LESSON LEARNED

Focus: Representing Decimal Numbers

Nova Scotia Assessment: Mathematics Grade 6

“For learners to succeed, teachers must assess students’ individual abilities and characteristics and choose appropriate and effective instructional strategies accordingly.”
– Helene J. Sherman

Purpose of this Document

This Lessons Learned document was developed based on an analysis of the Item Description Reports for the Nova Scotia Assessment: Mathematics in Grade 6 (NSA-M6). This document is intended to support all classroom teachers at grades 3 - 6, and administrators at the school, region, and provincial levels. The focus of the document is to help educators work through the process of taking in the information provided by the data analysis and see how it can inform lesson design and assessment in the classroom.

It is suggested that school teams make use of this resource in concert with their school's Item Description Report provided by the Department of Education and Early Childhood Development to all regional centres for education. These reports include student achievement data at the school, regional centre, and provincial level for all questions appearing on the Mathematics in Grade 6 Assessment. By analyzing their own performance on groupings of questions dealing with similar outcomes, schools can identify areas of strength and areas where changes in instruction and/or assessment might be made. This process is designed to foster continued discussions, explorations, and support for mathematics focus at the classroom, school, regional centre, and provincial levels that are all based on valid and reliable data.

This document specifically addresses some of the areas that students across the province found challenging based on provincial assessment data. It is essential that teachers consider assessment evidence from a variety of sources to inform the next steps most appropriate for their students. Effective classroom instruction and assessment strategies are responsive to the individual learners within a classroom.

This document highlights those outcomes where students seem to require additional support. It provides some information about student performance on the assessment in addition to suggested classroom instruction strategies. Sample assessment items are included for each topic explored.

Overview of the Nova Scotia Assessment: Mathematics in Grade 6

Nova Scotia Assessments are large-scale assessments that provide reliable data about how well all students in the province are learning the mathematics curricula. It is different from many standardized tests in that all questions are written by Nova Scotia teachers to align with curriculum outcomes and the results reflect a snapshot of how well students are learning these outcomes. These results can be counted on to provide a good picture of how well students are learning curriculum outcomes within schools, regions and in the province. Since the assessments are based on the Nova Scotia curriculum, and are developed by Nova Scotia teachers, results can be used to determine whether the curriculum, approaches to teaching and allocation of resources are effective. Furthermore, because individual student results are available, these, in conjunction with other classroom assessment evidence, help classroom teachers understand what each student has under control and identify next steps to inform instruction.

The assessment provides information about mathematics for each student and complements assessment data collected in the classroom. This assessment is administered at the beginning of grade 6. It is designed to provide detailed information for every student in the province regarding their progress in achieving a selection of mathematics curriculum outcomes at the end of Grade 5. Information from this assessment can be used by teachers to inform their instruction and next steps in providing support and intervention for their students.

Lessons Learned Overview

Provincial assessments and examinations generate information that teachers can use to help inform classroom instruction and assessment. Following the analysis of each assessment or examination, patterns and trends are identified. These include areas of strength and areas for growth. The Lessons Learned documents specifically highlight concepts where growth is still needed.

There are six areas that have been identified as the areas of focus for this Lessons Learned document.

They are:

- Solving whole number multiplication and division questions in context
- Representing decimals
- Relating fractions and decimal
- Generalizing to extend patterns
- Understanding the relationship between area and perimeter
- Identifying and describing the attributes of objects and shapes

This section specifically addresses representing decimals. It begins with an overview of the student errors and misconceptions identified through the provincial assessment. These include:

- Models for decimal values
- Counting in tenths
- Interpreting and comparing decimal digits

Strategies are then outlined that are designed to enhance student comprehension, drawing from researched best practices. The strategies emphasize the integration of essential models, tools, and interconnections to facilitate the transition between concrete, pictorial, and abstract representations of concepts, highlighted by the importance of deliberate planning and purposeful questioning. To support both assessment and instruction, sample lesson activities are presented alongside a series of cognitive-level questions, providing educators with ideas for addressing knowledge gaps and fostering strategic reasoning and problem-solving skills. Each section culminates with a selection of print and online resources, as well as recommended manipulatives to support professional learning and student understanding of that topic.

