Nova Scotia Assessment: Mathematics in Grade 4

Lessons Learned

> "For learners to succeed, teachers must assess students' individual abilities and characteristics and choose appropriate and effective instructional strategies accordingly."
>
> – Helene J. Sherman

NOVA SCOTIA

Contents

Purpose of this document

This *Lessons Learned* document was developed based on an analysis of the Item Description Reports for the Nova Scotia Assessment: Mathematics in Grade 4 (2013–2014, 2014–2015). It is intended to support all elementary classroom teachers (in particular grades Primary–3) and administrators at the school, board, and provincial levels, in using the information gained from this assessment to inform next steps for numeracy focus. The analysis of these items form the basis of this document, which was developed to support teachers as they further explore these areas through classroom-based instruction and assessment across a variety of mathematical concepts.

After the results for each assessment become available, an Item Description Report is developed to describe each item of the mathematics assessment in relation to the curriculum outcomes and cognitive processes involved with each mathematical strand. The percentage of students across the province who answered each item correctly is also connected to each item. Item description reports for mathematics are made available to school boards for distribution to schools, and they include provincial, board, and school data. Schools and boards should examine their own data in relation to the provincial data for continued discussions, explorations, and support for mathematics focus at the classroom, school, board, and provincial levels.

This document specifically addresses areas that students across the province found challenging based on provincial assessment evidence. It is essential that teachers consider assessment evidence from a variety of sources to inform the next steps most appropriate for their students. Effective classroom instruction and assessment strategies are responsive to the individual learners within a classroom.

The M4 generates information that is useful in guiding classroom-based instruction and assessment in mathematics. This document provides an overview of the mathematics tasks included in the assessment, information about this year's mathematics assessment results, and a series of Lessons Learned for mathematics. Suggested next steps for classroom instruction and assessment are included for each Lesson Learned.

Overview of the Nova Scotia Assessment: Mathematics in Grade 4

The assessment provides information about mathematics for each student and complements assessment data collected in the classroom. This assessment is administered at the beginning of Grade 4. It is designed to provide detailed information for every student in the province regarding his or her progress in achieving a selection of mathematics curriculum outcomes at the end of Grade 3. Information from this assessment can be used by teachers to inform their instruction and next steps in providing support and intervention for their students.

The design of the assessment includes the following:

- mathematical tasks that reflect a selection of outcomes aligned with the curriculum from the end of grade 1 to the end of grade 3 from across all strands of the mathematics curriculum
- all items are in selected response format
- all items are designed to provide a broad range of challenge, thereby providing information about a range of individual student performance

Cognitive levels of questions in mathematics are defined as:

- *Knowledge questions* (Level 1) may require students to recall or recognize information, names, definitions, or steps in a procedure.
- *Application/comprehension questions* (Level 2) may require students to make connections, represent a situation in more than one way (translating between representations), or solve contextual problems.
- *Analysis questions* (Level 3) may require students to go beyond comprehension and application to higher order thinking skills, such as generalizations and non-routine problem-solving.

These are the percentages of the questions on the Nova Scotia provincial assessments for M4 and M6:

- Knowledge (Level 1) 20–30%
- Application (Level 2) 50–60%
- Analysis (Level 3) 10–20%

These percentages are also recommend for classroom-based assessments.

Please refer to Appendix A at the end of this document for further information about cognitive levels of questioning.

Performance Levels

Below are the Nova Scotia Assessment: Mathematics in Grade 4 Performance Levels

Level 1: Students at Level 1 can generally solve problems when they are simple and clearly stated or where the method to solve the problem is suggested to them. They can do addition and subtraction of whole numbers, but may not understand when each operation should be used. They can recognize some math terms and symbols, mainly from earlier grades.

Level 2: Students at Level 2 can generally solve problems similar to problems they have seen before. They depend on a few familiar methods to solve problems. They can do addition and subtraction of whole numbers and usually understand when each operation should be used. They can understand and use some math terms and symbols, especially those from earlier grades.

Level 3: Students at Level 3 can generally solve problems that involve several steps and may solve problems they have not seen before. They can apply number operations (+, –, as well as x, ÷ to 5 x 5) correctly and can judge whether an answer makes sense. They can understand and use many math terms and symbols, including those at grade level.

Level 4: Students at Level 4 can solve new and complex problems. They can apply number operations (+, –, x, ÷) with ease. They can think carefully about whether an answer makes sense. They find math terms and symbols easy to use and to understand.

Assessment Results

The Nova Scotia Assessment: Mathematics in Grade 4 has been administered since the 2013-2014 school year. The following percentage of students performed at the expectation of level 3 or above on the assessment: 74% (2013–2014), and 74% (2014–2015).

The following is a breakdown of the 2014–2015 M6 results for each performance level (7496 grade 4 students participated in the M4 assessment):

- 74% of grade 4 students in the province have a performance level of 3 or above
- Performance Level 1: 9.6% of students in the province are below the expectations of this assessment
- Performance Level 2: 16.2% of students in the province are approaching the expectations of this assessment
- Performance Level 3: 61.5% of students in the province are at the expectations of this assessment
- Performance Level 4: 12.7% of students in the province are above the expectations of this assessment

Mathematics in Grade 4 Lessons Learned

The assessment information gathered from the Nova Scotia Assessment: Mathematics in Grade 4 data has been organized into 5 ***Lessons Learned***, Translating Between and Among Representations, Representing and Partitioning Whole Numbers, Whole Number Operations, Patterns and Relations and Problem Solving.

Each Lesson Learned is divided into four sections that address the following questions

A. What conclusions can be drawn from the Nova Scotia Assessment: Mathematics in Grade 4?
B. Do students have any misconceptions or errors in their thinking?
C. What are the next steps in instruction for the class and for individual students?
D. What are the most appropriate methods and activities for assessing student learning?

Lessons Learned

1. **Translating Between and Among Representations**: Students were challenged when translating and moving flexibly between and among all representations of a concept. Students need to be encouraged to translate between the name of the shape (words), pictures and symbols. If asked to represent a question using representations, (words, symbols and/or pictures), students usually only provide symbols. They do not seem to realize that they can use all three representations (words, symbols and/or pictures) when solving a question.

2. **Representing and Partitioning Whole Numbers:** Students were challenged when asked to apply their knowledge of basic facts, skills, represent a situation or the steps in a procedure when given Level 2 questions. They struggled with translating between representations of a concept (contextually, concretely, pictorially, symbolically, and verbally). When students are working with partitioning whole numbers and when performing operations, it is very important for students to understand that numbers can be broken down into two or more parts, in many different ways.

3. **Whole Number Operations**: Students were quite challenged when asked to apply basic skills, knowledge, and computational procedures to Level 2 and Level 3 questions. Students need to be exposed to more than knowledge (Level 1) items in order to apply the higher order thinking skills to do application (Level 2) and analysis (Level 3) items. Students need to review the Story Structures for Addition and Subtraction. Students also need to be encouraged to estimate before calculating an answer to a question.

4. **Patterns and Relations**: Students were challenged when asked to transfer their visual knowledge of patterns to numerical patterns. Students should be able to describe either an increasing shape pattern or a decreasing shape pattern, but need to recognize that each term has a numeric value. Students forget that a pattern rule must have a starting point or the pattern rule is incomplete. They need to be encouraged to provide all the information for a pattern rule. Students need to continue to work with representations of patterns, contextually, pictorially, symbolically, and verbally.

5. **Problem Solving**: Students need more exposure to application (Level 2) and analysis (Level 3) items in order to apply these higher order thinking skills when problem solving. Students need to be encouraged to understand that problems may have more than one entry point and there are many strategies to solve a problem. Students need to continue to work on translating between and among representations when problem solving. They do not seem to realize that they can use all three representations when asked to solve a word problem. These other representations may support their problem solving and their reasoning.

Key Messages

The following key messages should be considered when using this document to inform classroom instruction and assessment.

The Nova Scotia mathematics curriculum is based upon several key assumptions or beliefs about mathematics learning that have grown out of research and practice. These beliefs include the following:

- Mathematics learning is an active and constructive process.
- Learning is most effective when standards of expectation are made clear with ongoing assessment and feedback.
- Learners are individuals who bring a wide range of prior knowledge and experiences and who learn via various styles and at different rates.
- Learning is most likely to occur when placed in meaningful contexts and in an environment that supports exploration, risk taking, and critical thinking and that nurtures positive attitudes and sustained effort.
- Students are curious, active learners with individual interests, abilities, and needs. They come to classrooms with varying knowledge, life experiences, and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.
- Students learn by attaching meaning to what they do and need to construct their own meaning of mathematics. This meaning is best constructed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of models and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students and enhance the formation of sound, transferable, mathematical concepts. At all levels, students benefit from working with and translating through a variety of materials, tools, and contexts when constructing meaning about new mathematical ideas. Meaningful discussions can provide essential links among concrete, pictorial, contextual, and symbolic representations of mathematics.

  (EECD, 2013b, p. 23)

Research has shown that assessment for learning (formative assessment) practices produce significant and often substantial learning gains, close achievement gaps, and build students' ability to learn new skills (Black & Wiliam 1998; OECD 2006). Student involvement in assessment promotes learning. Timely and effective teacher feedback and student self-assessment allow students to reflect on and articulate their understanding of mathematical concepts and ideas.

- Provincial assessment results form part of the larger picture of assessment for each student and complements assessment data collected in the classroom. Ongoing assessment for learning (formative assessment) is essential to effective teaching and learning. Assessment for learning can and should happen every day as part of classroom instruction. Assessment of learning (summative assessment) should also occur regularly and at the end of a cycle of learning. Both forms of assessment are necessary to guide teaching, stimulate learning, and produce achievement gains.
- It is important to construct assessment activities that require students to complete tasks across the cognitive levels. While it is important for students to be able to answer factual and procedural type questions, it is also important to embed activates that require strategic reasoning and problem-solving.

- Ongoing assessment for learning involves the teacher focusing on how learning is progressing during the lesson and the unit, determining where improvements can be made and identifying the next steps. "*Such assessment becomes 'formative assessment' when the evidence is actually used to adapt the teaching to meet learning needs," (*Black, Harrison, Lee, Marshall & Wiliam, 2003, p.2). Effective strategies of assessment for learning during a lesson include: strategic questioning, observing, conversing (conferring with students to "hear their thinking"), analyzing student's work (product), engaging students in reviewing their progress, as well as, providing opportunities for peer and self-assessment.
- Assessment of learning involves the process of collecting and interpreting evidence for the purpose of summarizing learning at a given point in time, and making judgments about the quality of student learning on the basis of established criteria. The information gathered may be used to communicate the student's achievement to students, parents, and others. It occurs at or near the end of a learning cycle.
- All forms of assessment should be planned with the end in mind, thinking about the following questions:
  - What do I want students to learn? (identifying clear learning targets)
  - What does the learning look like? (identifying clear criteria for success)
  - How will I know they are learning?
  - How will I design the learning so that all will learn?
- Before planning for instruction using the suggestions for instruction and assessment, it is important that teachers review individual student results in conjunction with current mathematics assessment information. A variety of current classroom assessments should be analyzed to determine specific strengths and areas for continued instructional focus or support.

