one object to estimate the mass of another. For example, a tennis ball and an orange are about the same size, but the orange is much heavier than a tennis ball.

It is important for students to know that 1000 grams is equal to a kilogram. Model for students how 1000 g is equal to 1 kg using a balance scale; use food items of various benchmark masses, such as 2 bags of 500 g, 4 boxes of 250 g, or a pre-counted bag of 1000 jellybeans.

Perimeter: Students learn perimeter best when they can make connections to real life examples. Ideas include fencing a yard, and actually measuring the perimeter of the classroom or gym using metre sticks.

It is important that students have many opportunities to find the perimeter of many different regular, irregular, and shapes concretely before being introduced to pictorial forms. Pentominoes may be used to illustrate measuring and recording the perimeter of a given composite shape. Pentominoes are shapes each made up of five squares, all of which must have at least one side matching up with the side of another.

In addition to composite shapes with straight sides, it is important to expose students to other shapes such as their handprint. Working with a partner, ask students to trace around their closed hand. Using string they can outline their handprint and then cut the string to determine the perimeter of their handprint by measuring the length of the string with their ruler. Again, students need to be able to explain the strategies they used for finding perimeter as they proceed.

In the early grades, students should explore their own methods for determining the perimeter of a shape and should not develop or follow a formula for calculating perimeter. For example, provide students with various regular and irregular polygons (e.g., squares, rectangles, and triangles), some string, and a ruler. Small groups of students should be asked to find the perimeter of the various regular and irregular polygons in a variety of ways. Some may use the string, while others go directly to measuring the sides with the ruler.

Students need to draw more than one shape for the same given perimeter. Students may use a geo-board or centimetre grid paper to explore various shapes with the same perimeter. They may explore various rectangles before exploring other shapes.

Students should be given opportunities to construct shapes of a given perimeter. Discuss with students that when constructing shapes for a given perimeter, they must remember that their shapes should be completely enclosed. It would be easier for students to begin their constructions drawing rectangles using centimetre grid paper and horizontal and vertical lines only.

D. What are the most appropriate methods and activities for assessing student learning?

Below are some sample questions related to measurement which will be used to represent some of the appropriate methods and activities for assessing student learning.

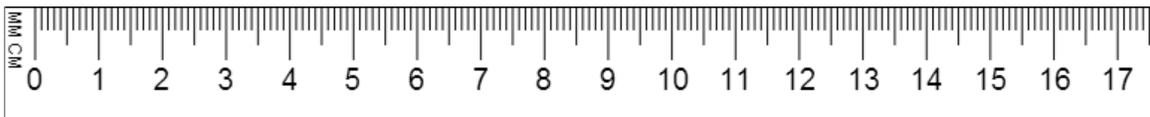
Encourage students to share their thinking, their strategies, and their solutions. Rich discussion fosters the development of alternative problem-solving strategies.

Examples:

1. What is the best unit to choose to measure the length of a season?

- Days
- Weeks
- Months

2. What is the length of the pencil below?



- 4 cm
- 11 cm
- 12 cm
- 15 cm

3. The width of your thumb is about

- 1 cm
- 10 cm
- 100 cm
- 1 m



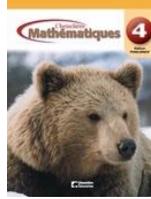
4. The mass of a box of table salt is about

- 1 g
- 100 g
- 1 kg
- 100 kg



5. The mass of your mathematics book is about

- 10 g
- 100 g
- 200 g
- 1 000 g



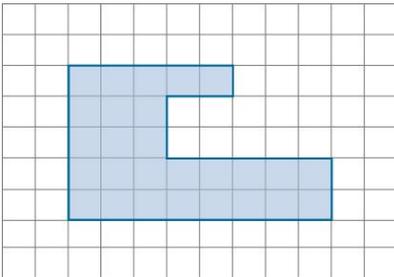
6. Which of the following statements is **true**:

- My chair is lighter than my marker.
- The mass of an orange is about 200 grams.
- The length of my arm is about 5 m.
- My pencil is longer than my mathematics book.

7. A garden is in the shape of a regular hexagon. One side of this hexagon is 2 m long. What is the perimeter of this garden?

- 12 m
- 10 m
- 8 m
- 6 m

8. What is the perimeter of the figure below? It is drawn on the 1 cm grid paper.



- 37 cm
- 30 cm
- 29 cm
- 28 cm

9. How many rectangles can you draw that have a perimeter of 16 cm each?

- 4
- 3
- 2
- 1

Mathematics in Grade 4 Lesson Learned 7

Geometry

Students need to continue developing their knowledge of 2-D shapes and 3-D objects by describing and sorting them according to their geometric attributes. Students need experiences to identify the faces, edges, and vertices of 3-D objects including spheres, cones, cylinders, pyramids, cubes and other prisms. Students need to be provided with opportunities to explore these attributes through sorting and constructing activities. Students need to extend their knowledge of both regular and irregular polygons. They need to focus on comparing the number of sides as the key attribute for classifying polygons. Students need more experiences with regular and irregular polygons, so that they begin to realize that a polygon, regardless of its dimensions, or position in space, remains the same shape.

- A. What conclusions can be drawn from the Nova Scotia Assessment: Mathematics in Grade 4? Students did very well when presented with questions related to 3-D objects, such as cylinders and spheres, found in their everyday life. They were able to identify the object being described. These were all application questions.

In the 2015–2016 Item Description Report, the following descriptions describe the questions with which students had difficulty

- determining how many vertices a pyramid has
- identifying an irregular pentagon in a composite shape
- sorting 3-D objects according to a given sorting rule
- choosing the statement that is false concerning the number of faces, vertices, and edges on a 3-D object
- determining the sorting rule for two sets of polygons

Many of these questions were analysis questions.

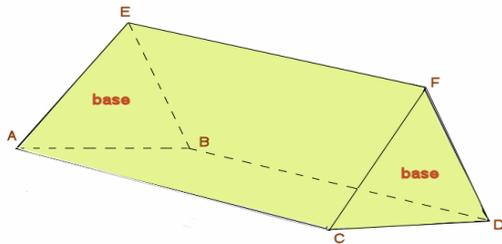
Students need to continue to develop their knowledge of geometry by describing and sorting 3-D objects according to their geometric attributes. Students require more experiences to identify the faces, edges, and vertices of 3-D objects including spheres, cones, cylinders, pyramids, and cubes and other prisms.

Students need to extend their knowledge to include both regular and irregular polygons. They need to focus on comparing the number of sides as the key attribute for classifying polygons. Students need be able to name the specific polygons including the triangle, quadrilateral, pentagon, hexagon, and octagon.

B. Do students have any misconceptions or errors in their thinking?

A common misconception that many students make is they confuse faces, edges, or vertices on a geometric object. They have difficulty distinguishing what determines the faces, edges, or vertices of an object.

Another common error or misconception that students have is understanding that the base of a geometric object is whatever face it is sitting on. A pyramid or prism is named according to the shape of its base. A pyramid has one base and triangular faces. A prism has two matching bases that are polygons. The other faces are rectangles.



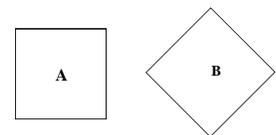
The two congruent triangles ABE and CDF are the two bases. The rectangles ABDC, ACFE and BDFE are the lateral faces.

When students are working with 3-D objects such as cylinders, cones, and spheres, they consider the rounded surfaces on these 3-D objects to be faces. This is an error or misconception. Another important concept for students to understand is that an edge occurs where two surfaces of a 3-D object meet. A vertex (vertices) is a point where three or more edges meet. **(Note: On a cone and a pyramid the highest point above the base is called the apex. In a pyramid the apex is also a vertex, but for a cone, it is a mistake to refer to the apex as a vertex as there are no edges that meet.)** As they become more familiar with identifying the attributes, students can determine the number of faces, edges, and vertices. Students will use informal language at this stage rather than precise mathematical language.