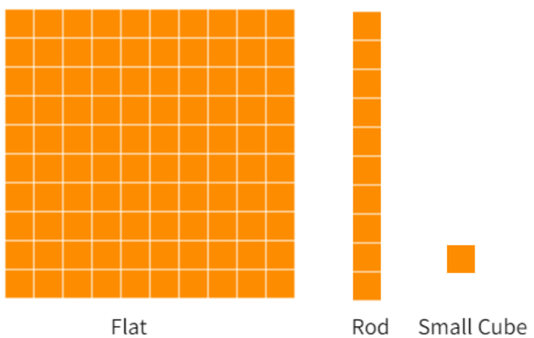
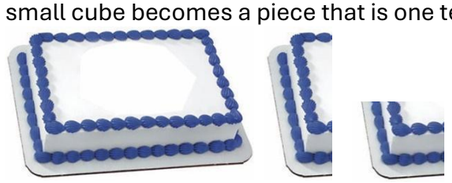
Representing Decimal Numbers

Alignment to previous Outcomes	Related Outcome	Alignment to upcoming Outcomes
4N09: Students will be expected to describe and represent decimals (tenths and hundredths), concretely, pictorially, and symbolically.	**5N08:** Students will be expected to describe and represent decimals (tenths, hundredths, and thousandths) concretely, pictorially, and symbolically. **5N10:** Students will be expected to compare and order decimals (to thousandths) by using benchmarks, place value, and equivalent decimals.	**6N01** Students will be expected to demonstrate an understanding of place value for numbers greater than one million and less than one thousandth.

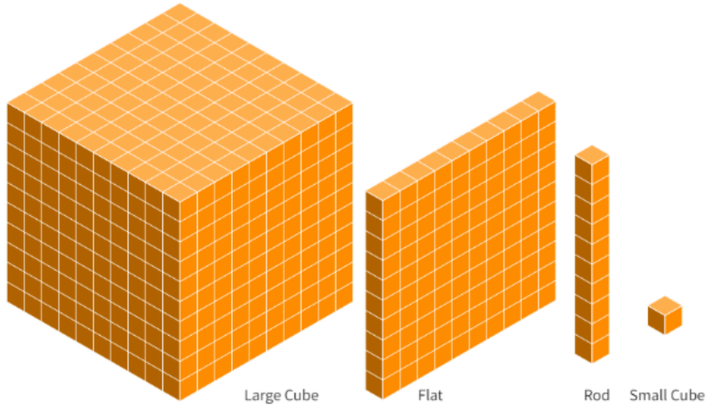
What conclusions can be drawn from the NSA: Mathematics in Grade 6?

When presented with base-ten blocks to represent decimal numbers, almost half of students were unable to identify the decimal number being represented. This was evident especially when the flat was not used to represent one. Students had difficulty thinking proportionally between the blocks to identify the decimal number being represented. Students also had challenges going between representing decimal numbers in words and symbolic form. This resulted in place value errors where students identify digits in the hundreds position rather than the hundredths and likewise in the thousands position rather than the thousandths.

Why is this an area of need and how can we support students?

Models for Decimal Values	
Misconceptions/Errors in Student Work	**Possible Next Steps in the Classroom**
Some students struggle with using the same models for whole numbers as for decimals. If the flat represents 100 with whole numbers, students have difficulty switching to a flat representing the whole, or 1, for decimals.  Decimals: If the flat is 1, the rod is 0.1, and the small cube is 0.01.	One important strategy to remember when transitioning to decimals using base ten materials is helping students to understand that the same proportional relationship still exists when working with decimal numbers. Whatever block represents the whole, proportionally we are still thinking in terms of tens or tenths. Like with whole numbers, students will need many opportunities to explore this concept and explain the relationship between the same digits in different numbers. When working with base ten materials, make sure that students do not refer to the flat as 100 but as 1 whole. Relate the flat to everyday items such as one whole cake. In this case, the rod becomes a slice that is one tenth of the cake, and the small cube becomes a piece that is one tenth of the slice and one hundredth of the whole cake. 