Balanced Assessment in Mathematics: Effective ways to gather information about a student's mathematical understanding

- Conversations/Conferences/Interviews: Individual, Group, Teacher-initiated, Child-initiated
- Products/Work Samples: Mathematics journals, Portfolios, Drawings, Charts, Tables, Graphs, Individual and classroom assessment, Pencil-and-paper tests, Surveys, Self-assessment
- Observations: Planned (formal), Unplanned (informal), Read-aloud (literature with mathematics focus), Shared and guided mathematics activities, Performance tasks, Individual conferences, Anecdotal records, Checklists, Interactive activities

(EECD, 2013a, p. 4)

Mathematics in Grade 4 Lesson Learned 1
Translating Between and Among Representations

Students were challenged when translating and moving flexibly between and among all representations of a concept. Students need to be encouraged to translate between the name of the shape (words), pictures and symbols. If asked to represent a question using representations, (words, symbols and/or pictures), students usually only provide symbols. They do not seem to realize that they can use all three representations (words, symbols and/or pictures) when solving a question.

Mathematics should be viewed as an integrated whole rather than as the study of separate strands or units. Connections must also be made between and among the different representational modes – contextual, concrete, pictorial, linguistic/verbal, and symbolic. The process of making connections, in turn, facilitates a child's learning.

"Children who have difficultly translating a concept from one representation to another are the same children who have difficulty solving problems and understanding computations. Strengthening the ability to move between and among these representations improves the growth of children's concepts" (Van De Walle*,* John A. 2001*, Elementary and Middle School Mathematics*, Fourth Edition, p. 34).

One way to encourage children to use multiple representations is to explicitly ask for them.
Ask questions such as

- How many ways can you show the number 20 using words, pictures, models, and numbers?
- How many ways can you write the number 75?
- Can you represent a rectangle as a combination of other shapes?
- Can you represent this line plot as a bar graph?
- Can you use an equation to represent how you thought about this story problem?

A. What conclusions can be drawn from the Nova Scotia Assessment: Mathematics in Grade 4?

Overall, our students are having issues when translating between and among representations of a concept. We found that students have a good understanding of basic facts and procedures, but when given Level 2 items, they appear to want to rush to symbolic. For example, when problem-solving, students are able to understand the context of the question but many are not able to translate between the representations (translating from words to pictures or symbolic to pictures, etc.).

A key concept of students' understanding is to know how to translate and move flexibly between and among all representations of a concept.

B. Do students have any misconceptions or errors in their thinking?

If asked to represent a question using words, symbols and/or pictures, students usually only provided symbols. They do not seem to realize that they can use all three representations (words, symbols and/or pictures) when solving a question. For example, when given a Level 3 question concerning the distance around (perimeter) a named shape, students struggle with knowing how many sides the named shape would have, unless the actual shape is shown in the question as a picture/diagram. Students should be encouraged to translate between the name of the shape (words) and a picture. Rather than relying on the question itself, students could draw the picture and check how many sides the named shape actually has.

C. What are the next steps in instruction for the class and for individual students?

Next steps in instruction should provide opportunities for students to use representations to communicate mathematical ideas, and to select, apply, model and translate among mathematical representations to solve problems. The five representations of a concept are contextual, concrete, pictorial, symbolic, and verbal (written/oral). All five representations should remain a focus for students.

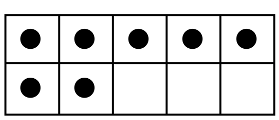
“Representational competence (Novick, 2004) for students is knowing how and when to use particular mathematical representations. A key aspect of understanding mathematics means not only knowing how to use a representation during problem-solving situations but also being able to make connections between representations” (*Teaching Children Mathematics*. NCTM, August 2010, p. 40). When students are able to move between and among the different representations of a concept, we say that they have concept attainment.

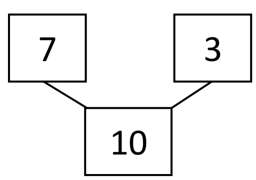
There are three specific strategies that provide opportunities to support students’ development of representational competency

- “Engaging in dialogue about the explicit connections between representations
- Alternating directionality of the connections made among representations
- Encouraging purposeful selection of representation”
  (*Teaching Children Mathematics*. NCTM, August 2010, p. 40)

For example, during instruction students focus on specific parts of one representation and think about the correspondence with parts in another. Teachers should ask questions that require students to translate between parts. Alternating directionality supports students’ thinking with various representations.
For example, consider the activity below, All About Ten (*Teaching Children Mathematics*. NCTM, August 2010, p. 44).

All About Ten: Fill in the box diagram to show the two parts of ten shown in the ten-frame. Write number sentences to match. [Below is the completed activity.]





7 + 3 = 10	10 = 7 + 3
3 + 7 = 10	10 = 3 + 7
10 – 7 = 3	3 = 10 – 7
10 – 3 = 7	7 = 10 – 3

Instructional Strategies

"Three specific instructional strategies create opportunities that may support students' development of representational competence …" (*Teaching Children Mathematics*. NCTM, August 2010, p. 40).

1. Students need to **engage** in dialogue about the explicit connections between representations. Pose challenges/questions to students such as
   - How is the number 10 represented in each diagram?
   - How are the three representations of the number the same? How are they different?
   - Can you show the number 10 in a different way?

2. Students need to be encouraged to **alternate directionality** in order to make connections among representations. When discussing multiple representations, ask focused questions such as
   - Can you describe the 3 in the ten-frame, in the box diagram, and in each of the equations?
   - What meaning does the 3 have in each of the diagrams?

   "The **directionality** of the connections made between the representations and the problem situation is another important feature of representational competence. For example, translating from a box diagram to a ten-frame, and vice versa, promotes the use of different mathematical thought processes**.** An important aspect of developing understanding of mathematics means not only knowing how to use a representation during problem solving situations but also being able to move flexibly between different representations, making connections from one representation to the other, and vice versa." (*Teaching Children Mathematics*. NCTM, August 2010, p. 44)

3. Students need to be **encouraged** to purposefully select the most appropriate representation. "Encourage students to consider the suitability of a representation. Discuss a variety of reasons to use particular representations, including but not limited to the following: efficiency, accuracy, ease of use, appropriateness with respect to the problem context, and student preference. By comparing the use of multiple representations for the same problem, students can more easily see the suitablility of one representation over another" (*Teaching Children Mathematics*. NCTM, August 2010, p. 46).

D. What are the most appropriate methods and activities for assessing student learning?

The strands Measurement (M) and Geometry (G) will be used to represent some of the appropriate **methods and activities for assessing student learning.**

Encourage students to share their thinking, their strategies, and their solutions. Rich discussion fosters the development of alternative problem-solving strategies.

Below are some sample questions that represent translating between and among representations.

1. Peter found a stick that was more than a 1 m long.
How many centimetres long could the stick be?

- ○ 80 cm
- ○ 90 cm
- ○ 100 cm
- ○ 115 cm

Show how you know using words, pictures, and/or symbols.

2. The perimeter (distance around) of a pentagon is 20 cm.
How long is each side?

Show how you know using words, pictures, and/or symbols.

3. I am a 3-D object.
I have 5 faces.
I have 5 vertices.
I have 8 edges.
Which shape am I?

- ○ cube
- ○ sphere
- ○ square-based pyramid
- ○ triangular-based pyramid

Show how you know using words, pictures, and/or symbols.

Mathematics in Grade 4 Lesson Learned 2
Representing and Partitioning Whole Numbers

Students were challenged when asked to apply their knowledge of basic facts, skills, represent a situation or the steps in a procedure when given Level 2 questions. They struggled with translating between representations of a concept (contextually, concretely, pictorially, symbolically, and verbally). When students are working with partitioning whole numbers and when performing operations, it is very important for students to understand that numbers can be broken down into two or more parts, in many different ways.

A. What conclusions can be drawn from the Nova Scotia Assessment: Mathematics in Grade 4?

We noticed that students were able to do well when explicitly given all the information needed to do the question. Students performed well on knowledge questions (Level 1) that required them to use basic skills, symbolic procedures, and factual knowledge. For example, students were successful when asked to calculate 61 + 35. Students were successful problem solvers and performed well on questions that required analysis and non-routine problem-solving.

However, our assessment information also shows that many students experienced challenges in applying their knowledge (Level 2). Our students were challenged when asked to apply their knowledge of basic facts and skills. They also struggled with translating between representations of a concept (contextually, concretely, pictorially, symbolically, and verbally).

When students are working with partitioning whole numbers and when performing operations, it is very important for students to understand that numbers can be broken down into two or more parts, in many different ways.

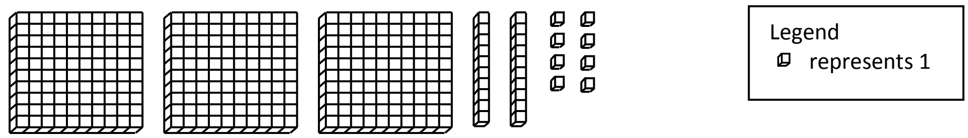
B. Do students have any misconceptions or errors in their thinking?

Students were very capable of performing well with partitioning numbers when given Level 1 questions. For example, students represented a number using base-ten blocks in a conventional display. However, the difficulty appeared when students were asked to apply their knowledge, or to represent a situation or the steps in a procedure when given Level 2 questions. For example, 75 can be partitioned into 70 + 5, 50 + 25, or 60 + 12 + 3. Many students have the misconception that these are three expressions that have an answer of 75, and do not understand that this is also three ways of writing 75. An expression names a number. Sometimes an expression is a number such as 150. Sometimes an expression shows an arithmetic expression, such as 125 + 25. 150 may also be represented by its partition, such as 80 + 70, 100 + 50, and 50 + 50 + 50. Numbers can also be represented by a difference expression, such 175 – 25.

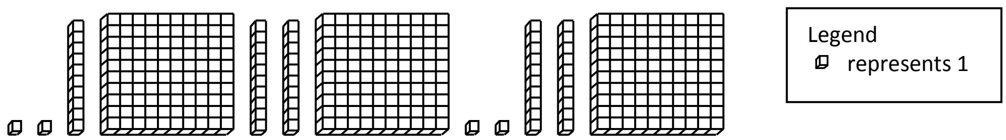
C. What are the next steps in instruction for the class and for individual students?

When students translate between and among the five representations of a concept (contextual, concrete, pictorial, symbolic and verbal), we say that they have concept attainment. Students need numerous experiences representing numbers to 1000 and translating between and among these representations of a concept to strengthen their knowledge. They need many experiences with pictures, coins, base-ten materials, tallies, ten-frames, words, and contexts to conceptualize a number being made up of two or more parts. It is extremely important that students have opportunities to view and create numbers using conventional and non-conventional displays of base-ten blocks.

Conventional Display



Non-conventional Display

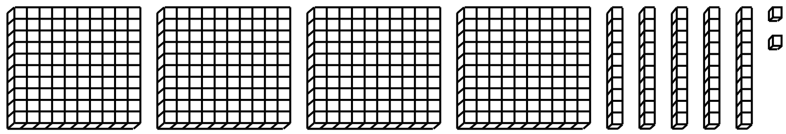


Partitioning numbers using the models above supports student ability to recognize that any number can be partitioned into two or more parts. It also helps students develop part-part-whole thinking. Although it is important for students to experience a variety of partitions including traditional expanded notation (425 = 400 + 20 + 5), they should also continue to experience partitions such as 424 + 1, 325 + 100, 200 + 200 + 10 + 10 + 5.This is the most important understanding that can be developed about number relationships.

