LESSON LEARNED

Focus: Relating Fractions and Decimal Numbers

Nova Scotia Assessment: Mathematics Grade 6

“For learners to succeed, teachers must assess students’ individual abilities and characteristics and choose appropriate and effective instructional strategies accordingly.”
– Helene J. Sherman

Purpose of this Document

This Lessons Learned document was developed based on an analysis of the Item Description Reports for the Nova Scotia Assessment: Mathematics in Grade 6 (NSA-M6). This document is intended to support all classroom teachers at grades 3 - 6, and administrators at the school, region, and provincial levels. The focus of the document is to help educators work through the process of taking in the information provided by the data analysis and see how it can inform lesson design and assessment in the classroom.

It is suggested that school teams make use of this resource in concert with their school's Item Description Report provided by the Department of Education and Early Childhood Development to all regional centres for education. These reports include student achievement data at the school, regional centre, and provincial level for all questions appearing on the Mathematics in Grade 6 Assessment. By analyzing their own performance on groupings of questions dealing with similar outcomes, schools can identify areas of strength and areas where changes in instruction and/or assessment might be made. This process is designed to foster continued discussions, explorations, and support for mathematics focus at the classroom, school, regional centre, and provincial levels that are all based on valid and reliable data.

This document specifically addresses some of the areas that students across the province found challenging based on provincial assessment data. It is essential that teachers consider assessment evidence from a variety of sources to inform the next steps most appropriate for their students. Effective classroom instruction and assessment strategies are responsive to the individual learners within a classroom.

This document highlights those outcomes where students seem to require additional support. It provides some information about student performance on the assessment in addition to suggested classroom instruction strategies. Sample assessment items are included for each topic explored.

Overview of the Nova Scotia Assessment: Mathematics in Grade 6

Nova Scotia Assessments are large-scale assessments that provide reliable data about how well all students in the province are learning the mathematics curricula. It is different from many standardized tests in that all questions are written by Nova Scotia teachers to align with curriculum outcomes and the results reflect a snapshot of how well students are learning these outcomes. These results can be counted on to provide a good picture of how well students are learning curriculum outcomes within schools, regions and in the province. Since the assessments are based on the Nova Scotia curriculum, and are developed by Nova Scotia teachers, results can be used to determine whether the curriculum, approaches to teaching and allocation of resources are effective. Furthermore, because individual student results are available, these, in conjunction with other classroom assessment evidence, help classroom teachers understand what each student has under control and identify next steps to inform instruction.

The assessment provides information about mathematics for each student and complements assessment data collected in the classroom. This assessment is administered at the beginning of grade 6. It is designed to provide detailed information for every student in the province regarding their progress in achieving a selection of mathematics curriculum outcomes at the end of Grade 5. Information from this assessment can be used by teachers to inform their instruction and next steps in providing support and intervention for their students.

Lessons Learned Overview

Provincial assessments and examinations generate information that teachers can use to help inform classroom instruction and assessment. Following the analysis of each assessment or examination, patterns and trends are identified. These include areas of strength and areas for growth. The Lessons Learned documents specifically highlight concepts where growth is still needed.

There are six areas that have been identified as the areas of focus for this Lessons Learned document.

They are:

- Solving whole number multiplication and division questions in context
- Representing decimals
- Relating fractions and decimal
- Generalizing to extend patterns
- Understanding the relationship between area and perimeter
- Identifying and describing the attributes of objects and shapes

This section specifically addresses relating fractions and decimals. It begins with an overview of the student errors and misconceptions identified through the provincial assessment. Specifically, understanding that fractions and decimals are different types of numbers that represent parts of a whole and that they can represent the same quantity.

Strategies are then outlined that are designed to enhance student comprehension, drawing from researched best practices. The strategies emphasize the integration of essential models, tools, and interconnections to facilitate the transition between concrete, pictorial, and abstract representations of concepts, highlighted by the importance of deliberate planning and purposeful questioning. To support both assessment and instruction, sample lesson activities are presented alongside a series of cognitive-level questions, providing educators with ideas for addressing knowledge gaps and fostering strategic reasoning and problem-solving skills. Each section culminates with a selection of print and online resources, as well as recommended manipulatives to support professional learning and student understanding of that topic.