As such, students also struggle when another block is used to represent 1, such as a large cube or the rod. Essentially, students are unable to think proportionally or move flexibly between representations in knowing how many parts make up the whole.

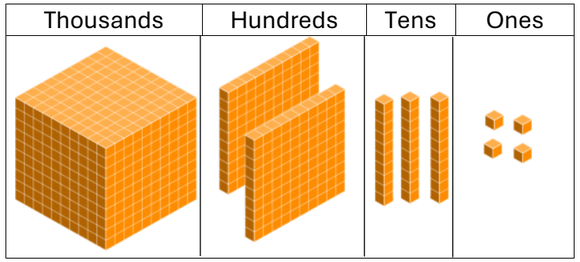


For thousandths: If the large cube is 1, the flat is 0.1, the rod is 0.01, and the small cube is 0.001.

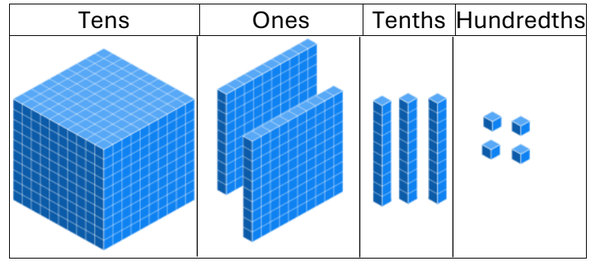
Another analogy may also include thinking about the large cube as a big piece of cheese, the flat as a cheese slice, the rod as a "cheese string", and the small cube as a bite or a small piece.

Using a place value chart along with base-ten blocks may help to support student understanding like whole numbers. If available, using a set of base-ten blocks of a different colour to differentiate decimal blocks from whole number blocks.

Example (whole numbers)



Example (decimals)



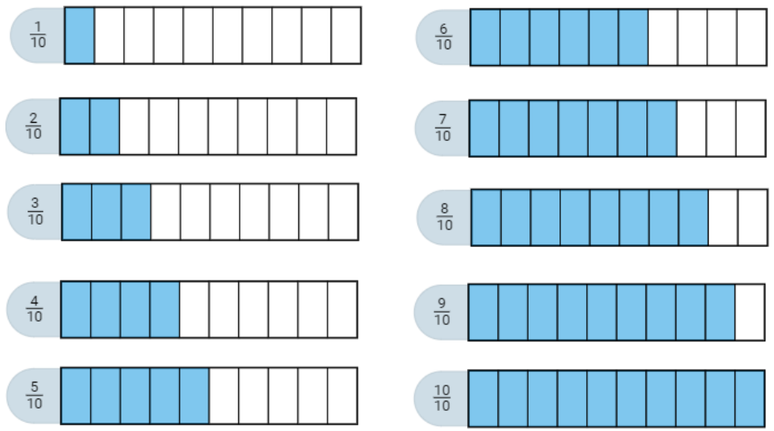
If students struggle with this model for decimals, they might feel more comfortable working with hundredths grids instead. Students can relate how many small squares (hundredths) fill the grid (one whole).

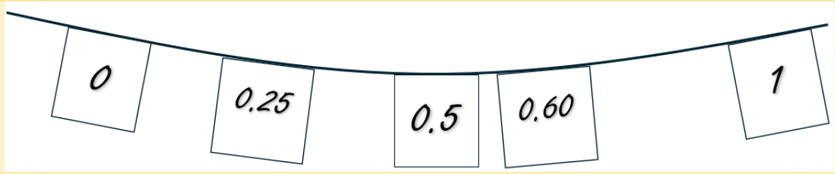
When working with hundredths, another strategy is to use coins. A loonie can represent 1, a dime for 0.1, and a penny for 0.01. Students can build an understanding of tenths and hundredths by relating the number of coins to the decimal value.