D. What are the most appropriate methods and activities for assessing student learning?

Below are some sample questions that represent partitioning whole numbers.

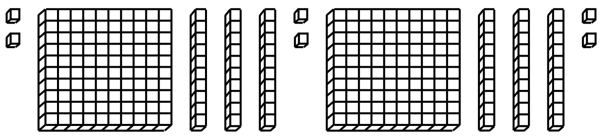
1. What number do these base-ten blocks represent?



Legend
represents 1

Write the number, _________.

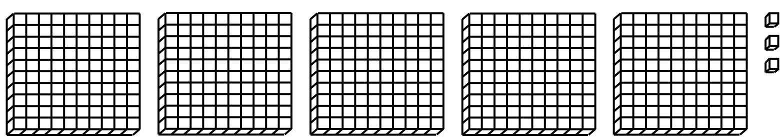
2. What number do these base-ten blocks represent?



Legend
represents 1

Write the number, __________.

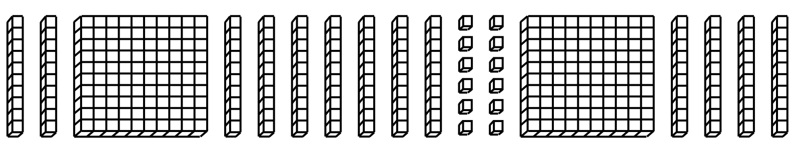
3. What number do these base-ten blocks represent?



Legend
represents 1

Write the number, _________.

4. What number do these base-ten blocks represent?



Legend
represents 1

Write the number, _________.

5. Draw a picture of base-ten blocks to show 236 in 3 different ways.

Legend
represents 1

6. The number 358 is the same as:

- ○ 100 + 100 + 50 + 8 + 100
- ○ 300 + 5 + 8
- ○ 400 – 58
- ○ 3 + 5 + 8

7. Choose the number that is equal to thirty-one tens.

- ○ 31
- ○ 301
- ○ 310
- ○ 3010

8. The number 642 is the same as:

- ○ 5 hundreds, 2 tens, and 14 ones
- ○ 64 tens and 2 ones
- ○ 6 tens and 42 ones
- ○ 6 hundreds, 20 tens and 4 ones

9. Write three expressions that can be used to represent 53.

53 is the same as ____

53 is the same as ____

53 is the same as ____

Mathematics in Grade 4 Lesson Learned 3
Whole Number Operations

Students were quite challenged when asked to apply basic skills, knowledge, and computational procedures to Level 2 and Level 3 questions. Students need to be exposed to more than knowledge (Level 1) items in order to apply the higher order thinking skills to do application (Level 2) and analysis (Level 3) items. Students need to review the Story Structures for Addition and Subtraction. Students also need to be encouraged to estimate before calculating an answer to a question.

The Mathematics curriculum documents for grades Primary to 3 note that a true sense of number goes well beyond the skills of simply counting, memorizing facts, and the situational rote use of algorithms. Number sense develops when students connect numbers to real-life experiences and use benchmarks and referents. This results in students who are computationally fluent, flexible with numbers, and have intuition about numbers. The evolution of number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing rich mathematical tasks that allow students to make connections. Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects, and concepts.

A. What conclusions can be drawn from the Nova Scotia Assessment: Mathematics in Grade 4?

We noticed that students were able to do well when explicitly given all the information needed to do the question. Students performed well on knowledge questions (Level 1) that required them to use basic facts and skills, symbolic procedures, and factual knowledge. For example, students were able to solve 487 – 37 when the problem was presented symbolically.

However, when students were asked to apply basic skills, knowledge, and computational procedures to Level 2 and Level 3 questions, they were quite challenged. This appeared to be a theme throughout the assessment data. At times, students were not sure whether they should add, subtract, multiply, or divide when questions were presented in the context of a story problem. The assessment analysis also showed that our students did not understand the relationship between addition and subtraction. Many of the students were doing addition and subtraction questions as procedures and were not making any connection between these two operations.

B. Do students have any misconceptions or errors in their thinking?

Addition and Subtraction

When solving computation questions, many students performed better when solving the computation using an alternative algorithm, such as presenting the digits to be added or subtracted horizontally. Students did less well, when the traditional algorithm was used. This may result because the traditional algorithm focuses on single digits within the computation rather than thinking about the number as a whole. Below are two examples.

$67 - 32 = 35$

$$\begin{array}{r} 650 \\ \underline{-\ 421} \\ 229 \end{array}$$

Many students have the misconception that they always subtract the smaller number from the larger number. They apply this thinking regardless of whether the position of the number is in the subtrahend (a number which is to be subtracted from another number) or the minuend (a number from which another number is to be subtracted) in the question. Below are two examples.

$$\begin{array}{rl} 451 & \text{(minuend)} \\ \underline{-\ 231} & \text{(subtrahend)} \\ 220 & \text{(difference)} \end{array} \qquad \begin{array}{rl} 5\mathbf{0}9 & \text{(minuend)} \\ \underline{-\ 3\mathbf{8}9} & \text{(subtrahend)} \\ 2\mathbf{8}0 & \text{(difference)} \end{array}$$

In the first question, when subtracting the tens, 5 – 3 = 2, (50 – 30 = 20) the student completes the question correctly using his/her understanding of subtracting the smaller number from the larger number. But in the second question, the student tries to use the same method (smaller number 0 subtracted from larger number 8) and he/she gets an answer of 8 tens, which is incorrect.

At times, students forgot to regroup when adding. They often wrote a two-digit number where there should have only been one digit. Below is an example.

$$\begin{array}{r} 14\mathbf{5} \\ \underline{+\ 24\mathbf{7}} \\ 38\mathbf{12} \end{array}$$

Some students misaligned the digits when recording their calculations and computed incorrectly. A suggested strategy for dealing with this misconception is for students to use grid paper or lined paper turned sideways to align the digits and to focus on the place value of the digits being added or subtracted. Another way to address these errors or misconceptions is to focus on developing personal strategies and alternative algorithms which tend to focus on the meaning of the number, rather than on individual digits.

Multiplication and Division

It is important when developing multiplication and division concepts that children are able to explain the connection between their models and the story problems using verbal expressions such as "groups of," "rows of" and "jumps of".

Early in developing the division concept, students should meet situations involving remainders. Also the connection between words and symbols must be carefully developed for division. For example, when writing 12 ÷ 4, we should say aloud, "How many groups of 4 are in 12?" As well, students should be familiar with both forms of the symbolic expression, and below are two examples.

$$4\overline{)12} \qquad\qquad 12 \div 4$$

A misconception that students have about multiplication is that the product of two numbers is always greater than the sum of those two numbers. So, when they encountered expressions like 8 x 1, 8 x 0, and 2 x 2 where this did not apply, they were puzzled.

C. What are the next steps in instruction for the class and for individual students?

Students were challenged when applying estimation strategies. They knew how to round numbers in isolation but could not estimate sums and differences in context. One of the first steps in instruction is making sure that children are exposed to and understand how to estimate sums and differences. Estimating allows students to predict answers, check their calculations, and ask themselves if their actual answer is reasonable. The factors that may influence estimating sums and differences is the context and the numbers and operations involved.

Addition and Subtraction

Students need to learn that addition and subtraction are related. They undo each other. The basic facts of addition and subtraction do not have to be learned as separate facts. Below are some examples.

6 + 5 = 11, so 5 + 6 = 11	11 – 5 = 6, so 11 – 6 = 5
11 = 6 + 5, so 11 = 5 + 6	6 = 11 – 5, so 5 = 11 – 6

Addition and subtraction problems can be categorized based on the kinds of relationships they represent. It is important that all of the story problem structures are presented and developed from students' experiences. Initial work with the story structures will focus on join and separate types of problems because students associate the actions in these problems with the operations. However, they must also experience addition and subtraction in part-part-whole and comparison situations. Please refer to the grade level appropriate curriculum documents (*Mathematics 1*, p. 64; *Mathematics 2*, p. 68; and *Mathematics 3*, p. 71) for more information about the story structures and instructional strategies.

Students will be expected to use and describe strategies to determine sums and differences using manipulatives and visual aids. Initial strategies include

- counting on or counting backwards
- one more or one less
- making ten
- doubles
- near doubles

Other strategies are described in the curriculum documents, *Mathematics 1*, *Mathematics 2*, and *Mathematics 3*.

Manipulatives can and should be used to model not only the above strategies but also model the story structures. Examples of manipulatives that can be used for this purpose include

- two-sided counters
- linking cubes
- game materials (number cubes)
- ten-frames
- walk-on number line

Multiplication and Division
When introducing multiplication, students should be introduced to multiplication through situations (equal-group story problems) that lend themselves to modelling with sets, arrays, and linear measurement models, such as number lines. For example, 3 x 5 can be represented with the

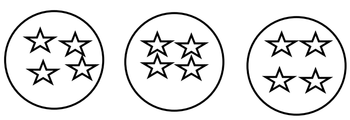
- Set Model – represent 3 plates of 5 cookies by making 3 groups of 5 counters
- Array Model – represent 3 rows of 5 cadets on parade by making 3 rows of 5 counters
- Linear Model – represent 3 jumps of 5 on a number line

The formal writing of multiplication sentences should be delayed until students understand the meaning of multiplication. That is, they can correctly interpret and create story problems, model them correctly, record them pictorially, and write repeated addition number sentences to represent them. It is important not to begin too soon using the word "times" and the multiplication symbol because this may interfere with students' understanding of multiplication situations.

Students should understand and use correct mathematical terms when describing multiplication situations. The numbers being multiplied are called factors and the answers are called products.
For example, in 3 x 5 = 15 both 3 and 5 are factors, and 15 is the product.

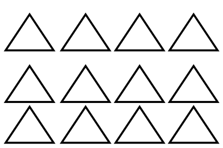
Students should understand the different ways that factors and products can be represented. Below are some examples.

Repeated addition: 3 x 4 means 4 + 4 + 4



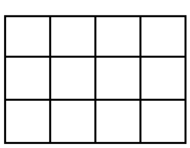
The factor 3 represents the number of groups, the factor 4 represents the quantity within each group, and the product 12 is the total quantity in all groups.

Sets of equal groups: 3 groups of 4 items is equal to 12 (3 x 4 = 12)



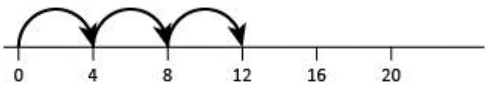
The factor 3 represents the number of groups, the factor 4 represents the quantity within each group, and the product 12 is the total quantity in all groups.

An array: 3 rows of 4 columns is equal to 12 (3 x 4 = 12)



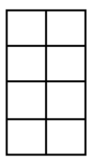
The factor 3 represents the number of rows, the factor 4 is the quantity within each row, or the number of columns and the product 12 is the total quantity in all rows.

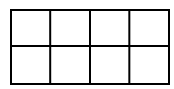
Jumps on a number line: 3 jumps of 4 on a number line



The factor 3 represents the number of jumps, the factor 4 represents the length of each jump, and the product 12 is the total length of all jumps.