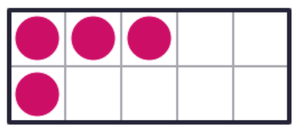
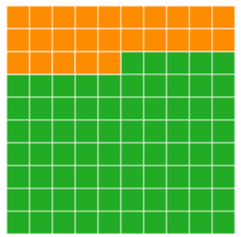
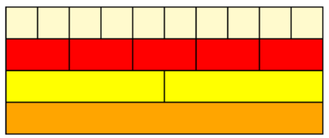
Relating Fractions and Decimal Numbers

Alignment to previous Outcomes	Related Outcome	Alignment to upcoming Outcomes
4N10: Students will be expected to relate decimals to fractions and fractions to decimals (to hundredths).	**5N09:** Students will be expected to relate decimals to fractions and fractions to decimals (to thousandths).	**6N06**: Students will be expected to demonstrate an understanding of percent (limited to whole numbers) concretely, pictorially, and symbolically.

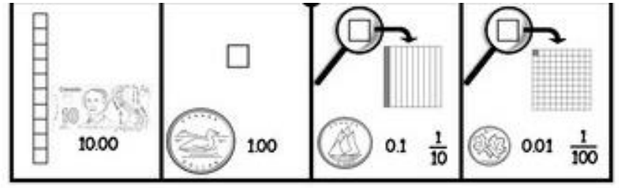
What conclusions can be drawn from the NSA: Mathematics in Grade 6?

Students had some difficulty identifying decimals that are equivalent to a given fraction. This included fractions represented as tenths and thousandths. Students commonly identified incorrect decimal values related to misconceptions in place value. In these questions, no images were provided to students to support their thinking.

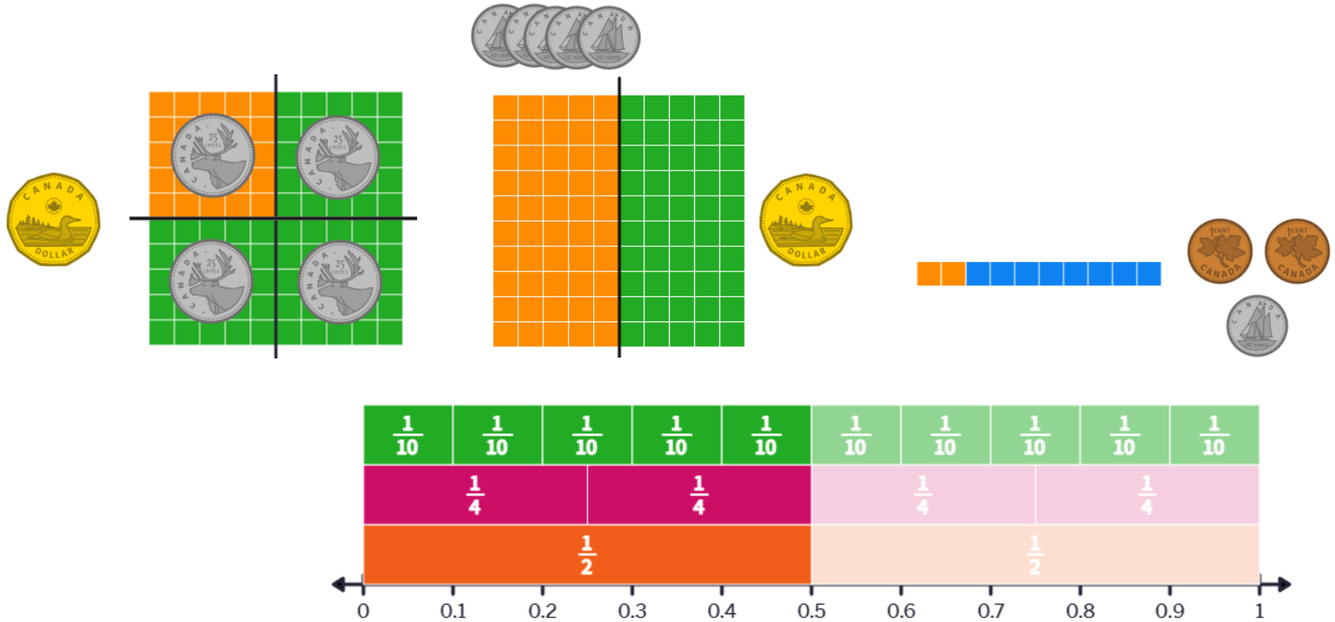
Why is this an area of need and how can we support students?