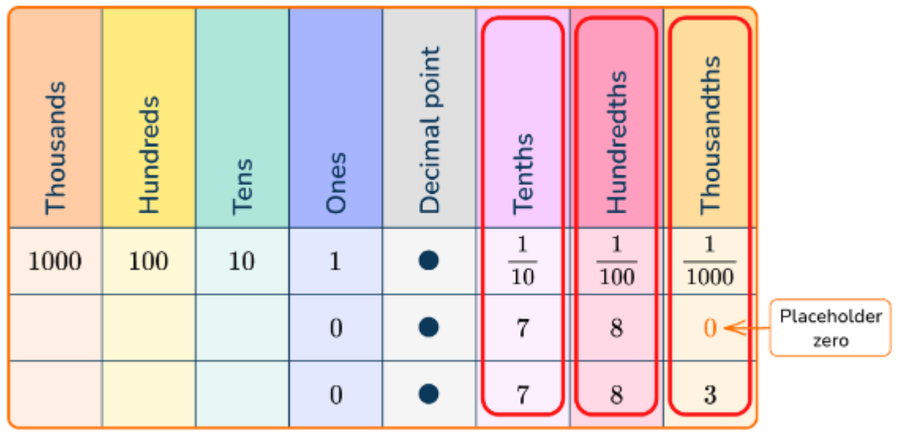
Read decimals and encourage students to read decimals as tenths, hundredths, thousandths, and so on and not as digits. This can include activities where students count by tenths or hundredths and relate to their fractional representations.

	Penny	$0.01
	Nickel	$0.05
	Dime	$0.10
	Quarter	$0.25
	Loonie	$1.00

Sample Activities to Support Lesson Planning		
Grade 4	**Grade 5**	**Grade 6**
Ask students to draw a picture on a decimal grid. Each colour or figure used represents a decimal value including tenths and hundredths. The activity could also include the use or a discussion on money where students relate pennies, nickels, dimes and quarters to a dollar or loonie; e.g. “My figure is worth 30 cents or $0.30.” Knowledge: Draw two different shapes on your grid worth the same decimal amount. Draw two shapes on your grid worth different decimal amounts. Application: What decimals are represented in your drawing? What are ways to write and say this decimal using the words tenths and hundredths? Analysis: Which part of your drawing contains the largest decimal and the smallest decimal? How do you know? Other possibilities: Application: Draw something to match the value _____. Analysis: How do you know you are representing _____? How does it compare to your partners drawing?	Ask students to draw a picture on a thousandths grid. Each colour or figure used represents a decimal value. OR Ask students to create a picture using pattern blocks or tangrams. Each shape/colour/size represents a decimal value. Knowledge: Draw two different shapes on your grid worth the same decimal amount. Draw two shapes on your grid worth different decimal amounts. Application: What decimals are represented in your drawing? What are ways to write and say this decimal using the words tenths, hundredths, or thousandths? Analysis: Which part of your drawing contains the largest decimal and the smallest decimal? How do you know? Other possibilities: Application: Draw something to match the value _____. Analysis: How do you know you are representing _____? How does it compare to your partners drawing?	Ask students to draw a picture on a thousandths grid. Each colour or figure used represents a decimal value. OR Ask students to create a picture using pattern blocks or tangrams. Each shape/colour/size represents a decimal value. Knowledge: Draw two different shapes on your grid worth the same decimal amount. Draw two shapes on your grid worth different decimal amounts. Application: What decimals are represented in your drawing? What are ways to write and say this decimal using the words tenths, hundredths, or thousandths? Analysis: Which part of your drawing contains the largest decimal and the smallest decimal? How do you know? Other possibilities: Application: Draw something to match the value _____. Analysis: How do you know you are representing _____? How does it compare to your partners drawing?