Although pattern blocks are frequently used for geometric inquiry, most of these shapes are regular. Students may develop the misconception that only certain familiar polygons meet the criteria for these shapes. For example, students may not initially recognize all of the shapes below as hexagons.



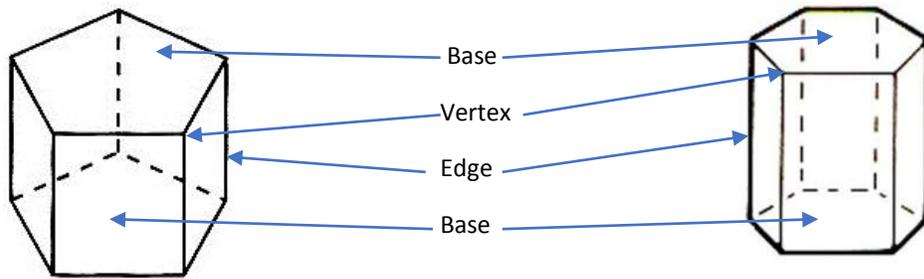
Some students incorrectly believe that the orientation of a geometric figure, changes the figure itself. Students recognize that shape A is square, but think that shape B is not a square.



The student results of the 2016–2017 Nova Scotia Assessment Mathematics in Grade 4 shows the following data:

- 44% of the students were not able to correctly determine the number of vertices of a regular hexagonal pyramid.
- 54% of the students had difficulty recognizing a 3-D object described by its attribute (number of vertices, edges and faces).
- 57% of the students were not able to determine the sorting rule of polygons.
- 58% of the students were not able to recognize a polygon made by 2 geometric shapes (a triangle and a trapezoid).
- 74% of the students struggled when asked to sort a set of 3-D objects containing prisms and pyramids.

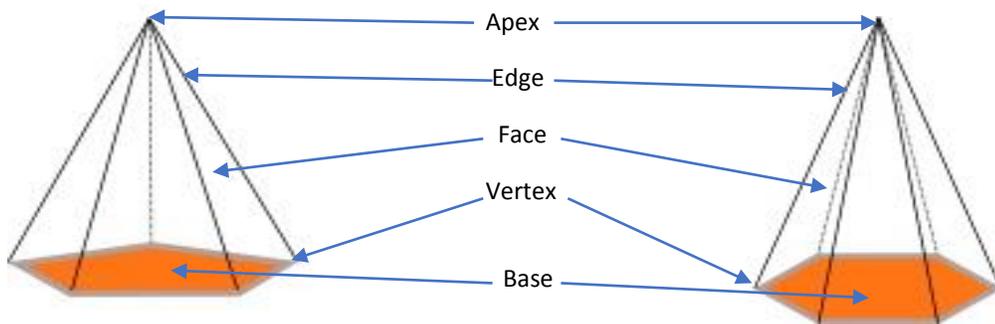
C. What are the next steps in instruction for the class and for individual students?
 Students should be able to identify the faces, edges, and vertices as well as the shape of the faces of a given 3-D object.



This pentagonal prism has 7 faces (5 rectangular lateral faces and 2 pentagonal faces which are the 2 bases), 15 edges and 10 vertices.

This hexagonal prism has 8 faces (6 rectangular lateral faces and 2 hexagonal faces which are the 2 bases), 18 edges and 12 vertices.

Note: The lateral faces of a prism are **rectangles**. The two bases, that are also faces, are **polygons**. A prism is named by its bases.



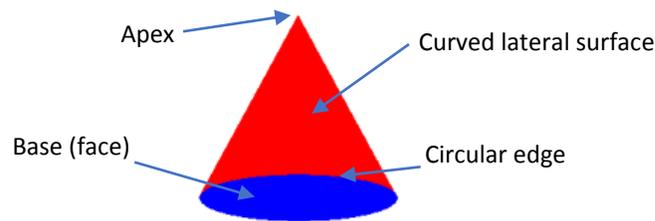
This pentagonal pyramid has 6 faces (5 lateral triangular and 1 pentagonal face, the base), 10 edges, 6 vertices (5 base vertices and 1 vertex or apex).

This hexagonal pyramid has 7 faces (6 lateral triangular and 1 hexagonal face, the base), 12 edges, 7 vertices (6 base vertices and 1 vertex or apex).

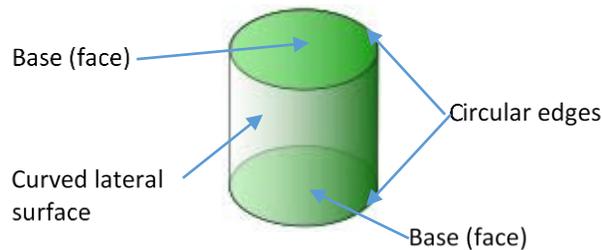
Note: The lateral faces of a pyramid are **triangles**. The base, that is also a face, is a **polygon**. A pyramid is named by its bases.

It is essential that students explore the attributes of a cone, a cylinder, and a sphere using manipulatives and concrete geometric materials. They must visualize these 3-D objects in order to understand their attributes.

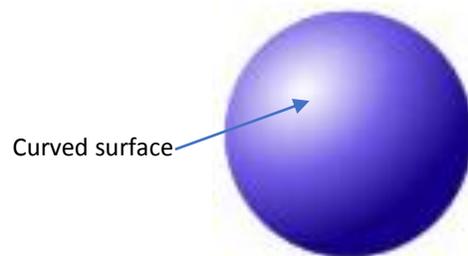
A cone has a circular flat face which is its base, a lateral curved surface starting from the base to the apex and a circular edge.



A cylinder has two circular flat faces which are its bases, a lateral curved surface joining the two bases and two circular edges.



A sphere has a curved surface. It does not have faces, edges or vertices.



Note: The curved surface of a cone, a cylinder or a sphere is not a face.

A suggested strategy to help students identify faces, edges, and vertices is to put a different colour of a small ball of modelling clay on each face, edge or vertex as they count. This should help students describe 3-D objects according to the shape of the faces and the number of edges and vertices.

A pyramid or prism is named according to the shape of its base. A strategy that may help with identifying the base of a geometric object is to remember that the base of a pyramid may not be a triangle. For a rectangular prism, any of the faces can be the base.

When working with 3-objects such as cylinders, cones, and spheres, students consider the rounded surfaces on these 3-D objects to be faces. The strategy suggested to help with this error or misconception is to help students with the conceptual understanding that a face is a flat surface on geometric objects.

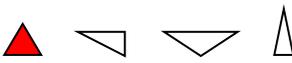
Show students models and real-life objects of cylinders, cones, and spheres. Ask students what the difference is between these solids and the prisms and pyramids already studied. Show students the faces, edges, and vertices of each solid. Brainstorm, with the students, what each term means.

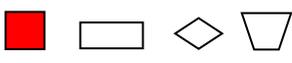
Students should be able to determine the number of faces, edges, and vertices of a given 3-D object.

- A cylinder is a 3-D object with 2 faces, 1 curved surface, 2 edges, and 0 vertices.
- A cone is a 3-D object with 1 face, 1 curved surface, 1 edge, and 1 apex.
- A sphere is a 3-D object with 1 curved surface, 0 faces, 0 edges, and 0 vertices.

Students should compare and sort 3-D objects by observing the number of faces, edges, and vertices. A student may sort objects in various ways, such as those that have all square faces, those that have circular faces, those that have 8 vertices, or those that have straight edges. Students should play games with their peers in which they sort objects and ask their peers to guess the sorting rule according to the number of faces, edges, and vertices.