Fractions and Decimals are Different Types of Numbers	
Misconceptions/Errors in Student Work	**Possible Next Steps in the Classroom**
Some students think that fractions and decimals are entirely different and unrelated concepts. They have yet to understand that decimals and fractions are two different ways of representing the same concept: parts of a whole. Some errors can also stem from a lack of understanding of fractions and when decimals are introduced, requires familiarity with the concept of fractional tenths. This works the other way as well, where students may misunderstand the relationship between place value and the size of decimals in relation to a whole. Understanding equivalence in terms of base ten is essential for moving between fractions and decimals. Errors and misconceptions may be a result of not having enough exposure to the language of place value and decimals and the various representations to help define the meaning of parts of a whole.	Before diving into exploring the relationship between fractions and decimal numbers, students should have a good understanding of each in so far as they both represent parts of whole. A decimal number is expressed using the place value system but can be expressed in an equivalent fractional form. To start, it is natural to begin with items that come in tens to begin renaming fraction tenths as decimal tenths. Using ten frames, Cuisenaire rods, hundred grids and base-ten blocks are strategies to connect fractions to decimals and decimals to fractions concretely and pictorially. Concrete tools and images are great ways to help students visualize how fractions and decimals are connected. Frequent use of visuals and tools will support how student begin to internalize or imagine fractions and decimals (internalize) in their minds. Note how the values are presented, ensuring students use and practice with conventional and non-conventional representations. Examples:   

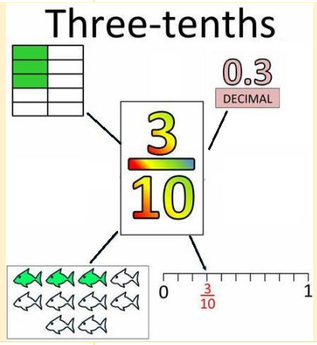
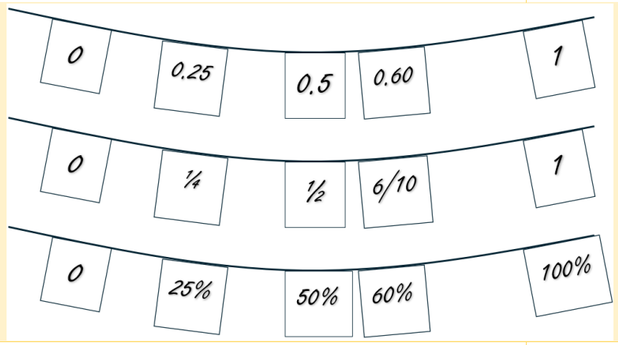
Another strategy is to use money models. Since 100 cents is analogous to 1 dollar, students can begin to show money amounts where the whole is 100 and the equivalent decimal form which many may already be familiar with through real-life experiences.



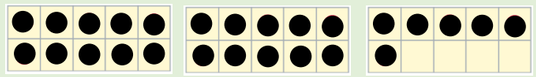
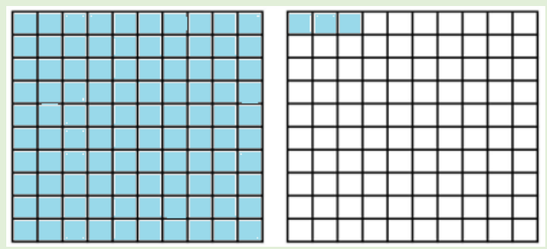
Do not begin dividing the digits in the fractions or multiplying to find equivalencies too quickly. Students need to develop the understanding of how the two types of numbers are related using images and concrete materials first. Common fractions like fourths, halves, tenths, or hundredths can be used to illustrate their equivalence.