Many students were not successful when working with the array model. The array is a powerful model to illustrate the order or commutative property in multiplication. For example, the first array below has 4 rows of 2 columns and therefore is a model for 4 x 2. The second array has 2 rows and 4 columns and therefore is a model for 2 x 4. Both have a product of 8. This becomes important when studying matrices in further grades. These arrays can also be used to represent 8 ÷ 4 = 2 and 8 ÷ 2 = 4.

4 x 2 = 8 

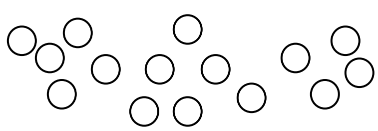
2 x 4 = 8 

Students should be introduced to division through story problems. For this instruction, there are two types of situations, equal-sharing and equal-grouping, which need to be considered. Equal-sharing problems are those in which the number of groups is known and the number in each group needs to be found.

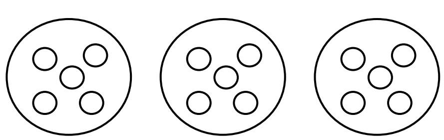
Students should see that dividing the class into two groups, sharing 12 pieces of paper with 4 students, and sharing a large bag of candy into 3 small bags are all examples of equal-sharing situations.

Below are some sample questions that represent division through story problems.

1. Three friends want to share 15 candies. How many candies will each friend get?

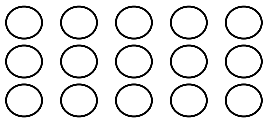


Each friend will get 5 candies. This may be described that when 15 is divided into 3 groups, there are 5 in each group.

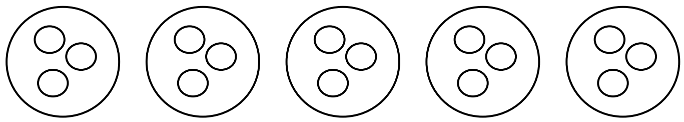


Equal-grouping problems are those in which the number in each group is known and the number of groups needs to be found. Students should also see that dividing the class into groups of 5, giving each student 4 pencils, and placing books into stacks of 4 are all examples of equal-grouping situations.

2. Friends want to share 15 candies by each taking 3 candies. How many friends will get candies?



Five friends will get candies. This may be described that when 15 is divided into groups of 3, there are 5 groups.

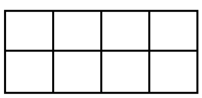


Students should solve several examples of both of these types of division problems by modelling them concretely, recording them pictorially, and describing the division in words before they are introduced to division sentences.

D. What are the most appropriate methods and activities for assessing student learning?

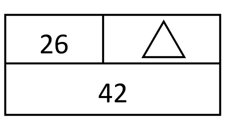
Below are some sample questions that represent operations.

1. This array represents which multiplication fact?



- ○ 1 x 8
- ○ 2 x 4
- ○ 2 x 8
- ○ 4 x 2

2. Forty-two students were in the gym. Twenty-six of them were girls. How many were boys? Choose the equation that shows how to work out this problem.



- ○ 42 = 26 + △
- ○ 26 + 42 = △
- ○ △ – 26 = 42
- ○ 26 – △ = 42

3. 7 + 9 = ☐

- ○ 19
- ○ 16
- ○ 13
- ○ 6

4. 3 + ☐ = 12 – 6

- ○ 1
- ○ 3
- ○ 6
- ○ 9

5. Mila made 2 rectangular cakes. Each cake can be cut into 37 pieces.
About how many pieces of cake did Mila make?

- ○ 40
- ○ 70
- ○ 80
- ○ 90

6. Amir has 12 tomatoes. He puts the tomatoes into 3 boxes in equal groups.
How many tomatoes are in each box?

- ○ 12 + 3 = 15
- ○ 12 – 3 = 9
- ○ 12 x 3 = 36
- ○ 12 ÷ 3 = 4

7.

$$\begin{array}{r} 809 \\ -\ 489 \\ \hline \end{array}$$

- ○ 320
- ○ 420
- ○ 480
- ○ 1298

8. 363 + 25 =

- ○ 308
- ○ 388
- ○ 618
- ○ 5118

9. Lu has 72 stickers. Lilly has 29 stickers.
About how many more stickers does Lu have than Lilly?
Choose the best estimate.

- ○ 100
- ○ 90
- ○ 40
- ○ 30

10. Marley did this subtraction.

675 – 346 = 329

Which expression could help her check her work?

- ○ 675 + 329
- ○ 675 + 346
- ○ 329 + 346
- ○ 346 – 329

11. A vegetarian pizza was cut into slices.
Choose the fraction that would represent the smallest slice of pizza.

- ○ $\frac{1}{4}$
- ○ $\frac{1}{2}$
- ○ $\frac{1}{8}$
- ○ $\frac{1}{10}$

Mathematics in Grade 4 Lesson Learned 4
Pattern and Relations

Students were challenged when asked to transfer their visual representation of patterns to numerical patterns. Students should be able to describe either an increasing shape pattern or a decreasing shape pattern, but need to recognize that each term has a numeric value. Students forget that a pattern rule must have a starting point. Without the starting point, the pattern rule is incomplete. They need to be encouraged to provide all the information for a pattern rule. Students need to continue to work with representations of patterns, contextually, pictorially, symbolically, and verbally.

Patterns are the foundation for many mathematical concepts. Patterns should be taught throughout the year in situations that are meaningful to students. Patterns are explored in all the strands and are also developed through connections with other disciplines, such as science, social studies, English language arts, physical education, and music. Providing students with the opportunity to discover and create patterns, and then describe and extend those patterns, will result in more flexible thinking across strands. Students should initially describe non-numerical patterns, such as shape, action, sound, and then incorporate numerical patterns by connecting them to the non-numerical patterns.

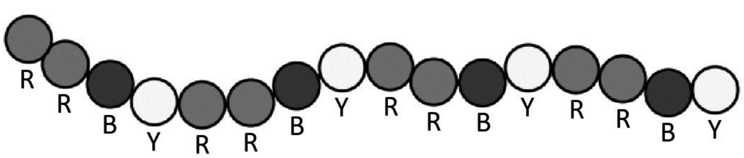
A large focus in Mathematics 3 is the introduction and development of decreasing patterns. Students use their knowledge of increasing patterns to make connections to the concept of decreasing patterns, since similar understandings are developed. Several of the same tasks that were suggested for work on increasing patterns can be used with modifications to represent decreasing patterns.

A. What conclusions can be drawn from the Nova Scotia Assessment: Mathematics in Grade 4?

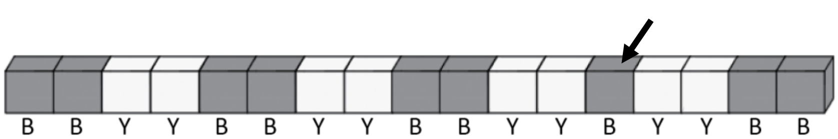
We noticed that our students did very well recognizing simple errors in increasing number patterns, identifying a pattern rule used to create a given increasing pattern, and identifying the next term in an increasing pictorial pattern. These types of items were either knowledge questions (Level 1) or application questions (Level 2). Analysis questions (Level 3) challenged our students. For example, ask students to create an increasing pattern in which a specific element is identified (e.g., the 7th element is 56).

Students do extremely well when the patterns that they are working with are a visual representation of a pattern. See below for examples:

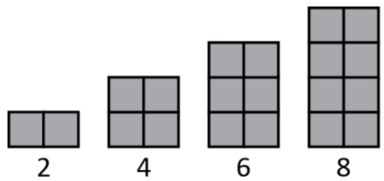
"The pattern for my bead is red (R), red (R), blue (B), yellow (Y)"



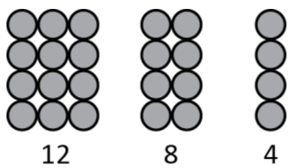
"I see a mistake in this block pattern. It needs another blue block here."



Students were challenged when they were transferring their knowledge of visual patterns to numerical patterns. Students should be able to describe an increasing pattern made of shapes, but need to recognize that each term in the pattern also has a numeric value. For example,



This expectation also applies to decreasing patterns. For example,



B. Do students have any misconceptions or errors in their thinking?

> One of the most fundamental concepts in pattern work, but also one not clear to all students, is that, although the part of the pattern that they see is finite, when mathematicians talk about a pattern, they are talking about something that continues beyond what the student sees. (Small, 2009, p.568)

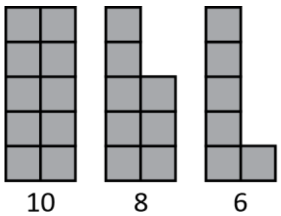
In Mathematics 2, students describe, reproduce, extend, and create repeating, increasing and, decreasing patterns. They use ordinal numbers (to tenth) to describe elements of repeating patterns. Students in Mathematics 3 explore increasing and decreasing patterns, both numerical patterns with numbers to 1000 and non-numerical patterns with concrete materials, pictures, sounds, and actions. They use ordinal numbers (to 100^{th}) to refer to or to predict terms within an increasing pattern.

> A common misconception students have when increasing number patterns or decreasing number patterns, is that they have difficulty extending an increasing number pattern or a decreasing number pattern. A suggested strategy is to have students locate the numbers in the pattern on a hundred chart and place a transparent counter over each number. Have students use the visual pattern in the counters to extend the pattern. Help students relate the visual pattern to the starting point and the number added each time in the number pattern. (Pearson, 2009b, p. 14)

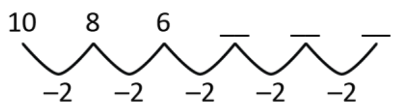
For example, when skip counting by 3, use only starting points that are multiples of 3 (3, 6, 9, 12, ...). This will result in a diagonal representation on a hundred chart. Skip counting with 5, starting at 0, results in a pattern that is two vertical columns with numbers ending in the digits 5 and 0. Students should also explore hundred charts to 1000 (1–100, 101–200, 201–300, …) and look for patterns when counting by 2s, 5s, 10s, 25s, and 100s.

Another common misconception when exploring decreasing patterns is that students do not extend a decreasing pattern correctly. A suggested strategy is to help students identify how each figure in the pattern differs from each previous figure.

For example, in the pattern below, the pattern rule is to start with 10 squares and decrease by 2 squares each time.



As students describe decreasing shape patterns, help them recognize that each term has a numeric value. The above pattern can be expressed as 10, 8, 6, … by counting the number of squares in each term. Students may also find it useful to record the change from one term to the next as shown below:



Remind students that a pattern rule must have a starting point or the pattern rule is incomplete. For example, if a student describes the pattern 10, 8, 6, ... as a decrease by 2 without indicating that the pattern starts at 10, the pattern rule is incomplete.

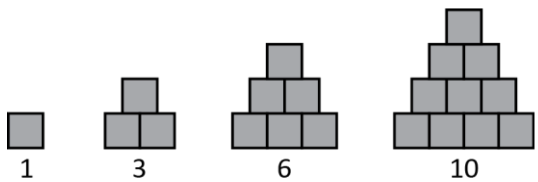
C. What are the next steps in instruction for the class and for individual students?