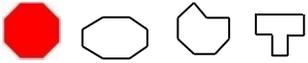
Students should focus on comparing the number of sides as the key attribute for classifying polygons. Students should be able to name the specific polygons—triangle, quadrilateral, pentagon, hexagon, and octagon. In the diagram below, the shaded polygons are regular polygons, and all others are irregular polygons.

3 straight sides: triangles 

4 straight sides: quadrilaterals  Regular polygons are shaded.

5 straight sides: pentagons 

6 straight sides: hexagons 

8 straight sides: octagons 

Students should be given opportunities to explore both regular and irregular polygons while varying the positions and dimensions of the shapes. Provide students with a polygon to trace as they experiment with different positions by turning (rotating), flipping (reflecting), and sliding (translating). Through many experiences with identifying polygons in a variety of positions, students should begin to realize that a polygon, regardless of its position, remains the same shape.

Provide students with various sizes of a particular polygon. Have students count the number of sides and identify the polygon. Having a variety of these experiences with different polygons, students should begin to realize that a polygon, regardless of its dimensions, remains the same shape.

Students should find examples of polygons in the world around them, perhaps even collect as many types of a shape as they can find. Sort the shapes according to the number of sides. By sorting polygons according to the number of sides, students can learn the names for the polygons. The following are examples:



D. What are the most appropriate methods and activities for assessing student learning?

Below are some sample questions related to geometry which will be used to represent some of the appropriate methods and activities for assessing student learning.

Encourage students to share their thinking, their strategies, and their solutions. Rich discussion fosters the development of alternative problem-solving strategies.

Examples:

1. I am 3-D object. I have 5 faces, 8 edges and 5 vertices.

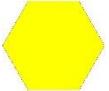
Which object am I?

- A rectangular prism
- A triangular pyramid
- A square prism
- A square pyramid

2. Which figure is **not** a polygon?

- 
- 
- 
- 

3. Which figure is a quadrilateral?

- 
- 
- 
- 

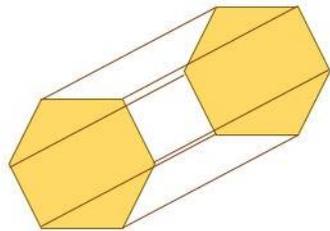
4. Look at the following pyramid:



Which statement is **true**?

- The pyramid has 9 faces, 9 edges and 9 vertices.
- The pyramid has a hexagonal base and 8 triangular faces.
- The pyramid has 8 triangular faces, 8 edges and 8 vertices.
- The pyramid has an octagonal base and 16 edges.

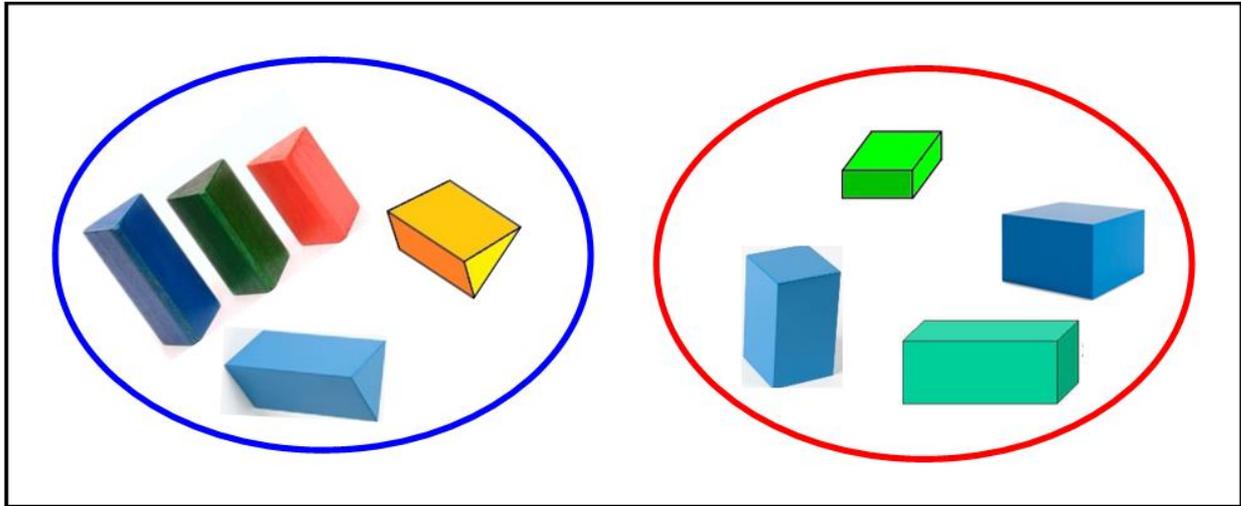
5. Look at the following prism:



Which statement is **true**?

- The prism has 8 faces, 18 edges and 12 vertices.
- The prism has 6 faces, 18 edges and 12 vertices.
- The prism has bases and 8 rectangular faces.
- The prism has an octagonal base, 8 faces and 12 vertices.

6. Robert has sorted the following prisms using a Venn diagram:

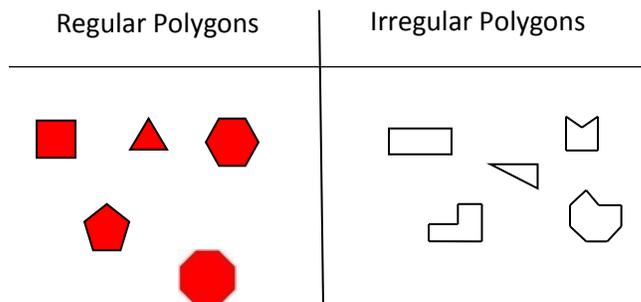


Which sorting rule did he use?

- Prisms that have 6 faces and prisms that have more than 6 faces.
- Prisms that have 6 edges and prisms that have 6 faces.
- Triangular prisms and rectangular prisms.
- Prisms which have 6 vertices and prisms which have 8 edges.

7. Lucie sorted these polygons.

What sorting rule did Lucie use?



- Polygons that have 4 sides and polygons that have 5 sides.
- Polygons that are regular and polygons that are irregular.
- Polygons that have 4 sides and polygons that have more than 4 sides.
- Polygons that have 3 sides and polygons that have more than 5 sides.

8. Which object resembles a cone?



Mathematics in Grade 4 Lesson Learned 8

Statistics and Probability

Students were challenged using tally marks, lists, charts, line plots, and bar graphs to organize data relevant to their everyday life. Students need opportunities and experiences to interpret information collected, organized, and displayed in tally charts, charts, line plots and bar graphs. Students need to develop the skill of interpreting graphs, and answering questions and to draw conclusions from those tally charts, line plots and bar graphs. They need to be encouraged to ask or write questions that go beyond simplistic reading of a graph. Both literal and inferential comprehension questions need to be asked.

- A. What conclusions can be drawn from the Nova Scotia Assessment: Mathematics in Grade 4? Students did very well interpreting data from line plots. Line plots contain information that can be used to answer questions using a visual comparison, counting, and reading labels. It offers students a visual comparison of the different quantities of every piece of data. The cognitive level of these questions is application and analysis.

We noticed that students did very well when interpreting the data from a vertical bar graph when asked to answer a straight forward literal comprehension question about the data. They understood that they had to add all of the data together to find the correct answer. This was an application question.

Students had difficulty when asked to interpret a table showing the results of a survey using tally marks. They understood the concept of the tally table and what it represented, but could not interpret what they were being asked to answer for the data displayed. They did not realize that to answer the question, they had to compare two quantities and then subtract the smallest quantity from the largest in order to determine their answer. This was a knowledge question.

Students also had difficulty when asked to interpret a question displaying information in a bar graph. Again, they understood the concept of the bar graph and what it represented, but they did not realize that in order to answer the question, they had to perform an operation of addition with all the information given in each bar being displayed. This was an application question.