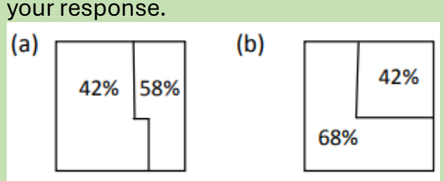


When working with fractions and decimals with students, make sure to consistently use place value language. This involves saying values in tenths, hundredths, and thousandths. For example, instead of saying, “zero decimal four five” or “zero decimal forty-five”, use place value language, “forty-five hundredths”. This is very important in supporting the understanding of the relationship between these types of numbers.

Sample Activities for Lesson Planning		
Grade 4	**Grade 5**	**Grade 6**
Provide students in the class with a card. There are three types of cards: a decimal value, a fraction, and a picture of a representation of a decimal value. Students are to find their equivalent form and end up in groups of three. When they get into their group, they need to prove to themselves that all representations of the value are equivalent and be prepared to share with others. Once all the groups have been formed, you can also have the students place their values on a number line (clothesline). They must then reflect to see if they notice any patterns with the representations. Examples of decimals can include, 0.1, 0.2, 0.5, 0.01, 0.02, 0.9, 0.05, 0.25, 0.75.	Provide students with a similar activity for that in grade 4. Except students can be challenged with cards that also have values into the thousandths. Similarly, students are to find their equivalent form and end up in groups of three. When they get into their group, they need to prove to themselves that all representations of the value are equivalent and be prepared to share with others. Once all the groups have been formed, you can also have the students place their values on a number line (clothesline). They must then reflect to see if they notice any patterns with the representations. Examples of decimals can include, 0.250, 0.5, 0.01, 0.020, 0.900, 1.05, 0.75, 1.45.	Provide students with a similar activity for that in grade 4 or 5. Except students can be challenged with cards that also have percent values. Similarly, students are to find their equivalent form and end up in groups of three or four. When they get into their group, they need to prove to themselves that all representations of the value are equivalent and be prepared to share with others. Once all the groups have been formed, you can also have the students place their values on a number line (clothesline). They must then reflect to see if they notice any patterns with the representations. Examples of decimals can include, 0.250, 0.5, 0.01, 0.020, 0.900, 0.75, 1.000, 1.5.
		
Knowledge: Find an equivalent representation of your number. Place your value on the number line in relation to other values. Application: How does your value compare to your group's value? How do you know they are equivalent? Which card(s) supported your understanding the most. How does your value compare to the others on the number line. What patterns/relationships do you notice between the values on the number line? Analysis: Provide another card with a decimal value to all students. Complexity may differ based on readiness. Have students create their own matching pictorial representation of that decimal as well as fraction. They can then share and prove to their partner of the equivalence. Students can also compare their values and then place their new values on the number line. What further patterns/relationships do you notice between the values on the number line? What has supported your understanding the most?	Knowledge: Find the equivalent representation of your number. Place your value on the number line in relation to other values. Application: How does your value compare to your group's value? How do you know they are equivalent? Which card(s) supported your understanding the most. How does your value compare to the others on the number line. What patterns/relationships do you notice between the values on the number line? Analysis: Provide another card with a decimal value to all students. Complexity may differ based on readiness. Have students create their own matching pictorial representation of that decimal as well as fraction. They can then share and prove to their partner of the equivalence. Students can also compare their values and then place their new values on the number line. What further patterns/relationships do you notice between the values on the number line? What has supported your understanding the most?	Knowledge: Find the equivalent representation of your number. Place your value on the number line in relation to other values. Application: How does your value compare to your group's value? How do you know they are equivalent? Which card(s) supported your understanding the most. How does your value compare to the others on the number line. What patterns/relationships do you notice between the values on the number line? Analysis: Provide another card with a decimal value to all students. Complexity may differ based on readiness. Have students create their own matching pictorial representation of that decimal as well as fraction. They can then share and prove to their partner of the equivalence. Students can also compare their values and then place their new values on the number line. What further patterns/relationships do you notice between the values on the number line? What has supported your understanding the most?