As students identify the core of a pattern, be sure to use appropriate patterning vocabulary, such as **core** (the repeating part of the pattern) and **elements** (the actual objects used in the pattern) with students. It is important to create patterns that have the core repeated at least three times. To help students identify the pattern core, it is suggested students highlight, or isolate, the core each time it repeats. Remind students that repeating patterns can be extended in both directions. Encourage students to reference the position of the elements of the pattern using ordinal numbers. The core of the shape pattern below is: circle, square, triangle. There are three elements in this pattern, namely a circle, a square, and a triangle.

The pattern above is also a three-element pattern. The core of this three-element pattern is circle (1st element), square (2nd element), and triangle (3rd element).

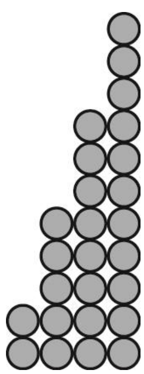
Increasing Numeric Patterns

Students should be able to describe an increasing pattern. An increasing pattern is a growing pattern where the size of the term increases in a predictable way. The terms in an increasing pattern grow by either a constant amount or by an increasing amount each time. Students need sufficient time to explore increasing patterns using various manipulatives, such as cube-a-links, tiles, toothpicks, counters, pattern blocks, base-ten blocks, ten-frames, bread tags, stickers, or buttons, to realize they increase in a predictable way. As students describe increasing shape patterns, help them recognize that each term has a numeric value. For example,

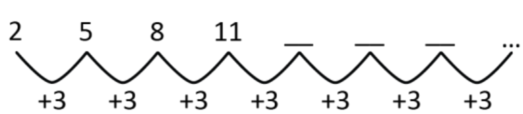


A counting sequence is an increasing pattern where each number represents a term in the pattern. For example, in the counting sequence 1, 2, 3, 4, …, 1 represents the first term, 2 the second term, 3 the third term … This counting sequence can be connected to ordinal numbers where students should be able to recognize that the 34th term is 34 and that 57 is the 57th term in the sequence. These ordinal number patterns should be investigated for numbers up to 100.

Students should be able to describe a given increasing pattern by stating the pattern rule. A pattern rule tells how to make the pattern and can be used to extend an increasing pattern. Give students the first three or four terms of an increasing pattern. Ask them to state the pattern rule by identifying the term that represents the starting point and describing how the pattern continues. For example, in the pattern below, the pattern rule is, start with 2 counters and add 3 counters each time.



As students describe concrete or pictorial patterns, help them recognize that each term has a numeric value. For example, the above pattern can be expressed as, 2, 5, 8, 11, … by counting the number of counters in each term. Students may also find it useful to record the change from one number to the next as shown below.



Students should be able to extend a pattern by identifying the rule, and then use the rule to build and draw the next three terms. Initially, students should replicate the first three terms with concrete materials and then extend the pattern. The use of the concrete materials allows them to make changes if necessary and to build onto one term to make the next term. Students should be able to explain why their extension follows the pattern. It is important to note that for some patterns, there may be more than one way to extend the pattern. For example, If only one term is given, such as the third term 12, some possible solutions could be

4, 8, 12, 16, …

3, 7, 12, 18, …

2, 6, 12, 20, …

6, 9, 12, 15, …

Students need opportunities to compare numeric patterns, discussing how they are the same and how they are different. When comparing increasing patterns, compare the starting points and how each term increases. For example, one way students may address this is by using a page with four small hundred charts. Ask them to skip count starting a 0 and shade one chart by 2s, one chart by 5s, one chart by 10s, and one chart by 25s. Then discuss the pattern rule in each chart comparing the starting points and the amount of increases.

Students should be able to create various representations of an increasing pattern that follow a given pattern rule. Initially students will create patterns with concrete materials, then pictures, and then numbers. Some students may create a simple pattern, while other students may create a more complex pattern.

Students should be able to create increasing patterns, concretely, pictorially, and symbolically, and be able to describe the pattern rule they used to create their representations. When creating increasing patterns, initially students need to choose a starting point and then decide on the amount of increase. The amount of increase may be either a constant amount or an increasing amount. Students should be able to describe their pattern by clearly explaining how it changes from one term to the next. Students may share their patterns and the strategies they used to create their pattern.

Students should have frequent experiences using increasing patterns to solve real-world problems that interest and challenge them. They should use concrete materials or pictures to model the problem before determining the pattern rule. A variety of strategies may be used to solve the problem such as using a number line, a hundred chart, a picture, concrete materials, or skip counting.

Students should be able to identify and describe the strategy used to determine a missing term in a given increasing pattern. Since patterns increase in a predictable way, to determine a missing term, students should first look at the term that comes before and after. One strategy may be to identify and use the pattern rule. Other possible strategies may include using a number line, a hundred chart, a picture, concrete materials, or skip counting.

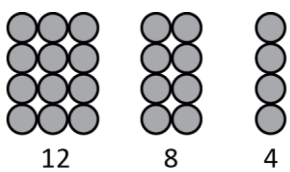
Decreasing Numeric Patterns

Students should be able to describe a decreasing pattern. A decreasing pattern is a shrinking pattern that decreases by a constant amount each time. Students need sufficient time to explore decreasing patterns using various manipulatives, such as cube-a -links, tiles, toothpicks, counters, pattern blocks, base-ten blocks, ten-frames, bread tags, stickers, and buttons. Sometimes students are more comfortable during the exploration stage if they can experiment first, using manipulatives, then pictures, and eventually numbers.

Students should be able to identify and describe various decreasing patterns such as horizontal, vertical, and diagonal patterns found on a hundred chart. Working with decreasing patterns can be connected to skip counting in outcome N01. Provide copies of hundred charts. Ask students to begin at 100 and skip count backward by a given number, shading in the number for each count all the way to 1. Then they write a description of the pattern. For example, if they chose 5, the pattern is two vertical columns, with numbers ending in the digits 5 or 0.

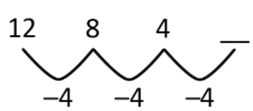
As students begin to investigate patterns, they sometimes confuse repeating patterns with decreasing patterns. Remind them to look for a core first. If they cannot find a core, then the pattern is not a repeating pattern.

Earlier, students became familiar with assigning a numeric value to each element in an increasing pattern. This expectation also applies to decreasing patterns.



Students should be able to describe a given decreasing pattern by stating the pattern rule. A pattern rule includes a term representing a starting point and a description of how the pattern continues. A pattern rule tells how to make the pattern and can be used to extend a pattern. For example, in the pattern above, the pattern rule is to start with 12 squares and decrease by 4 squares each time.

As students describe decreasing shape patterns, help them recognize that each term has a numeric value. The above pattern can be expressed as 12, 8, 4, … by counting the number of squares in each term. Students may also find it useful to record the change from one term to the next as shown below.



Remind students that a pattern rule must have a starting point or the pattern rule is incomplete. For example, if a student describes the pattern 12, 8, 4, ... as a decrease by 4 pattern without indicating that it starts at 12, the pattern rule is incomplete.

Students need opportunities to compare numeric patterns and to discuss how they are the same and how they are different. When comparing decreasing patterns, compare the starting points and how each term decreases using a variety of representations such as shape patterns, hundred charts, and number patterns. For example, give students a page with four small hundred charts. Ask them to skip count backward starting at 100 and shade one chart by 2s, one chart by 5s, one chart by 10s, and one chart by 25s. Then discuss the pattern rule in each chart indicating the starting point and the amount of decrease.

Students should be able to create various representations of a decreasing pattern that follow a given pattern rule. Initially students will create patterns with concrete materials, then pictures, and then numbers. Some students may create a simple pattern, while other students may create a more complex pattern.

Students should be able to create decreasing patterns, concretely, pictorially, and symbolically, and be able to describe the pattern rule they used to create their representations. When creating decreasing patterns, initially students need to choose a starting point and then decide on the amount of the decrease. The amount of decrease may be either a constant amount or an amount that increases each time. Students should be able to describe their pattern by clearly explaining how it changes from one term to the next. Students may share their patterns and the strategies they used to create their pattern.

Students should have frequent experiences using decreasing patterns to solve real-world problems that interest and challenge them. They should use concrete materials or pictures to model the problem before determining the pattern rule. A variety of strategies may be used to solve the problem, such as using a number line, a hundred chart, a picture, concrete materials, or skip counting.

Students should be able to identify and describe the strategy used to determine a missing term in a given decreasing pattern. Since patterns decrease in a predictable way, to determine a missing term the students should first look at the term that comes before and after. One strategy may be to identify and use the pattern rule. Other possible strategies include using a number line, a hundred chart, a picture, concrete materials, or skip counting.

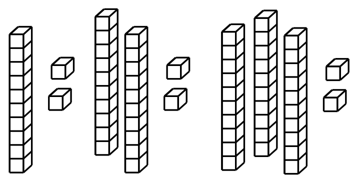
D. What are the most appropriate methods and activities for assessing student learning?

Below are some sample questions related to patterns.

1. Create an increasing pattern in which a specific element is identified.

For example, the 5th element is 50.

2. Look at the base-10 blocks below.

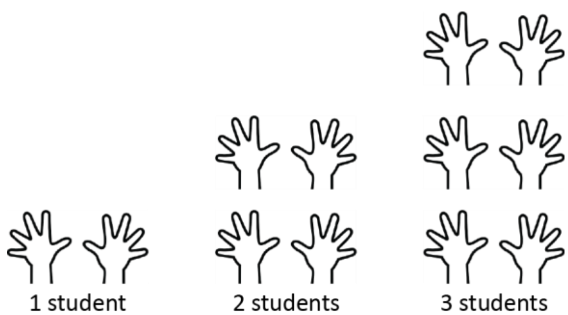


a) Identify what would be next in the pattern. How do you know?
b) Use a number line to show an increasing and decreasing pattern. Describe the pattern.

Have a whole class discussion/debriefing for parts a) and b).

3. How many hands are there in our classroom? How can we use increasing patterns to find the answer?

Model the process using digital pictures of students' hands and then translate them to numbers, for example, see below.



Number of Students	Number of Hands
1	2
2	4
3	6
4	?

4. You are having a birthday party. One square table will seat 4 of your friends. If 2 square tables were put together, you could seat 6 of your friends. How many friends could you seat with 6 square tables put together? 8 square table put together? 10 square tables put together? Have students explain their reasoning.

5. Show different ways these patterns could be extended.

20, 40, ___, ___, ___, ...

1, 4, ___, ___, ...

6. Identify the pattern rule of the following increasing patterns and extend the pattern 3 more terms.

4, 7, 10, 13, 16, …

13, 18, 23, 28, 33, …

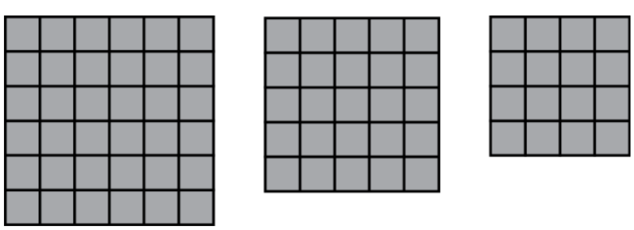
7. Explore hundreds charts to 1000 (1–100, 101–200, 201–300, etc.).
Look for increasing patterns when counting forward by 2s, 5s, 10s, 25s, and 100s.
Shade the different patterns with different colours and compare the patterns.

8. Palo is counting coins. He says, "15, 20, 25, 30, 35, 40. I have 40 cents."
What coins do you think he has? Explain your thinking.