Students found it difficult to draw conclusions by comparing data represented in different types of displays. They were asked to draw a conclusion concerning the common attributes of line plots, horizontal bar graphs, pictographs, and vertical bar graphs with the same given set of data. They could not determine which data display did not represent the data correctly.

- B. Do students have any misconceptions or errors in their thinking?

Although, students did well when working with line plots, they did make errors when reading or counting the Xs on a line plot.

The difficulty with bar graphs, whether horizontal or vertical, is that students interpret data from the wrong bar on the graph. Therefore, when asked to answer questions concerning the bar graph data, it is not correct.

A misconception or error that students have is concerning the common attributes of line plots, horizontal bar graphs, pictographs, and vertical bar graphs with the same given set of data. They should notice that the attributes that are common include the title, the labels, the horizontal axis, and the use of dots or crosses. They should also notice that the attributes can differ; for example, there could be different titles, different use of the horizontal axis, and different labels.

The student result of the 2016–2017 Nova Scotia Assessment Mathematics in Grade 4 shows the following data:

- 52% of the students were not able to interpret a bar graph.
- 54% of the students had difficulty to recognize the graph that represents a given set of data incorrectly.
- 66% of the students had difficulty to recognize the attributes of a bar graph.

C. What are the next steps in instruction for the class and for individual students?

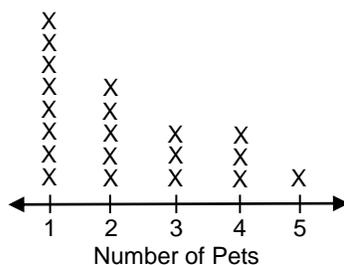
Students should be encouraged to collect, organize, and record their data using a tally system, line plots, charts, and lists to answer questions relevant to their everyday life. Using tally marks is a simple way for students to keep track of information as they collect it. Lists are a way for students to record the objects collected. A list can be made into a chart on which students would record their tally marks. Grouping the tally marks in fives makes it easier for students to total the numbers in each category by skip counting. When making a chart, students should always give it a title or heading to inform the reader about the meaning of the data. Students could then organize the data on line plots.

A line plot provides a bridge from tally charts to bar graphs. At first, students should create their line plots using grid paper, with one dot or cross per grid paper square. The dots or crosses are placed one above the other for each tally mark for each item in the list or chart. Students should then count and write the total number of Xs under each column shown on the line plot. Students should notice that the attributes that are common include the title, the labels, the horizontal axis, and the use of dots or crosses. They should also notice that the common attributes can differ; for example, there could be different titles, different use of the horizontal axis, and different labels. They might also notice that when a line plot does not have a title as in the following picture, it is hard to make sense of the graph.

Tally system

Number of Pets	Number of Students
1	TTTT III
2	TTTT
3	III
4	III
5	I

Line plot



Chart/List

Number of Pets	Number of Students
1	8
2	5
3	3
4	3
5	1

After a display of the data is constructed, discussing the information that can be obtained from the display is a valuable exercise. Students should work together to formulate questions that can be answered by other students using the data in the line plot, chart, or list. For example, from a line plot that displays the number of letters in your last name, students might formulate questions such as, What is the most common number of letters in a name? How many letters does the longest name in the class have? Shortest?

A good graph should communicate some overall impressions of the data to a reader. Students should be able to answer questions using the display of the collected data. It is also important that each graph accurately represents the data and includes clear labelling and a title.

Present students with vertical and horizontal bar graphs that represent two different sets of data. Discuss what common attributes the two bar graphs have, such as title, axes, and labels for the axes, numerical scale, and bars. Discuss how the two bar graphs are different; for example, the titles of the graphs, labels for the axes, lengths and widths and spacing of the bars, and how some graphs have horizontal bars and others have vertical bars.

Once data has been collected, it should be organized and displayed so that questions can be asked and answered. Students have made line plots that have common attributes and the appearance of a bar graph. Students can discuss how they can change their line plots into bar graphs. As they create their bar graphs, have them check to see if all the attributes of the bar graph are visible.

Although students may be able to create bar graphs, they have trouble matching the length of the bars to the numbers on the scale, whether it is a vertical or horizontal bar graph. To help students with this error, have them use a ruler to line up the tops of the bars with the scale. Students should also be asked to match the information they are looking for with one of the categories on the axis with the correct title/category of the bar.

Students should also be presented with vertical and horizontal bar graphs that represent two different sets of data. Discuss what common attributes the two bar graphs have, such as title, axes, and labels for the axes, numerical scale, and bars. Discuss how the two bar graphs are different; for example, the titles of the graphs, labels for the axes, lengths and widths and spacing of the bars, and how some graphs have horizontal bars and others have vertical bars.

Students also experienced some difficulty drawing conclusions from graphs. To develop the skill of interpreting graphs, students should be given bar graphs and be asked to draw conclusions. They should be encouraged to ask or write questions that go beyond simplistic reading of a graph. Both literal and inferential comprehension questions should be asked, such as, What can you tell about _____ by looking at this graph? How many more/less than ...? Based on the information presented in the graph, what other conclusions can you make? Why do you think _____? For example, when trying to solve the problem about what foods should be offered in the cafeteria, students would examine a bar graph that has the title, What Foods Should Be Available on the Cafeteria Menu? and be able to tell what food selections were considered the favourites by noticing which bars are the highest or longest. They may also draw conclusions that more students want healthy food for lunch than not healthy food.

Students should understand that to solve some problems, collecting and organizing graphs can help people to reach conclusions. Data is usually collected to answer questions, to discover something of interest, or most importantly, to solve a problem. Some examples of problems students might be interested in include, What should students be allowed to do during the lunch break? What foods should be available on the cafeteria menu? What foods should be removed from the cafeteria menu? What activities would you like to do in the gym? To answer questions like these, or to solve these kinds of problems, students could collect and display data, then interpret it.

D. What are the most appropriate methods and activities for assessing student learning?

Below are some sample questions related to statistics and probability which will be used to represent some of the appropriate methods and activities for assessing student learning.

Encourage students to share their thinking, their strategies, and their solutions. Rich discussion fosters the development of alternative problem-solving strategies.

Examples:

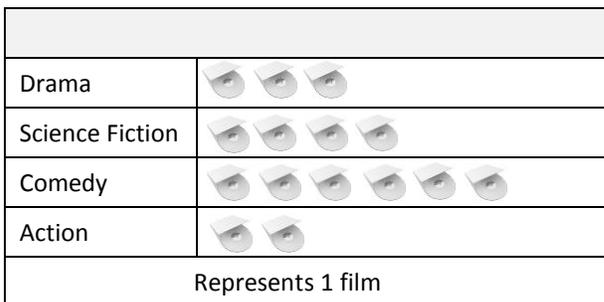
1. Emma asked her classmates a question to collect data. She recorded their answers in the following list:

Drama	3
Science Fiction	4
Comedy	6
Action	2

Which question did Emma ask her classmates?

- Do you prefer films about the police?
- Do you prefer comedy more than action films?
- What kind of films do you prefer?
- Do you prefer drama more than action films?