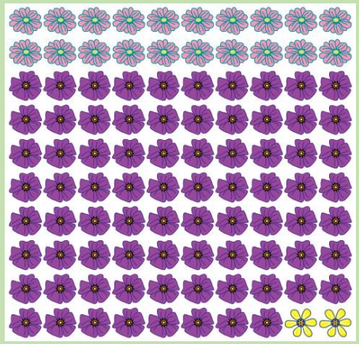
Counting in Tenths	
Misconceptions/Errors in Student Work	**Possible Next Steps in the Classroom**
Some students have difficulty counting in tenths as they bridge whole numbers. For example, … 0.8, 0.9, 0.10, 0.11, 0.12 …instead of 0.8, 0.9, 1.0, 1.1, 1.2. While students may know through counting that they are 8 tenths, 9 tens, 10 tenths, 11 tenths, 12 tenths, representing them in decimal form is difficult.	If students are reading 0.10 as 10 tenths, present them with a 10 by 10 grid, and ask them to show you 10 hundredths and to write the decimal. Have them look at the decimal that they read as 10 tenths to see and hear the contradiction. When students are reading decimals, encourage them to use proper decimal terminology. When instructing, use the decimal name rather than what is typically used in society. 3.04 would be read as “three and 4 hundredths” and not “three decimal zero four.” 0.56 would be read as “fifty-six hundredths” and not “decimal fifty-six.” Have students practice counting in decimals in a similar way students learn how to count in early years. Match the count with the visual and/or symbolic representation of the decimal. Making a connection to how you count in unit fractions supports and strengthens this relationship as well.  Think about what students will say when they get to ten tenths. Will they see this as one? What will happen with eleven tenths or one and one tenth? Reinforce further with images illustrating decimals that go beyond one whole.

Sample Activities for Lesson Planning		
Grade 4	**Grade 5**	**Grade 6**
Have students create a concrete number line or use a clothesline. Provide each student with a decimal value and have them place or stand on the number line. Each student must say their number as they place it. When the number line is complete students count up or count back. Initially, use values less than one, then have students work with values greater than one as well.	Have students create a concrete number line or use a clothesline. Provide each student with a decimal value and have them place or stand on the number line. Each student must say their number as they place it. When the number line is complete students count up or count back. Include numbers that represent the same value (e.g. 0.5 and 0.50) and are greater than and less than one.	Have students create a concrete number line or use a clothesline. Provide each student with a decimal value and have them place or stand on the number line. Each student must say their number as they place it. When the number line is complete students count up or count back. Include numbers that represent the same value (e.g. 0.55 and 0.550 or fractions) and are greater than and less than one.
		
Knowledge: Write a decimal and explain what each digit means. Where could you see your decimal outside of school? Application: Represent your decimal value using base-ten blocks or hundredths grid. How does your value compare to your partner's value? What decimal values are between your two numbers? Name them and represent them. Analysis: What are some decimal values that are less than/greater than yours and your partners? How can you prove these decimals are less or greater? Place the numbers in order and label them using numbers and words.	Knowledge: Write a decimal and explain what each digit means. Where could you see your decimal outside of school? Application: Represent your decimal value using a thousandths grid. How does your value compare to your partner's value? What decimal values are between your two numbers? Name them and represent them. Analysis: What are some decimal values that are less than/greater than yours and your partners? How can you prove these decimals are less or greater? Place the numbers in order and label them using numbers and words.	Knowledge: Write two decimals that are equivalent and greater than 1. Explain how the values are equivalent. Application: Represent your decimal value using a thousandths grid. How does your value compare to your partner's value? What decimal values are between your two numbers? Name them and represent them. Analysis: What are some decimal values that are less than/greater than yours and your partners? How can you prove these decimals are less or greater? Place the numbers in order and label them using numbers and words.

Interpreting and Comparing Decimal Digits	
Misconceptions/Errors in Student Work	**Possible Next Steps in the Classroom**
Some students think that decimal numbers with a greater number of digits or with digits that are greater are “bigger” than they really are. For example, some students believe that 0.248 is larger because it has more digits. 0.248 > 0.79 2.45 < 2.358 Likewise, if the whole numbers of both decimal numbers being compared are the same, students read the values after the decimal as whole numbers (e.g., 45.380 as forty-five and three hundred eighty).	Place value language and estimation is helpful. When working with decimals, the teacher should model and use the decimal name and encourage the students to do the same. Instead of saying, “zero decimal two four eight”, say, “two hundred forty-eight thousandths”. Images and concrete tools that can be used to represent decimal values should be continually used to help reinforce quantity and comparison. Another strategy is to allow students to record zeros to align the numbers along the decimal. 0.248 can be compared to 0.79 by placing a zero to the right of the digit 9. 0.248 0.79**0** Similarly, zeros can be placed to the right of the decimal with a whole number. 4.000 0.591 