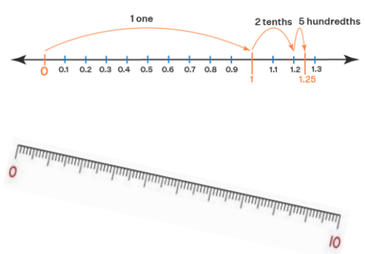
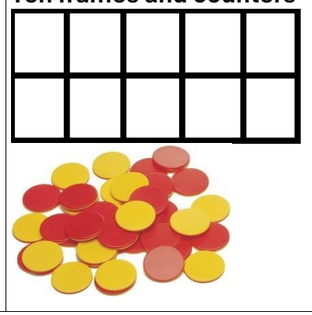
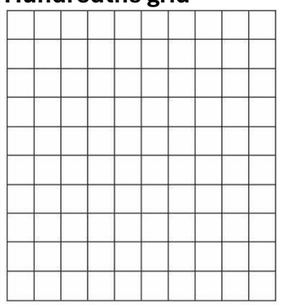
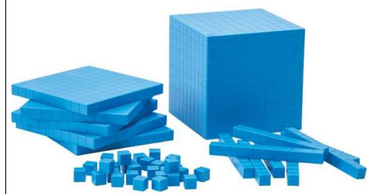
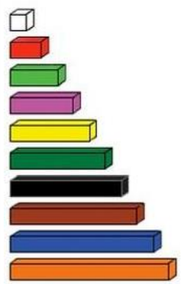
What are some sample questions to help support assessment?

Cognitive Level	Grade 4	Grade 5	Grade 6
Knowledge	Give an example of a fraction with a value less than 0.5. Give an example of a fraction with a value greater than 0.5. What fraction is equivalent to 0.50?	Determine the fraction that is equivalent to the decimal number 0.004? If a ten-frame represents one whole or 1, which decimal number will be represented by the following illustration? 	Which is the least? The greatest? Explain your answer. $\frac{1}{20}$ 20% 0.02 Which one does not belong? Explain your choice. $\frac{3}{4}$ 0.75 0.34 75%
Application	In the diagram below, a ten-frame represents one whole or 1. What decimal number is represented?  Show how 0.43 is the same as 43/100?	How can you show that $\frac{3}{4}$ is the same as 0.75? If a hundredths chart represents one whole or 1, what decimal number is represented by the following illustration? 	60 new floor tiles are being installed in one room. The tiles used must be the following colours: 25% must be blue; half must be red; 0.20 must be green; the rest are yellow. How many would there be of each colour? Draw a picture to support your answer. Changing to newer, more energy-efficient light bulbs can save up to 50% on your electric bill for lighting. If a person's electric bill was $30 before changing light bulbs, what would the bill be with the newer light bulbs? Use a number line to help model. Which of the following representations are equivalent to 20%?  2/10 0.020

Analysis	Pick a fraction and a decimal you find easier to compare. What makes them easy for you to compare? Pick a fraction and a decimal you find harder to compare. What makes them hard for you to compare? What would make it easier to compare them? Create a story problem involving a decimal value and a fraction. Solve your problem.	How many ways can you represent the value 0.25? How do you know all your representations are correct? What patterns exist between the decimal representations of the following fractions? 1/4, 2/4, 3/4, 4/4? How does this help you understand equivalent fractions and decimals? What tools support your understanding? Create a story problem involving a decimal value and a fraction. Solve your problem.	Pick a fraction and a percent that are easy/hard for you to compare. What makes it easy/hard for you to compare? What is incorrect about each of the following diagrams? Justify your response.  When could 45% be greater than 90%?

Supporting Resources

Manipulatives and Models to Support Learning

Open number line or a Ruler 	**Ten frames and counters** 	**Hundredths grid** 	**Base-ten blocks** 	**Cuisenaire rods** 	**Play money** 

Print and Electronic Resources

Bay-Williams, Jennifer M, et al. (2021). *Figuring out Fluency in Mathematics Teaching and Learning, Grades K-8: Moving beyond Basic Facts and Memorization*. Thousand Oaks, California, Corwin.

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Van De Walle, J.A. (2001). *Elementary and middle school mathematics teaching developmentally fourth edition*. New York, NY: Addison Wesley Longman.

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