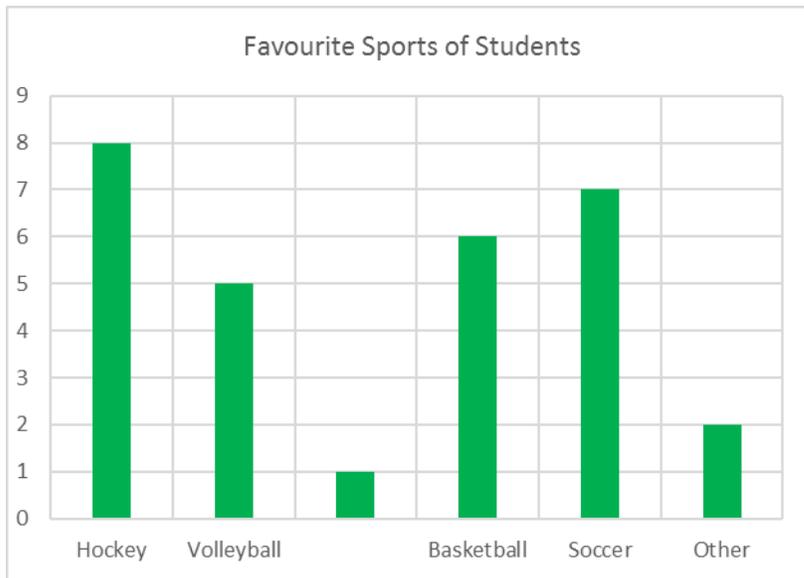
2. Emma constructed the following pictograph to present the collected data about her classmates. She made some errors.



What error did Emma make?

- She forgot the title.
- The number of comedy films is not equal to the number recorded in the list above.
- She forgot the legend.
- She forgot the title and the legend.

7. Christine asked her 29 classmates to name their favourite sport. The bar graph below shows the results of her survey.



What errors did Christine make?

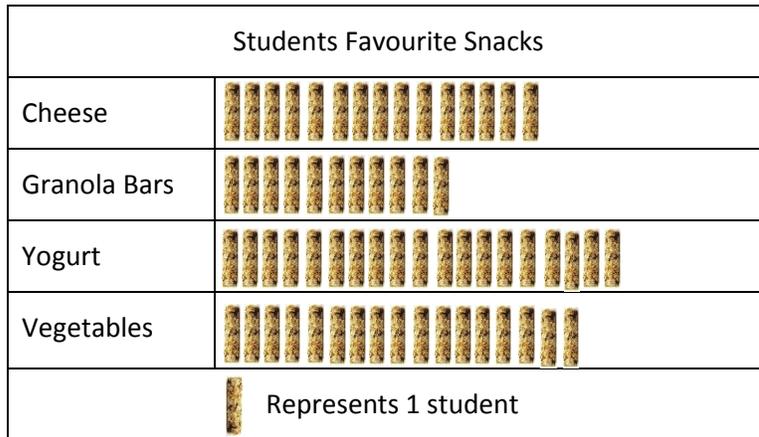
- The labels of the axes and the title is missing.
 - The labels of the axes and the name of a sport are missing.
 - The bars are not the same width.
 - The total number of students in the graph is not equal to 29.
8. The table below shows the results of a survey of students favourite snacks.

Favourite Snacks of Students	
Snacks	Number
Cheese	
Granola Bars	
Yogurt	
Vegetables	

How many more students prefer vegetables than cheese?

- 17
- 15
- 11
- 2

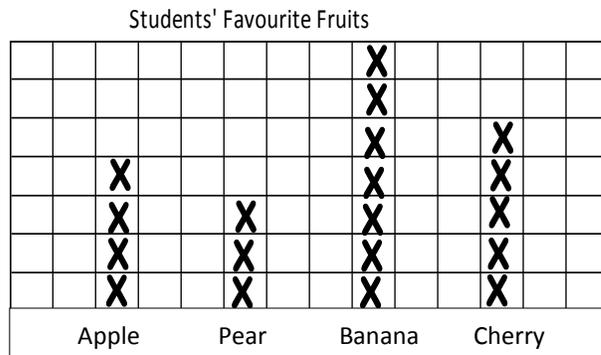
9. Sherene uses the data above to construct the pictograph below to show the results of students favourite snacks.



Which statement is true?

- There are 30 students who prefer granola bars and yogurt.
- There are 23 students who prefer cheese and yogurt.
- There are 27 students who prefer granola bars and vegetables.
- There are 35 students who prefer yogurt and vegetables.

10. Kyle surveyed the Grade 2 students about their favourite fruits. The line plot below shows the results of this survey.



Which statement is **true**?

- The students prefer apples to bananas.
- The students prefer pears to cherries.
- The apple is the most popular fruit.
- The pear is the least popular fruit.

References

- Black, P., Harrison, C., Lee, C., Marshall, B., & Wiliam, D. (2003). *Assessment for learning: Putting it into practice*. Buckingham, UK: Open University Press.
- Davies, A. (2009). *Assessment for Learning*. Courtenay, BC: Connections Publishing.
- Department of Education and Early Childhood Development (EECD), Province of Nova Scotia (2001). *Toward a coherent mathematics program—A study document for educators*. Halifax, NS: Author.
- Department of Education and Early Childhood Development (EECD), Province of Nova Scotia (2008). *Mathematics 3: A teaching resource*. Halifax, NS: Author.
- Department of Education and Early Childhood Development (EECD), Province of Nova Scotia (2013a). *Mathematics 1 curriculum guide, implementation draft*. Halifax, NS: Author.
- Department of Education and Early Childhood Development (EECD), Province of Nova Scotia (2013b). *Mathematics 2 curriculum guide, implementation draft*. Halifax, NS: Author.
- Department of Education and Early Childhood Development (EECD), Province of Nova Scotia (2013c). *Mathematics 3 curriculum guide, implementation draft*. Halifax, NS: Author.
- National Council of Teachers of Mathematics (NCTM). (2010, August). *Star students make connections, teaching children mathematics* Vol 17 (Issue # 1). Reston, VA: author.
- Novick, L. R. (2009, July). Diagram literacy in preservice math teachers, computer science majors, and typical undergraduates: The case of matrices, networks, and hierarchies. *Mathematical thinking and learning* 6, (3), pp. 307–420.
- Pearson Education Canada. (2009a). *Math makes sense, pro guide: Grade 3 planning and assessment support*. Toronto, ON: Pearson Education Canada.
- Pearson Education Canada. (2009b). *Math makes sense, pro guide: Grade 3 unit 1: Patterning*. Toronto, ON: Pearson Education Canada.
- Pearson Education Canada. (2009c). *Math makes sense, pro guide: Grade 3 unit 4: Measurement*. Toronto, ON: Pearson Education Canada.
- Pearson Education Canada. (2009d). *Math makes sense, pro guide: Grade 3 unit 6: Geometry*. Toronto, ON: Pearson Education Canada.
- Pearson Education Canada. (2009e). *Math makes sense, pro guide: Grade 3 unit 7: Data Analysis*. Toronto, ON: Pearson Education Canada.
- Sherman, H. J., Richardson, L. I., & Yard, G. J. (2005). *Teaching children who struggle with mathematics – A systematic approach to analysis and correction*. Upper Saddle River, NJ: Pearson Education Inc.
- Small, M. (2005). *Professional resources and instruction for mathematics educators (PRIME) number and operations background strategies*. Toronto, ON: Thomson Nelson Canada Ltd.
- Small, M. (2009). *Making mathematics meaningful to Canadian students, K–8*. Toronto, ON: Nelson Education Ltd.
- Van De Walle, J.A. (2001). *Elementary and middle school mathematics teaching developmentally fourth edition*. New York, NY: Addison Wesley Longman.
- Van de Walle, J.A. and Lovin, L. (2006a). *Teaching student-centered mathematics grades K–3*. Boston, MA: Pearson Allyn & Bacon.
- Van de Walle, J.A. and Lovin, L. (2006b). *Teaching student-centered mathematics grades 3–5*. Boston, MA: Pearson Allyn & Bacon.

Appendix A: Cognitive Levels of Questioning

Cognitive Levels of Questioning

Knowledge

Knowledge questions may require students to recall or recognize information, names, definitions, or steps in a procedure.