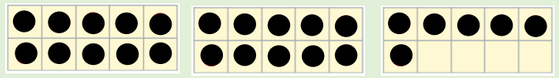
Sample Activities for Lesson Planning		
Grade 4	**Grade 5**	**Grade 6**
Ask students to build the number 0.35 using base-ten materials. Have them show it in more than one way. Do the same again but use the value of 0.4 for example where there are fewer digits, but a larger value. Discuss the various representations using place value language. Knowledge: Place the following decimals on a number line: 0.5, 0.9, 0.12, 0.35, 0.44, 0.76. Application: Represent the decimal values using base-ten blocks. Label each one. How many base-ten blocks did you use in each representation? Are your values the same/different? How do you know? Prove to me/partner which of your two values is greater. Analysis: Represent a different value of your choice. Prove to me/partner which value is greater. Order your values and your partner's values from smallest to greatest. How do you know you are correct? Note: The number line activity noted above in the "counting" section is also helpful at comparing the sequence and size of decimal values.	Ask students to build the number 0.358 using base-ten materials. Have them show it in more than one way. Do the same again but use the value of 0.45 for example where there are fewer digits, but a larger value. Discuss the various representations using place value language. You may also want to include a whole number as well, e.g. 1.358. Knowledge: Place the following decimals on a number line: 1.12, 0.35, 2.4, 0.545, 0.76, 0.09. Application: Represent the decimal values using base-ten blocks. How many base-ten blocks did you use in each representation? Are your values the same/different? How do you know? Prove to me/partner which of your two values is greater. Analysis: Represent a different value of your choice. Prove to me/partner which value is greater. Order your values and your partner's values from smallest to greatest. How do you know you are correct? Note: The number line activity noted above in the "counting" section is also helpful at comparing the sequence and size of decimal values.	Ask students to build the number 0.358 using base-ten materials. Have them show it in more than one way. Do the same again but use the value of 0.45 for example where there are fewer digits, but a larger value. Discuss the various representations using place value language. You may also want to include a whole number as well, e.g. 1.358. Knowledge: Place the following decimals on a number line: 1.12, 0.358, 2.4, 0.545, 0.76, 1.09. Application: Represent the decimal values using base-ten blocks. Label each one. How many base-ten blocks did you use in each representation? Are your values the same/different? How do you know? Prove to me/partner which of your two values is greater. Analysis: Represent a different value of your choice. Prove to me/partner which value is greater. Order your values and your partner's values from smallest to greatest. How do you know you are correct? Note: The number line activity noted above in the "counting" section is also helpful at comparing the sequence and size of decimal values.

What are some sample questions to help support assessment?

Cognitive Level	Grade 4	Grade 5	Grade 6
Knowledge	Why does the arrangement of flowers make it easy to describe 0.2 and 0.02 of the flowers?  *Eyes on Math g3–5 Identify the place value of each digit in the following number: 4.56 What is the number 6.87 in words?	What is the number 6.803 in words? Identify the place value of each digit of 14.352? Choose the appropriate symbol >, =, or < to compare 36.09 to 36.090.	What is the number 6.8039 in words? Identify the place value of each digit of 14.35279? Choose the appropriate symbol >, =, or < to compare: 36.09 to 36.090 123.456 to 123.4567

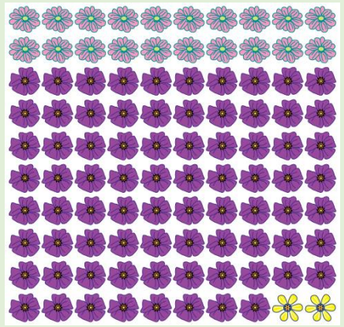
Application

If a ten-frame represents one whole or 1, which decimal number is represented by the following illustration?



Represent the value 0.27 on a hundredths grid or with base-ten blocks.

What other decimals of the flowers are easy to describe?