9. Explore hundreds charts to 1000 (1–100, 101–200, 201–300, etc.).
Look for decreasing patterns when skip counting backward by 2s, 5s, 10s, 25s, and 100s.
Shade the different patterns with different colours and compare the patterns.

10. Suppose you write this pattern: 78, 76, 74, 72, 70, …
Will you write 57 in this pattern?
How do you know?

11. Look at the decreasing pattern below,



Continue the pattern using tiles or grid paper.
Record the pattern numerically.

12. Look at these number patterns.

32, 30, 28, 26, 24, ...	95, 90, 85, 80, ...
Pattern 1	Pattern 2

a) Write the pattern rule for each number pattern.

b) How are the patterns the same? How are they different?

c) Write the next 5 numbers in each pattern.

d) Write a number pattern that decreases in a different way.

13. Use this hundred chart to answer the following questions.

100	99	98	97	96	95	94	93	92	91
90	89	88	87	86	85	84	83	82	81
80	79	78	77	76	75	74	73	72	71
70	69	68	67	66	65	64	63	62	61
60	59	58	57	56	55	54	53	52	51
50	49	48	47	46	45	44	43	42	41
40	39	38	37	36	35	34	33	32	31
30	29	28	27	26	25	24	23	22	21
20	19	18	17	16	15	14	13	12	11
10	9	8	7	6	5	4	3	2	1

a) Shade a decreasing number pattern on the hundred chart from 100–1.

b) Record your pattern rule.

c) Compare your pattern rule with that of a classmate.

d) How are the patterns the same?

e) How are they different?

14. You will need a hundred chart from 100–1 and a marker.
You have 85 cents.
The school store sells pieces of fruit for 5 cents each.
How many pieces of fruit can you buy?
Record your work on the hundred chart.
What pattern do you see?

Mathematics in Grade 4 Lesson Learned 5
Problem Solving

Students need more exposure to application (Level 2) and analysis (Level 3) items in order to apply these higher order thinking skills when problem solving. Students need to be encouraged to understand that problems may have more than one entry point and there are many strategies to solve a problem. Students need to continue to work on translating between and among representations when problem solving. They do not seem to realize that they can use all three representations when asked to solve a word problem. These other representations may support their problem solving and their reasoning.

Learning through problem solving should be the focus of mathematics at all grade levels. Problem solving is one of the critical mathematical processes that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics. When students encounter new situations and respond to questions such as, How would you …? or How could you …? the problem-solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.

In order for an activity to be problem-solving based, it must challenge students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. A true problem-solving activity requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement, perseverance, and collaboration. Problem solving is also a powerful teaching tool that fosters multiple, creative, and innovative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident mathematical risk takers.

Students need to be able to explore a wide variety of methods for solving and verifying problems in all areas of mathematics. They must be challenged to find multiple solutions for problems and be given opportunities to create and solve their own problems.

A. What conclusions can be drawn from the Nova Scotia Assessment: Mathematics in Grade 4 information?

We found that students have a good understanding of basic facts and procedures, when explicitly given all the information needed to do a knowledge (Level 1) question. But when given application (Level 2) and analysis (Level 3) items, they are not able to apply higher order thinking skills when problem solving. For example, students were not sure whether they should add or subtract when questions were presented in the context of a story problem. Overall, our students are experiencing challenges with problem solving across all mathematical strands. This appeared to be a theme throughout the assessment data when students were asked to "solve a word problem" or "solve a multi-step problem". The data also showed that, across all mathematics strands, students struggle with items that require translating among representations in problem-solving contexts.

B. Do students have any misconceptions or errors in their thinking?

Many students, when given a problem-solving task, especially a “word problem” (solving a problem in a context) have the misconception that is it always too hard for them to attempt. Putting words around the numbers seems to obstruct their ability to think about the question. Another misconception students have is that there is only one way to solve a word problem. When asked to solve a problem in a context, they struggle to identify a possible strategy and often fail to even attempt to solve the problem. Along with not having problem-solving strategies to help them attempt the problem, students also tend to forget any knowledge of translating between and among representations. If asked to solve a word problem using words, symbols and/or pictures, most students only provide symbols. They do not seem to realize that they can use all representations when asked to solve a word problem. These other representations may support their problem solving.

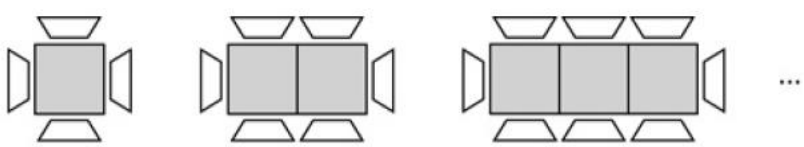
For example, when given an analysis (Level 3) problem in the context of an increasing pattern, students struggle with answering “what strategy could I use to solve this problem”? There are many strategies and ways to solve the problem. Some students find it difficult to find an entry point to begin to solve the problem.

Problem: Some friends are coming to your birthday party. A square table with 4 chairs can seat 4 of your friends. If 2 square tables are put together, you can seat 6 of your friends. How many friends can you seat with 5 square tables?

Solution 1: What do I know?

- I know that one square table can seat 4 of my friends.
- I know that if 2 square tables are put together, 6 of my friends can have seats.
- So if I continue this pattern, 3 square tables put together, can seat 8 of my friends.
- 4 square tables put together, can seat 10 of my friends, and 5 square tables put together, can seat 12 of my friends.
- So 5 square tables can seat 12 of my friends

Solution 2: Draw a picture and a chart



Tables	Chairs	Number of Friends
1	4	4
2	6	6
3	8	8
4	10	10
5	**?**	**?**

So 12 friends can be seated at 5 square tables when they come to my birthday party.

Students could have used concrete materials/manipulatives such as coloured tiles, two-sided counters, cube-a-link blocks, and pictures or numbers to solve this question.

C. What are the next steps in instruction for the class and for individual students?

A significant part of learning to solve problems is learning about the problem-solving process. It is generally accepted that the problem-solving process consists of four steps – understand the problem, devise a plan, carry out the plan, and look back to determine the reasonableness of an answer. Teachers need to teach their lessons through a problem-solving approach. Students learn mathematics as a result of solving problems. It is important to point out that not all lessons students encounter must be taught through problem solving. If the purpose of the lesson being taught is to develop a certain skill for conceptual understanding, then some practice is required.

> The teacher provides a context or reason for the learning by beginning the lesson with a problem to be solved. This approach contrasts with the more traditional approach of, for example, presenting a new procedure and then adding a couple of word problems at the end for students to solve. Instead, the teacher gives students the opportunity to think about the problem and work through the solution in a variety of ways, and only then draws the procedures out of their work. (Small, 2005, p. 154)

An important aspect of problem solving in grades 1–3 is addition and subtraction problems which can be categorized based on the kinds of relationships they represent. It is important that all of the story problem structures are presented and developed from students' experiences. Initial work with the story structures will focus on join and separate types of problems because students associate the actions in these problems with the operations. However, they must also experience addition and subtraction in part-part-whole and comparison situations. Please refer to the grade level appropriate curriculum documents (Mathematics 1, p. 64; Mathematics 2, p. 68; and Mathematics 3, p. 71) for more information about the story structures and instructional strategies.

Manipulatives can and should be used to model the strategies and the story structures. Examples of manipulatives that can be used for this purpose include

- two-sided counters
- linking cubes
- number cubes
- ten-frames
- walk-on number line

A Problem-Solving Approach

A problem-solving approach is embedded and expected throughout our curriculum, grades Primary to 12, in all strands.

As noted in Pearson (2009a), problem-solving is a key strategy:

> Problem-solving is a key instructional strategy that enables students to take risks, secure in the knowledge that their thoughts, queries, and ideas are valued. As students share their solutions and findings, the teacher has the opportunity to provide direct instruction on problem-solving strategies. After students share their solutions and justifications, teachers can elaborate on their methods and encourage students to comment or ask questions of their peers. Using student findings and solution methods as a means to guide instruction also allows students to see the value in their work, and encourages peers to share their strategies. While some strategies may be more efficient than others, several strategies may work and often a combination of strategies is required to solve a problem. Students must use strategies that are meaningful to them and make sense to them. (p. 13)

Problem-Solving Strategies
Students are already drawing on personal strategies for problem solving, in many of the activities they undertake. *Strategies Toolkit* lessons found in the *Pearson Math Makes Sense Series,* allows teachers to expand their students' personal repertoires through explicit instruction on a specific strategy. "When students develop a name for the strategy, they develop a stronger self-awareness of the personal strategies they are starting to use on their own" (Pearson, 2009a, p. 13).

The *Strategies Toolkit* lessons highlight these problem-solving strategies (Pearson, 2009a, p. 13):

- Make a chart or table
- Draw a picture
- Work backward
- Make an organized list
- Use a model
- Solve a simpler problem
- Guess and test
- Use a pattern

Van de Walle and Lovin (2006b), in their resource, *Teaching Student-Centered Mathematics Grades 3–5*, suggest a three-part lesson format for teaching through problem-solving. This same approach is used in our core resource, *Math Makes Sense* (Pearson, 2009b, pp. 13–14):

Before
Before students begin:

- Prepare meaningful problem scenarios for students. These should be sufficiently challenging, but easy to solve.
- Ensure the problem is understood by all.
- Explain the expectations for the process and the product.

During
As students work through the problem:

- Let students approach the problem in a way that makes sense to them.
- Listen to the conversations to observe thinking.
- Assess student understanding of her/his solution.
- Provide hints or suggestions if students are on the wrong path.
- Encourage students to test their ideas.
- Ask questions to stimulate ideas.

After
After students have solved their problem:

- Gather for a group meeting to reflect and share.
- Make the mathematics explicit through discussion.
- Highlight the variety of answers and methods.
- Encourage students to justify their solutions.
- Encourage students to comment positively or ask questions regarding their peer's solutions.

For more details on using a problem-solving approach to teach mathematics, see Van de Walle and Lovin (2006b), *Teaching Student-Centered Mathematics Grades 3–5,* from which these ideas are drawn (pp. 11–28).

Assessing Problem Solving
For information using a rubric to score problem-solving sample questions in either Mathematics in Grade 4 (M4) or Mathematics in Grade 6 (M6), please go to the Program of Learning Assessment for Nova Scotia (PLANS) website. On the Documents tab of each assessment page, you will find a Problem Solving document which includes the provincial rubric and sample questions (plans.ednet.ns.ca/grade4/documents).

From Reading Strategies to Mathematics Strategies
With a problem-solving approach embedded and expected throughout our curriculum grades Primary to 12 in all strands, there are definite implications for teaching reading strategies in mathematics. Concepts and skills should also be connected to everyday situations and other curricular areas. For example, when developing literacy skills students learn to make text-to-world, text-to-text, and text-to-self connections. Students can also make connections to make mathematics come alive through math-to-world, math-to-math, and math-to-self connections.

There are classroom strategies with suggestions of how to use the strategy in a mathematical context that support teachers as they develop students' mathematical vocabulary, initiate effective ways to navigate informational text, and encourage students to reflect on what they have learned. For example, the Frayer Model, Concept Circles, Three-Read Strategy, Exit Cards, etc. These are only a few of the strategies that are found in Appendix B at the end of this document.