Knowledge Verbs: identify, compute, calculate, name, find, evaluate, use, measure

Knowledge questions, items, and/or tasks:

- rely heavily on recall and recognition of facts, terms, concepts, or properties
- recognize an equivalent representation within the same form, for example, from symbolic to symbolic
- perform a specified procedure; for example, calculate a sum, difference, product, or quotient
- evaluate an expression in an equation or formula for a given variable
- draw or measure simple geometric figures
- read information from a graph, table, or figure

Application

Application/comprehension questions may require students to make connections, represent a situation in more than one way (translating between representations), or solve contextual problems.

Application Verbs: classify, sort, estimate, interpret, compare, explain

Application questions, items, and/or tasks:

- select and use different representations, depending on situation and purpose
- involve more flexibility of thinking
- solve a word problem
- use reasoning and problem-solving strategies
- may bring together skills and knowledge from various concepts or strands
- make connections between facts, terms, properties, or operations
- represent a situation mathematically in more than one way
- compare figures or statements
- explain and provide justification for steps in a solution process
- translate between representations
- extend a pattern
- use information from a graph, table, or figure to solve a problem
- create a routine problem, given data, and conditions
- interpret a simple argument

Analysis

Analysis questions may require students to go beyond comprehension and application to higher order thinking skills, such as generalizations and non-routine problem-solving.

Analysis Verbs: analyze, investigate, justify, compare and contrast, explain, describe, prove

Analysis questions, items, and/or tasks:

- require problem solving, reasoning, planning, analysis, judgment, and creative thought
- thinking in abstract and sophisticated ways
- explain relationships among facts, terms, properties, or operations
- describe how different representations can be used for different purposes
- analyze similarities and differences between procedures and concepts
- generalize a pattern
- solve a novel problem, a multi-step, and/or multiple decision point problem
- solve a problem in more than one way
- justify a solution to a problem and/or assumptions made in a mathematical model
- describe, compare, and contrast solution methods
- formulate a mathematical model for a complex situation, such as probability experiments
- provide a mathematical justification and/or analyze or produce a deductive argument

Below are the percentages of knowledge, application, and analysis questions in the Nova Scotia provincial assessments for Mathematics in Grade 4:

- Knowledge 20–30%
- Application 50–60%
- Analysis 10–20%

These percentages are also recommended for classroom-based assessments.

Appendix B: From Reading Strategies to Mathematics Strategies

The following table illustrates when strategies are to be used, and during what part of the three-part lesson format (as referenced in Lessons Learned 1).

Name of Strategy	Before	During	After	Assessment
1. Concept Circles	X	X	X	X
2. Frayer Model	X	X	X	X
3. Concept Definition Map	X	X	X	X
4. Word Wall	X	X	X	
5. Three-Read		X	X	
6. Graphic Organizer	X	X	X	X
7. K-W-L	X		X	X
8. Think-Pair-Share	X	X		
9. Think-Aloud	X	X		X
10. Academic Journal-Mathematics		X	X	
11. Exit Cards			X	X

1. Concept Circle

A concept circle is a way for students to conceptually relate words, terms, expressions, etc. As a “before” activity, it allows students to predict or discover relationships. As a “during” or “after” activity, students can determine the missing concept or attribute or identify an attribute that does not belong.

The following steps illustrate how the organizer can be used:

- Draw a circle with the number of sections needed.
- Choose the common attributes and place them in the sections of the circle.
- Have students identify the common concepts to the attributes.

This activity can be approached in other ways.

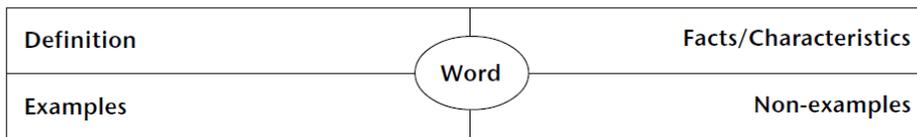
- Supply the concept and some of the attributes and have students apply the missing attributes.
- Insert an attribute that is not an example of the concept and have students find the one that does not belong and justify their reasoning.

2. Frayer Model

The Frayer Model is a graphic organizer used to categorize a word and build vocabulary. It prompts students to think about and describe the meaning of a word by

- giving a definition
- describing main characteristics
- providing examples and non-examples of the word or concept

It is especially helpful to use with a concept that might be confusing because of its close connections to another concept.

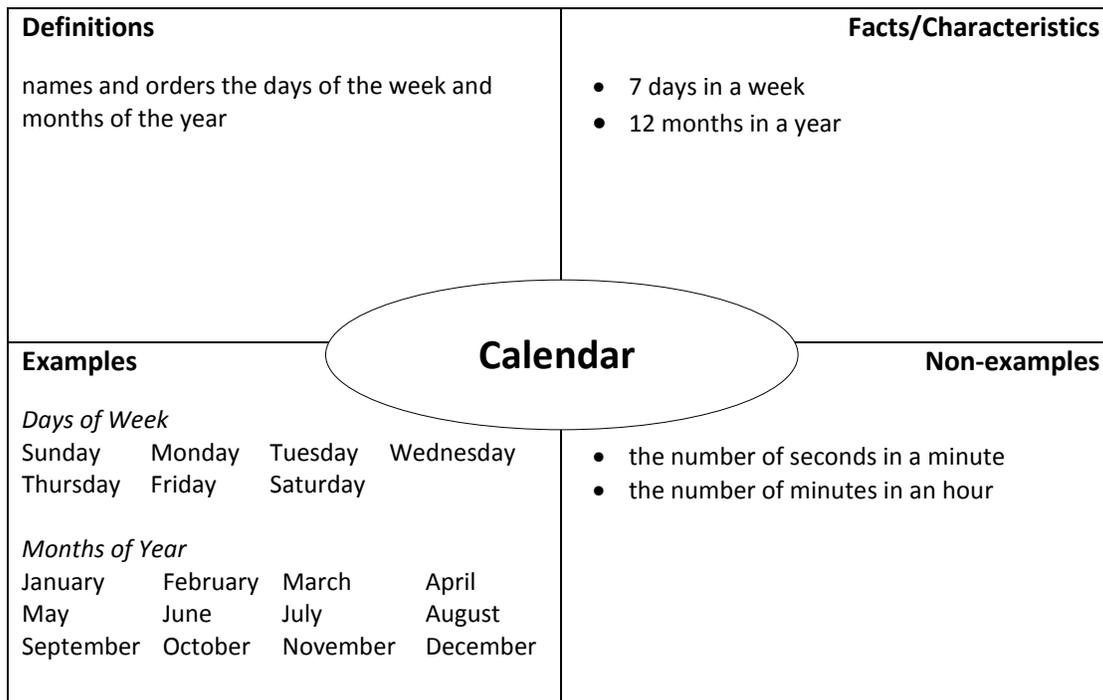


The following steps illustrate how the organizer can be used.

1. Display the template for the Frayer model and discuss the various headings and what is being sought.
2. Model how to use this example by using a common word or concept. Give students explicit instructions on the quality of work that is expected.
3. Establish the groupings (e.g. pairs) to be used and assign the concept(s) or word(s).
4. Have students share their work with the entire class.

This is an excellent activity to do in poster form to display in class. Each group might do the same word or concept, or different words or concepts could be assigned.