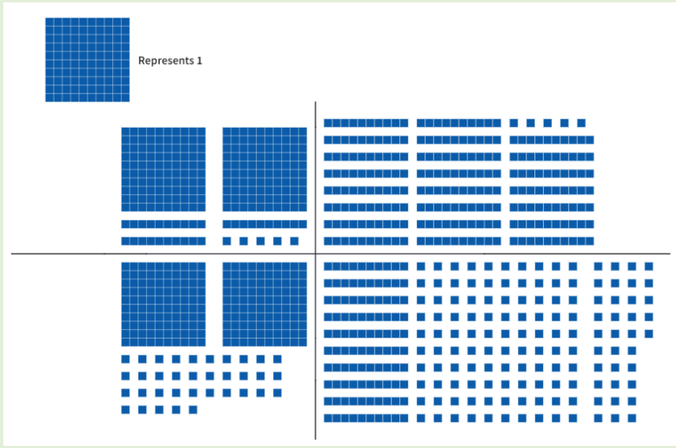


*Eyes on Math g3–5

Represent two values using base-ten blocks; one that is smaller than 1.34 and one that is larger. Prove it to your partner. Record all three values in increasing order.

The large cube  represents 1. Which set of base-ten blocks would you use to represent the decimal number 0.233?

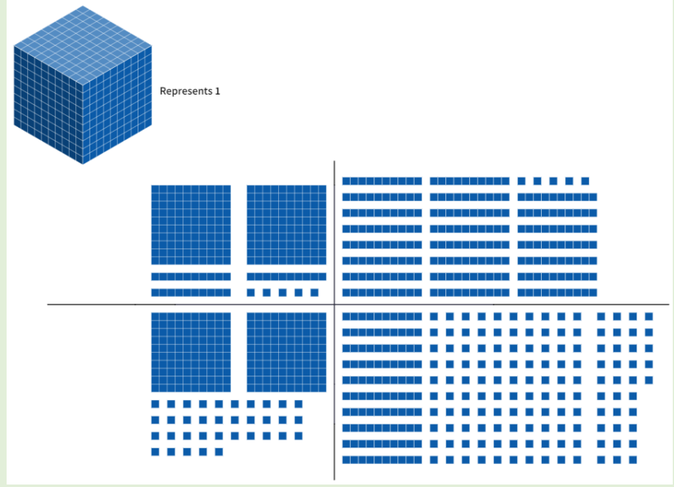
How are these representations the same and how are they different? Which representation is easiest to know the value and which is most difficult?

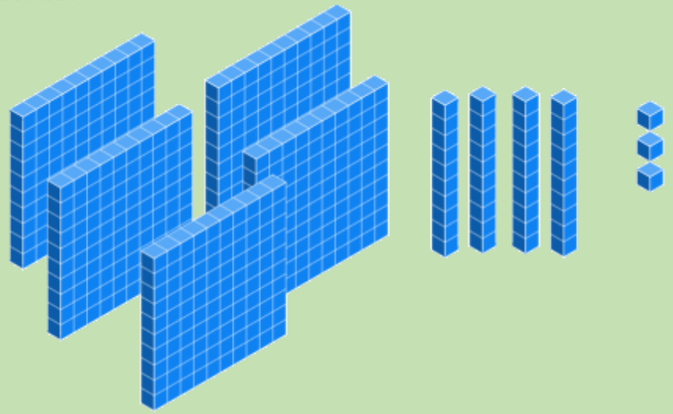
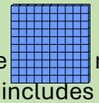
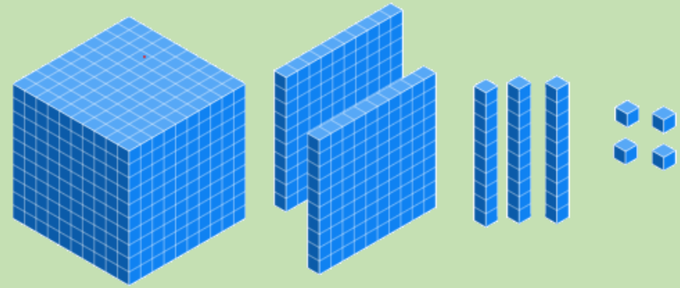


Represent two values using base-ten blocks; one that is smaller than 1.346 and one that is larger. Prove it to your partner. Record all three values in increasing order.

Place the following values on a number line:
3.8 2.1399 4.855 2.14 0.2358 0.98 1.25

How are these representations the same and how are they different? Which representation is easiest to know the value and which is most difficult?