When teachers use these strategies in the instructional process or embedded in assessment tasks, the expectations for students must be made explicit. The students' understanding of the mathematics involved and maintaining the integrity of the curriculum are still the foremost concerns.

Please refer to the Appendix B found at the end of this document for strategies with illustrative examples.

D. What are the most appropriate methods and activities for assessing student learning?

Below are some sample questions that represent problem solving.

Encourage students to share their thinking, their strategies, and their solutions. Rich discussion fosters the development of alternative problem-solving strategies.

1. Natasha has 4 T-shirts and 2 pairs of pants.
How many different outfits can Natasha make?
Show how you solved the problem and explain your strategy.

Possible Strategy: Make a chart, table or draw a picture; Strand: Number; Application (Level 2)

2. Megan and Danielle each ordered the same size pizza.
Megan asked to have her pizza cut into fourths. Danielle asked to have her pizza cut into sixths.
Who has the larger pieces of pizza?
Show how you solved the problem and explain your strategy.

Possible Strategy: Use a model; Strand: Number; Application (Level 2)

3. Pierre has a garden that has 3 sides.
The sides are 5 m, 7 m, and 5 m long.
Pierre planted tulips 1 m apart around the edge of his garden.
How many tulips did Pierre plant?
Show how you solved the problem and explain your strategy.

Possible Strategy: Draw a picture; Strand: Measurement; Application (Level 2)

4. Elise had nickels, dimes, and quarters.
She bought a scarf at the school fair for 45 cents.
She did not get any change back.
How many different ways could Elise have paid for her scarf?
Show how you solved the problem and explain your strategy.
How do you know you have found all the different ways?

Possible Strategy: *Solve a simpler problem*; Strand: Statistics and Probability; Analysis (Level 3)

5. Sebastian was at a bicycle sale.
There were bicycles and tricycles.
Altogether, there are 21 wheels.
How many bicycles and tricycles are there?
Show how you solved the problem and explain your strategy.

Possible Strategy: *Work Backward*; Strand: Number; Analysis (Level 3)

6. Read the "Wanted Poster" below and decide which 3-D object is being described.

Use the following 3-D objects to help you decide which 3-D object is being described: cube, prism, pyramid, and sphere.
Show how you solved the problem and explain your strategy.

Possible Strategy: *Guess and test*; Strand: Geometry; Application (Level 2)

7. Marbles come in packages of 10, 25, and 50.
You need 160 marbles.
Find 5 ways you could buy the marbles.
Show how you solved the problem and explain your strategy.

Possible Strategy: *Make an organized list*; Strand: Number; Application (Level 2)

8. Shade a deceasing pattern on a hundred chart from 1 to 100.
Tell the pattern rule.
Compare your pattern with your classmates.
How are the patterns the same?
How are they different?
Show how you solved the problem and explain your strategy.

Possible Strategy: *Use a pattern*; Strand: Patterns and Relations; Analysis (Level 3)

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Appendix A: Cognitive Levels of Questioning

Cognitive Levels of Questioning

Knowledge, Level 1
Knowledge questions (Level 1) may require students to recall or recognize information, names, definitions, or steps in a procedure.

Level 1 Verbs: identify, compute, calculate, name, find, evaluate, use, measure.

Level 1 questions, items, and/or tasks:

- rely heavily on recall and recognition of facts, terms, concepts, or properties
- recognize an equivalent representation within the same form, for example, from symbolic to symbolic
- perform a specified procedure; for example, calculate a sum, difference, product, or quotient
- evaluate an expression in an equation or formula for a given variable
- draw or measure simple geometric figures
- read information from a graph, table, or figure

Application, Level 2
Application/comprehension questions (Level 2) may require students to make connections, represent a situation in more than one way (translating between representations), or solve contextual problems.

Level 2 Verbs: classify, sort, estimate, interpret, compare, explain.

Level 2 questions, items, and/or tasks:

- select and use different representations, depending on situation and purpose
- involve more flexibility of thinking
- solve a word problem
- use reasoning and problem-solving strategies
- may bring together skills and knowledge from various concepts or strands
- make connections between facts, terms, properties, or operations
- represent a situation mathematically in more than one way
- compare figures or statements
- explain and provide justification for steps in a solution process
- translate between representations
- extend a pattern
- use information from a graph, table, or figure to solve a problem
- create a routine problem, given data, and conditions
- interpret a simple argument

Analysis, Level 3

Analysis questions (Level 3) may require students to go beyond comprehension and application to higher order thinking skills, such as generalizations and non-routine problem-solving.

Level 3 Verbs: analyze, investigate, justify, compare and contrast, explain, describe, prove.

Level 3 questions, items, and/or tasks:

- require problem solving, reasoning, planning, analysis, judgment, and creative thought
- thinking in abstract and sophisticated ways
- explain relationships among facts, terms, properties, or operations
- describe how different representations can be used for different purposes
- analyze similarities and differences between procedures and concepts
- generalize a pattern
- solve a novel problem, a multi-step, and/or multiple decision point problem
- solve a problem in more than one way
- justify a solution to a problem and/or assumptions made in a mathematical model
- describe, compare, and contrast solution methods
- formulate a mathematical model for a complex situation, such as probability experiments
- provide a mathematical justification and/or analyze or produce a deductive argument

Below are the percentages of questions at Levels 1, 2, and 3 in the Nova Scotia provincial assessments for Mathematics in Grade 4 and Mathematics in Grade 6:

- Knowledge (Level 1) 20–30%
- Application (Level 2) 50–60%
- Analysis (Level 3) 10–20%

These percentages are also recommend for classroom-based assessments.

Appendix B: From Reading Strategies to Mathematics Strategies

The following table illustrates when strategies are to be used, and during what part of the three-part lesson format (as referenced in Lessons Learned 1).

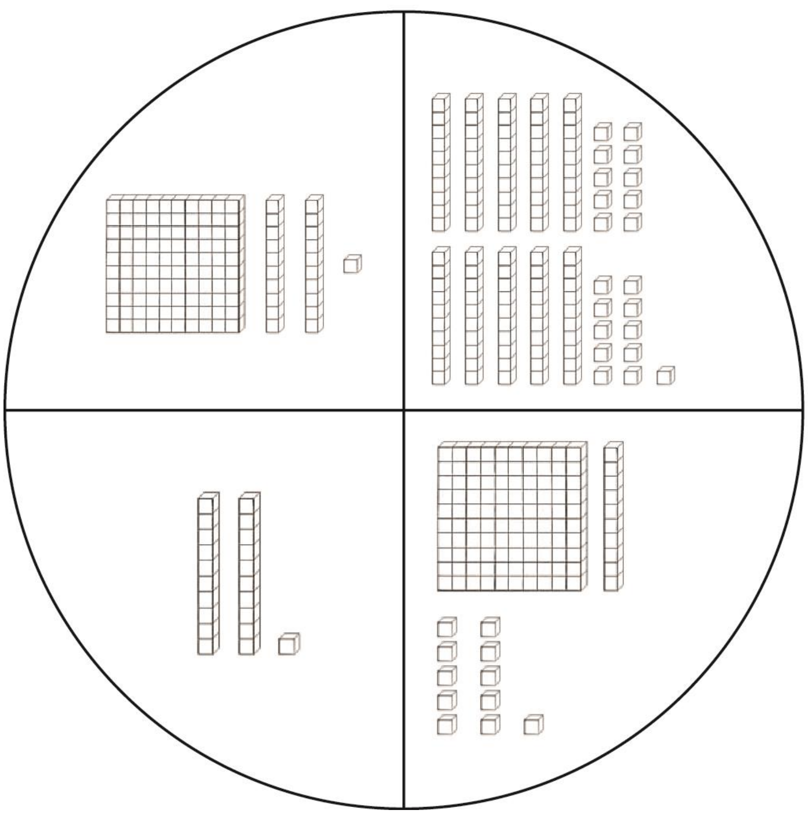
Name of Strategy	**Before**	**During**	**After**	**Assessment**
1. Concept Circles	X	X	X	X
2. Frayer Model	X	X	X	X
3. Concept Definition Map	X	X	X	X
4. Word Wall	X	X	X	
5. Three-Read		X	X	
6. Graphic Organizer	X	X	X	X
7. K-W-L	X		X	X
8. Think-Pair-Share	X	X		
9. Think-Aloud	X	X		X
10. Academic Journal-Mathematics		X	X	
11. Exit Cards			X	X

1. **Concept Circle**
   A concept circle is a way for students to conceptually relate words, terms, expressions, etc. As a "before" activity, it allows students to predict or discover relationships. As a "during" or "after" activity, students can determine the missing concept or attribute or identify an attribute that does not belong.

   The following steps illustrate how the organizer can be used:
   - Draw a circle with the number of sections needed.
   - Choose the common attributes and place them in the sections of the circle.
   - Have students identify the common concepts to the attributes.

   This activity can be approached in other ways.
   - Supply the concept and some of the attributes and have students apply the missing attributes.
   - Insert an attribute that is not an example of the concept and have students find the one that does not belong and justify their reasoning.

   Example of a Concept Circle
   Concept: Which pictures of base-ten blocks represent 121?



2. **Frayer Model**

The Frayer Model is a graphic organizer used to categorize a word and build vocabulary. It prompts students to think about and describe the meaning of a word by

- giving a definition
- describing main characteristics
- providing examples and non-examples of the word or concept

It is especially helpful to use with a concept that might be confusing because of its close connections to another concept.

Definition		**Facts/Characteristics**
	Word	
Examples		**Non-examples**

The following steps illustrate how the organizer can be used.

1. Display the template for the Frayer model and discuss the various headings and what is being sought.
2. Model how to use this example by using a common word or concept. Give students explicit instructions on the quality of work that is expected.
3. Establish the groupings (e.g. pairs) to be used and assign the concept(s) or word(s).
4. Have students share their work with the entire class.

This is an excellent activity to do in poster form to display in class. Each group might do the same word or concept, or different words or concepts could be assigned.

Example of a Frayer Model

Definitions names and orders the days of the week and months of the year	**Facts/Characteristics** • 7 days in a week • 12 months in a year
Calendar	
Examples *Days of Week* Sunday Monday Tuesday Wednesday Thursday Friday Saturday *Months of Year* January February March April May June July August September October November December	**Non-examples** • the number of seconds in a minute • the number of minutes in an hour

3. **Concept Definition Map**

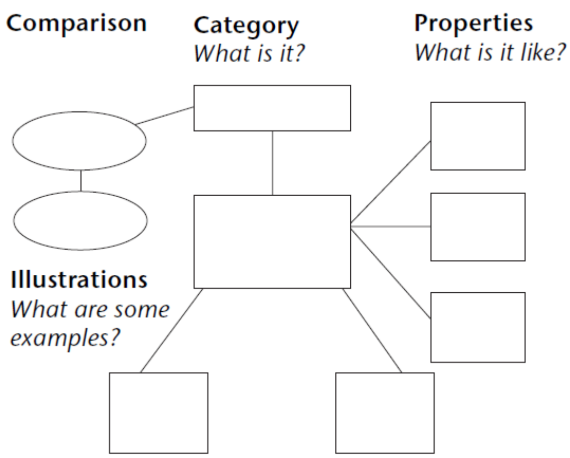
The purpose of a concept definition map is to prompt students to identify the main components of a concept, show the interrelatedness, and build vocabulary. Information is placed into logical categories, allowing students to identify properties, characteristics, and examples of the concept.