Example of a Frayer Model



3. Concept Definition Map

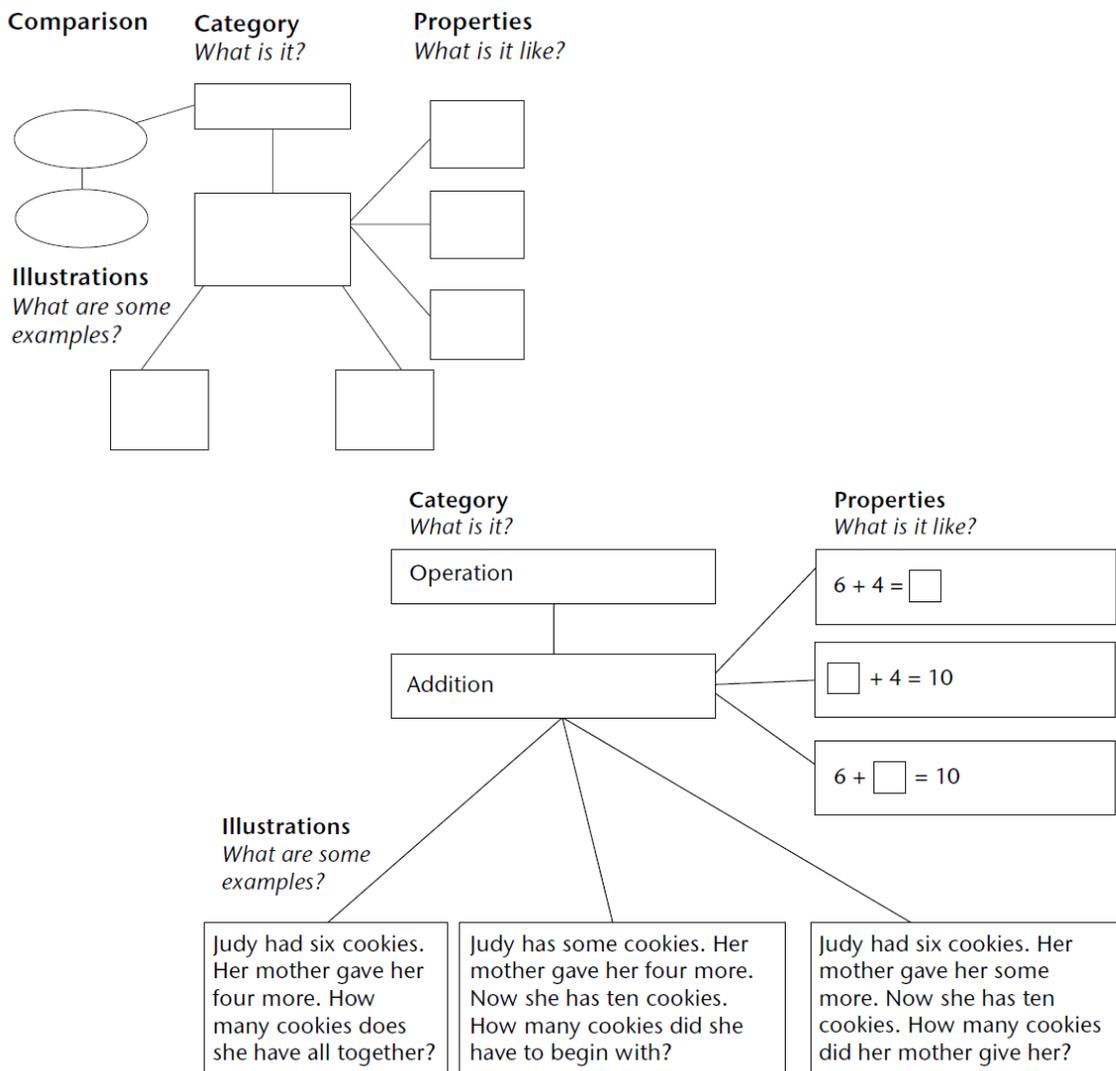
The purpose of a concept definition map is to prompt students to identify the main components of a concept, show the interrelatedness, and build vocabulary. Information is placed into logical categories, allowing students to identify properties, characteristics, and examples of the concept.

The following steps illustrate how this organizer can be used.

1. Display the template for the concept definition map.
2. Discuss the different headings, what is being sought, and the quality of work that is expected.
3. Model how to use this map by using a common concept.
4. Establish the concept(s) to be developed.
5. Establish the groupings (e.g. pairs) and materials to be used to complete the task.
6. Complete the activity by having the students write a complete definition of the concept.

Encourage students to refine their map as more information becomes available.

Example of a Concept Definition Map



4. Word Wall

A mathematics word wall is based upon the same principle as a reading word wall, found in many classrooms. It is an organized collection of words that is prominently displayed in the classroom and helps students learn the language of mathematics. A word wall can be dedicated to a concept, big idea, or unit in the mathematics curriculum. Words are printed in bold block letters on cards and then posted on the wall or bulletin board.

Illustrations placed next to the word on the word wall can add to the students' understanding. Students may also elaborate on the word in their journals by illustrating, showing an example, and using the word in a meaningful sentence or short paragraph. Students can be assigned a word and its illustration to display on the word wall. Room should be left to add more words and diagrams as the unit or term progresses.

As a new mathematical term is introduced to the class, students can define and categorize the word in their mathematics journal under an appropriate unit of study. Then the word can be added to the mathematics word wall so students may refer to it as needed. Students will be surprised at how many words fall under each category and how many new words they learn to use in mathematics.

Note: The word wall is developed one word at a time as new terminology is encountered.

Use the following steps to set up a word wall.

1. Determine the key words that students need to know or will encounter in the topic or unit.
2. Print each word in large block letters and add the appropriate illustrations.
3. Display cards when appropriate.
4. Regularly review the words as a warm-up or refresher activity.

5. Three-Read Strategy

Using this strategy, found in *Toward a Coherent Mathematics Program: A Study Document for Educators* (Nova Scotia Department of Education 2002), the teacher encourages students to read a problem three times before they attempt to solve it. There are specific purposes for each reading.

First Read

The students try to visualize the problem to get an impression of its overall context. They do not need specific details at this stage, only a general idea so they can describe the problem in broad terms.

Second Read

The students begin to gather facts about the problem to make a more complete mental image of it. As they listen for more detail, they focus on the information to determine and clarify the question.

Third Read

The students check each fact and detail in the problem to verify the accuracy of their mental image and to complete their understanding of the question.

During the Three-Read strategy, the students discuss the problem, including any information needed to solve it. Reading becomes an active process that involves oral communication among students and teachers; it also involves written communication as teachers encourage students to record information and details from their reading and to represent what they read in other ways with pictures, symbols, or charts. The teacher facilitates the process by posing questions that ask students to justify their reasoning, support their thinking, and clarify their solutions.

To teach the Three-Read Strategy, teachers should exaggerate each step as they model it. When they have students practice the strategy, teachers should ask questions that stimulate the kinds of questions that students should be asking themselves in their internal conversations. In every classroom, an ongoing discussion of this Three-Read strategy must be conducted and many students will need to be reminded to use this strategy.

6. Graphic Organizer

A graphic organizer can be of many forms: web, chart, diagram, etc. Graphic organizers use visual representations as effective tools to do such things as

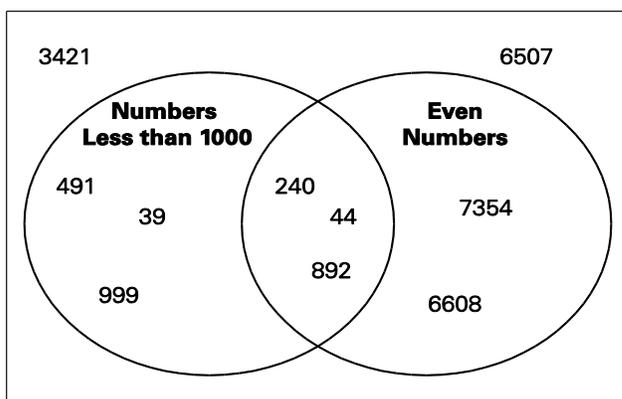
- activate prior knowledge
- analyze
- compare and contrast
- make connections
- organize
- summarize

The following steps illustrate how the organizer can be used.

1. Present a template of the organizer and explain its features.
2. Model how to use the organizer, being explicit about the quality of work that is expected.
3. Present various opportunities for students to use graphic organizers in the classroom.