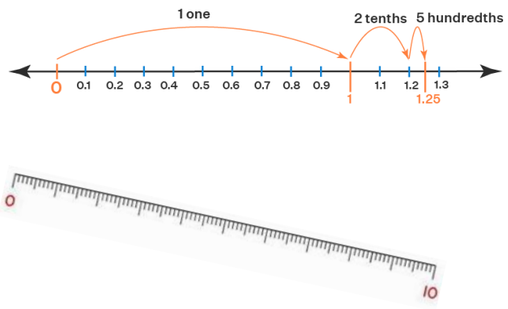


Analysis	Represent two values using base-ten blocks; one that is smaller than 1.34 and one that is larger. Prove it to your partner. Record all three values in increasing order. What number is 2 hundredths more than 4.89?	You represent a decimal number using all the following base-ten blocks.  Give two possible decimal numbers that can be represented. Can you think of a third number that can be represented? How are they the same and how are they different? If the  represents 1 whole, represent a decimal number that includes the use of the large cube, rod, and small cube. What number is it? How does it compare to your partner's number/representation? Which value is larger? How do you know?	You represent a decimal number using all the following base-ten blocks.  Give two possible decimal numbers that can be represented. Can you think of a third number that can be represented? How are they the same and how are they different? If the represents 1 whole, represent a decimal number that includes the use of the large cube, rod, and small cube. What number is it? How does it compare to your partner's number/representation? Which value is larger? How do you know?

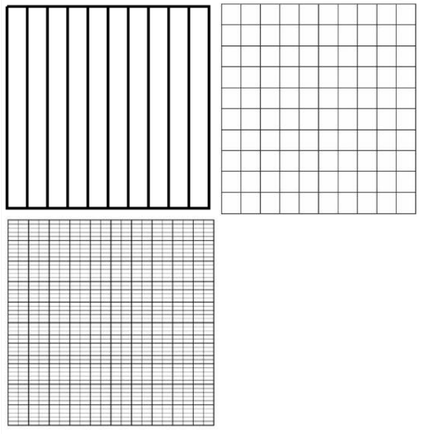
Supporting Resources

Manipulatives and Models to Support Learning

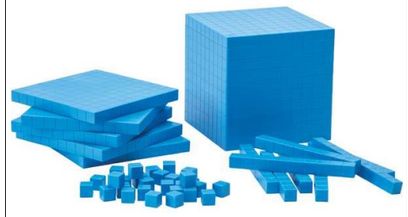
Open number line or a Ruler



Tenths, hundredths, thousandths grids



Base-ten blocks



Place value chart

Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths

Play money

Print and Electronic Resources

Bay-Williams, Jennifer M, et al. (2021). *Figuring out Fluency in Mathematics Teaching and Learning, Grades K-8: Moving beyond Basic Facts and Memorization*. Thousand Oaks, California, Corwin.

Cameron, Antonia. (2020). *Early Childhood Math Routines: Empowering Young Minds to Think*. Portsmouth, New Hampshire, Stenhouse Publishers.

Costello, D. (2021), *Making Math Stick: Classroom strategies that support the long-term understanding of math concepts*. Markham, ON: Pembroke Publishers.

Department of Education and Early Childhood Development (EECD), Province of Nova Scotia (2014a). Mathematics 4 Curriculum Guide. Halifax, NS: Author.

Department of Education and Early Childhood Development (EECD), Province of Nova Scotia (2014b). Mathematics 5 Curriculum Guide. Halifax, NS: Author.

Department of Education and Early Childhood Development (EECD), Province of Nova Scotia (2014c). Mathematics 6 Curriculum Guide. Halifax, NS: Author.

Fiore, Mary, and Maria Luisa Lebar. (2016). *The Four Roles of the Numerate Learner*. Pembroke Publishers Limited.

Marks Krpan, C., (2017), *Teaching Math with Meaning Cultivating Self-Efficacy Through Learning competencies, Grades K - 8*. Toronto, ON: Pearson Education Canada. (Chapters 5 and 6 – Communication and Thinking)

Newton, Nicki. (2021). *Guided Math in Action: Building Each Student's Mathematical Proficiency with Small-Group Instruction*. London, Routledge.

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