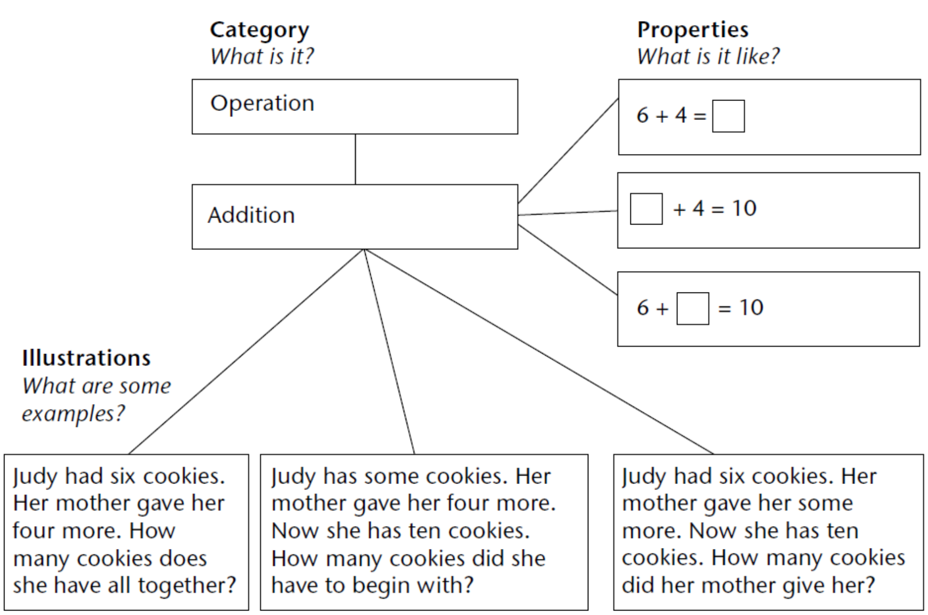
The following steps illustrate how this organizer can be used.

1. Display the template for the concept definition map.
2. Discuss the different headings, what is being sought, and the quality of work that is expected.
3. Model how to use this map by using a common concept.
4. Establish the concept(s) to be developed.
5. Establish the groupings (e.g. pairs) and materials to be used to complete the task.
6. Complete the activity by having the students write a complete definition of the concept.

Encourage students to refine their map as more information becomes available.

Example of a Concept Definition Map





4. Word Wall

A mathematics word wall is based upon the same principle as a reading word wall, found in many classrooms. It is an organized collection of words that is prominently displayed in the classroom and helps students learn the language of mathematics. A word wall can be dedicated to a concept, big idea, or unit in the mathematics curriculum. Words are printed in bold block letters on cards and then posted on the wall or bulletin board.

Illustrations placed next to the word on the word wall can add to the students' understanding. Students may also elaborate on the word in their journals by illustrating, showing an example, and using the word in a meaningful sentence or short paragraph. Students can be assigned a word and its illustration to display on the word wall. Room should be left to add more words and diagrams as the unit or term progresses.

As a new mathematical term is introduced to the class, students can define and categorize the word in their mathematics journal under an appropriate unit of study. Then the word can be added to the mathematics word wall so students may refer back to it as needed. Students will be surprised at how many words fall under each category and how many new words they learn to use in mathematics. Note: The word wall is developed one word at a time as new terminology is encountered.

Use the following steps to set up a word wall.

1. Determine the key words that students need to know or will encounter in the topic or unit.
2. Print each word in large block letters and add the appropriate illustrations.
3. Display cards when appropriate.
4. Regularly review the words as a warm-up or refresher activity.

5. Three-Read Strategy

Using this strategy, found in *Toward a Coherent Mathematics Program: A Study Document for Educators* (Nova Scotia Department of Education 2002), the teacher encourages students to read a problem three times before they attempt to solve it. There are specific purposes for each reading.

First Read

The students try to visualize the problem in order to get an impression of its overall context. They do not need specific details at this stage, only a general idea so they can describe the problem in broad terms.

Second Read

The students begin to gather facts about the problem to make a more complete mental image of it. As they listen for more detail, they focus on the information to determine and clarify the question.

Third Read

The students check each fact and detail in the problem in order to verify the accuracy of their mental image and to complete their understanding of the question.

During the Three-Read strategy, the students discuss the problem, including any information needed to solve it. Reading becomes an active process that involves oral communication among students and teachers; it also involves written communication as teachers encourage students to record information and details from their reading and to represent what they read in other ways with pictures, symbols, or charts. The teacher facilitates the process by posing questions that ask students to justify their reasoning, support their thinking, and clarify their solutions.

In order to teach the Three-Read Strategy, teachers should exaggerate each step as they model it. When they have students practice the strategy, teachers should ask questions that stimulate the kinds of questions that students should be asking themselves in their internal conversations. In every classroom, an ongoing discussion of this Three-Read strategy must be conducted and many students will need to be reminded to use this strategy.

6. Graphic Organizer

A graphic organizer can be of many forms: web, chart, diagram, etc. Graphic organizers use visual representations as effective tools to do such things as

- activate prior knowledge
- analyze
- compare and contrast
- make connections
- organize
- summarize

The following steps illustrate how the organizer can be used.

1. Present a template of the organizer and explain its features.
2. Model how to use the organizer, being explicit about the quality of work that is expected.
3. Present various opportunities for students to use graphic organizers in the classroom.

Students should be encouraged to use graphic organizers on their own as ways of organizing their ideas and work. If the graphic organizer being used is a Venn Diagram, it is important to draw a rectangle around Venn diagrams to represent the entire group that is being sorted. This will show the items that do not fit the attributes of the circle(s) outside of them, but within the rectangle. Therefore, elements of the set that do belong to Attribute A or Attribute B are shown within the rectangle but not within the circles in the rectangles.

7. **K-W-L (Know/Want to Know/Learned)**
K-W-L is an instructional strategy that guides students through a text or mathematics word problem and uses a three-column organizer to consolidate the important ideas. Students brainstorm what they know about the topic and record it in the K column. They then record what they want to know in the W column. During and after the reading, students' record what they have learned in the L column. The K-W-L strategy has several purposes:
    - to illustrate a student's prior knowledge of a topic
    - to give a purpose to the reading
    - to help a student monitor his or her comprehension

Know	Want to Know	Learned

The following steps illustrate how the K-W-L can be used.

1. Present a template of the organizer to students, explain its features, and be explicit about the quality of work that is expected.
2. Ask them to fill out the first two sections (what they know and what they want to know before proceeding).
3. Check the first section for any misconceptions in thinking or weakness in vocabulary.
4. Have the students read the text, and taking notes as they look for answers to the questions they posed.
5. Have students complete the last column to include the answers to their questions and other pertinent information.
6. Discuss this new information with the class, and address any questions that were not answered.

8. **Think-Pair-Share**

Think-Pair-Share is a learning strategy designed to encourage students to participate in class and keep them on task. It focusses students' thinking on specific topics and provides them with an opportunity to collaborate and have meaningful discussion about mathematics.

- First, teachers ask students to think individually about a newly introduced topic, concept, or problem. This provides essential time for each student to collect his or her thoughts and focus on his or her thinking.
- Second, each student pairs with another student, and together the partners discuss each other's ideas and points of view. Students are more willing to participate because they do not feel the peer pressure that is involved when responding in front of the class. Teachers ensure that sufficient time is allowed for each student to voice his or her views and opinions. Students use this time to talk about personal strategies, compare solutions, or test ideas with their partners. This helps students to make sense of the problem in terms of their prior knowledge.
- Third, each pair of students shares with the other pairs of students in large-group discussion. In this way, each student has the opportunity to listen to all of the ideas and concerns discussed by the other pairs of students. Teachers point out similarities, overlapping ideas, or discrepancies among the pairs of students and facilitate an open discussion to expand upon any key points or arguments they wish to pursue.

9. **Think-Aloud**

Think-aloud is a self-analysis strategy that allows students to gain insight into the thinking process of a skilled reader as he or she works through a piece of text. Thoughts are verbalized, and meaning is constructed around vocabulary and comprehension. It is a useful tool for such things as brainstorming, exploring text features, and constructing meaning when solving problems. When used in mathematics, it can reveal to teachers the strategies that are part of a student's experience and those that are not. This is helpful in identifying where a student's understanding may break down and may need additional support.

The think-aloud process will encourage students to use the following strategies as they approach a piece of text.

- Connect new information to prior knowledge.
- Develop a mental image.
- Make predictions and analogies.
- Self-question.
- Revise and fix up as comprehension increases.

The following steps illustrate how to use the think-aloud strategy.

1. Explain that reading in mathematics is important and requires students to be thinking and trying to make sense of what they are reading.
2. Identify a comprehension problem or piece of text that may be challenging to students; then read it aloud and have students read it quietly.
3. While reading, model the process verbalizing what you are thinking, what questions you have, and how you would approach a problem.
4. Then model this process a second time, but have a student read the problem and do the verbalizing.
5. Once students are comfortable with this process, a student should take a leadership role.

10. **Academic Mathematics Journal**

An academic journal in mathematics is an excellent way for students to keep personal work and other materials that they have identified as being important for their personal achievement in mathematics. The types of materials that students would put in their journals would include:

- strategic lessons – lessons that they would identify as being pivotal as they attempt to understand mathematics
- examples of problem-solving strategies
- important vocabulary

Teachers are encouraged to allow students to use these journals as a form of assessment. This will emphasize to the student that the material that is to be placed in his or her journal has a purpose. Mark these journals only on the basis of how students are using them and whether or not they have appropriate entries.

The goal of writing in mathematics is to provide students with opportunities to explain their thinking about mathematical ideas and then to re-examine their thoughts by reviewing their writing. Writing will enhance students' understanding of math as they learn to articulate their thought processes in solving math problems and learning mathematics concepts.

11. **Exit Card**

Exit cards are quick tools for teachers to become better aware of a students' mathematics understanding. They are written student responses to questions that have been posed in class or solutions to problem-solving situations. They can be used at the end of a day, week, lesson, or unit. An index card is given to each student (with a question that promotes understanding on it), and the student must complete the assignment before he or she is allowed to "exit" the classroom. The time limit should not exceed 5 to 10 minutes, and the student drops the card into some sort of container on the way out. The teacher now has a quick assessment of a concept that will help in planning instruction.

Appendix C: Cognitive Levels of Sample Questions

Translating Between and Among Representations		Representing and Partitioning Whole Numbers		Whole Number Operations		Patterns and Relations		Problem Solving	
Question	**Level**	**Question**	**Level**	**Question**	**Level**	**Question**	**Level**	**Question**	**Level**
1	2	1	2	1	2	1	3	1	2
2	2	2	2	2	2	2	2	2	2
3	3	3	2	3	2	3	2	3	2
		4	2	4	2	4	3	4	3
		5	3	5	2	5	2	5	3
		6	2	6	3	6	2	6	2
		7	1	7	2	7	2	7	2
		8	2	8	2	8	3	8	3
		9	2	9	2	9	2		
				10	2	10	2		
				11	3	11	3		
						12	3		
						13	3		
						14	2		