Students should be encouraged to use graphic organizers on their own as ways of organizing their ideas and work. If the graphic organizer being used is a Venn diagram, it is important to draw a rectangle around Venn diagrams to represent the entire group that is being sorted. This will show the items that do not fit the attributes of the circle(s) outside of them, but within the rectangle. Therefore, elements of the set that do belong to Attribute A or Attribute B are shown within the rectangle but not within the circles in the rectangles.

Example of a Venn diagram



7. K-W-L (Know/Want to Know/Learned)

K-W-L is an instructional strategy that guides students through a text or mathematics word problem and uses a three-column organizer to consolidate the important ideas. Students brainstorm what they know about the topic and record it in the K column. They then record what they want to know in the W column. During and after the reading, students' record what they have learned in the L column.

The K-W-L strategy has several purposes:

- to illustrate a student's prior knowledge of a topic
- to give a purpose to the reading
- to help a student monitor his or her comprehension

Know	Want to Know	Learned

The following steps illustrate how the K-W-L can be used.

1. Present a template of the organizer to students, explain its features, and be explicit about the quality of work that is expected.
2. Ask them to fill out the first two sections (what they know and what they want to know before proceeding).
3. Check the first section for any misconceptions in thinking or weakness in vocabulary.
4. Have the students read the text, and taking notes as they look for answers to the questions they posed.
5. Have students complete the last column to include the answers to their questions and other pertinent information.
6. Discuss this new information with the class, and address any questions that were not answered.

8. Think-Pair-Share

Think-Pair-Share is a learning strategy designed to encourage students to participate in class and keep them on task. It focusses students' thinking on specific topics and provides them with an opportunity to collaborate and have meaningful discussion about mathematics.

- First, teachers ask students to think individually about a newly introduced topic, concept, or problem. This provides essential time for each student to collect his or her thoughts and focus on his or her thinking.
- Second, each student pairs with another student, and together the partners discuss each other's ideas and points of view. Students are more willing to participate because they do not feel the peer pressure that is involved when responding in front of the class. Teachers ensure that sufficient time is allowed for each student to voice his or her views and opinions. Students use this time to talk about personal strategies, compare solutions, or test ideas with their partners. This helps students to make sense of the problem in terms of their prior knowledge.
- Third, each pair of students shares with the other pairs of students in large-group discussion. In this way, each student has the opportunity to listen to all of the ideas and concerns discussed by the other pairs of students. Teachers point out similarities, overlapping ideas, or discrepancies among the pairs of students and facilitate an open discussion to expand upon any key points or arguments they wish to pursue.

9. Think-Aloud

Think-aloud is a self-analysis strategy that allows students to gain insight into the thinking process of a skilled reader as he or she works through a piece of text. Thoughts are verbalized, and meaning is constructed around vocabulary and comprehension. It is a useful tool for such things as brainstorming, exploring text features, and constructing meaning when solving problems. When used in mathematics, it can reveal to teachers the strategies that are part of a student's experience and those that are not. This is helpful in identifying where a student's understanding may break down and may need additional support.

The think-aloud process will encourage students to use the following strategies as they approach a piece of text.

- Connect new information to prior knowledge.
- Develop a mental image.
- Make predictions and analogies.
- Self-question.
- Revise and fix up as comprehension increases.

The following steps illustrate how to use the think-aloud strategy.

1. Explain that reading in mathematics is important and requires students to be thinking and trying to make sense of what they are reading.
2. Identify a comprehension problem or piece of text that may be challenging to students; then read it aloud and have students read it quietly.
3. While reading, model the process verbalizing what you are thinking, what questions you have, and how you would approach a problem.
4. Then model this process a second time, but have a student read the problem and do the verbalizing.
5. Once students are comfortable with this process, a student should take a leadership role.

10. Academic Mathematics Journal

An academic journal in mathematics is an excellent way for students to keep personal work and other materials that they have identified as being important for their personal achievement in mathematics. The types of materials that students would put in their journals would include:

- strategic lessons – lessons that they would identify as being pivotal as they attempt to understand mathematics
- examples of problem-solving strategies
- important vocabulary

Teachers are encouraged to allow students to use these journals as a form of assessment. This will emphasize to the student that the material that is to be placed in his or her journal has a purpose. Mark these journals only on the basis of how students are using them and whether or not they have appropriate entries.

The goal of writing in mathematics is to provide students with opportunities to explain their thinking about mathematical ideas and then to re-examine their thoughts by reviewing their writing. Writing will enhance students' understanding of math as they learn to articulate their thought processes in solving math problems and learning mathematics concepts.

11. Exit Card

Exit cards are quick tools for teachers to become better aware of a students' mathematics understanding. They are written student responses to questions that have been posed in class or solutions to problem-solving situations. They can be used at the end of a day, week, lesson, or unit. An index card is given to each student (with a question that promotes understanding on it), and the student must complete the assignment before he or she is allowed to "exit" the classroom. The time limit should not exceed 5 to 10 minutes, and the student drops the card into some sort of container on the way out. The teacher now has a quick assessment of a concept that will help in planning instruction.

Appendix C: Cognitive Levels of Sample Questions

Translating Between and Among Representations		Representing and Partitioning Whole Numbers		Whole Number Operations		Patterns and Relations	
Question	What type of question?	Question	What type of question?	Question	What type of question?	Question	What type of question?
1	Application	1	Application	1	Knowledge	1	Knowledge
2	Application	2	Application	2	Application	2	Application
3	Analysis	3	Application	3	Application	3	Application
		4	Application	4	Application	4	Analysis
		5	Analysis	5	Application	5	Analysis
		6	Application	6	Application	6	Application
		7	Knowledge	7	Application	7	Analysis
		8	Application	8	Application	8	Analysis
		9	Application	9	Application	9	Application
		10	Knowledge	10	Application	10	Application
				11	Knowledge		
				12	Knowledge		
				13	Analysis		
				14	Knowledge		

Problem Solving		Measurement		Geometry		Statistics and Probability	
Question	What type of question?	Question	What type of question?	Question	What type of question?	Question	What type of question?
1	Application	1	Knowledge	1	Knowledge	1	Knowledge
2	Application	2	Application	2	Knowledge	2	Application
3	Application	3	Knowledge	3	Knowledge	3	Application
4	Application	4	Knowledge	4	Application	4	Analysis
5	Analysis	5	Knowledge	5	Application	5	Application
6	Analysis	6	Knowledge	6	Analysis	6	Application
7	Analysis	7	Application	7	Analysis	7	Analysis
8	Analysis	8	Application	8	Knowledge	8	Analysis
9	Analysis	9	Analysis			9	Application
						10	Application

Appendix D: Responses to the Sample Questions

Translating Between and Among Representations		Representing and Partitioning Whole Numbers		Whole Number Operations		Patterns and Relations	
Question	Response	Question	Response	Question	Response	Question	Response
1	D	1	452	1	B	1	B
2	A	2	266	2	A	2	D
3	C	3	503	3	A	3	C
		4	342	4	B	4	A
		5	Answers will vary	5	B	5	B
		6	A	6	C	6	D
		7	C	7	C	7	A
		8	B	8	B	8	D
		9	Answers will vary	9	B	9	D
		10	263, 373, 487 and 597 written in words	10	B	10	C
				11	B		
				12	C		
				13	D		
				14	D		

Problem Solving		Measurement		Geometry		Statistics and Probability	
Question	Response	Question	Response	Question	Response	Question	Response
1	8 outfits	1	C	1	D	1	C
2	Megan	2	B	2	B	2	A
3	37	3	A	3	B	3	B
4	Open-ended 3 possibilities (9–1), (6–3) and (3–5)	4	C	4	D	4	C
5	2	5	D	5	A	5	D
6	10 combinations	6	B	6	C	6	D
7	5 tables	7	A	7	B	7	B
8	121 cm	8	B	8	C	8	D
9	8 combinations	9	A			9	A
						10	D