LESSON LEARNED

Focus: Generalizing to Extend Patterns

Nova Scotia Assessment: Mathematics Grade 6

“For learners to succeed, teachers must assess students’ individual abilities and characteristics and choose appropriate and effective instructional strategies accordingly.”
– Helene J. Sherman

Purpose of this Document

This Lessons Learned document was developed based on an analysis of the Item Description Reports for the Nova Scotia Assessment: Mathematics in Grade 6 (NSA-M6). This document is intended to support all classroom teachers at grades 3 - 6, and administrators at the school, region, and provincial levels. The focus of the document is to help educators work through the process of taking in the information provided by the data analysis and see how it can inform lesson design and assessment in the classroom.

It is suggested that school teams make use of this resource in concert with their school's Item Description Report provided by the Department of Education and Early Childhood Development to all regional centres for education. These reports include student achievement data at the school, regional centre, and provincial level for all questions appearing on the Mathematics in Grade 6 Assessment. By analyzing their own performance on groupings of questions dealing with similar outcomes, schools can identify areas of strength and areas where changes in instruction and/or assessment might be made. This process is designed to foster continued discussions, explorations, and support for mathematics focus at the classroom, school, regional centre, and provincial levels that are all based on valid and reliable data.

This document specifically addresses some of the areas that students across the province found challenging based on provincial assessment data. It is essential that teachers consider assessment evidence from a variety of sources to inform the next steps most appropriate for their students. Effective classroom instruction and assessment strategies are responsive to the individual learners within a classroom.

This document highlights those outcomes where students seem to require additional support. It provides some information about student performance on the assessment in addition to suggested classroom instruction strategies. Sample assessment items are included for each topic explored.

Overview of the Nova Scotia Assessment: Mathematics in Grade 6

Nova Scotia Assessments are large-scale assessments that provide reliable data about how well all students in the province are learning the mathematics curricula. It is different from many standardized tests in that all questions are written by Nova Scotia teachers to align with curriculum outcomes and the results reflect a snapshot of how well students are learning these outcomes. These results can be counted on to provide a good picture of how well students are learning curriculum outcomes within schools, regions and in the province. Since the assessments are based on the Nova Scotia curriculum, and are developed by Nova Scotia teachers, results can be used to determine whether the curriculum, approaches to teaching and allocation of resources are effective. Furthermore, because individual student results are available, these, in conjunction with other classroom assessment evidence, help classroom teachers understand what each student has under control and identify next steps to inform instruction.

The assessment provides information about mathematics for each student and complements assessment data collected in the classroom. This assessment is administered at the beginning of grade 6. It is designed to provide detailed information for every student in the province regarding their progress in achieving a selection of mathematics curriculum outcomes at the end of Grade 5. Information from this assessment can be used by teachers to inform their instruction and next steps in providing support and intervention for their students.

Lessons Learned Overview

Provincial assessments and examinations generate information that teachers can use to help inform classroom instruction and assessment. Following the analysis of each assessment or examination, patterns and trends are identified. These include areas of strength and areas for growth. The Lessons Learned documents specifically highlight concepts where growth is still needed.

There are six areas that have been identified as the areas of focus for this Lessons Learned document.

They are:

- Solving whole number multiplication and division questions in context
- Representing decimals
- Relating fractions and decimal
- Generalizing to extend patterns
- Understanding the relationship between area and perimeter
- Identifying and describing the attributes of objects and shapes

This section specifically addresses generalizing to extend patterns. It begins with an overview of the student errors and misconceptions identified through the provincial assessment. These include:

- Extending a pattern
- Generalizing a pattern

Strategies are then outlined that are designed to enhance student comprehension, drawing from researched best practices. The strategies emphasize the integration of essential models, tools, and interconnections to facilitate the transition between concrete, pictorial, and abstract representations of concepts, highlighted by the importance of deliberate planning and purposeful questioning. To support both assessment and instruction, sample lesson activities are presented alongside a series of cognitive-level questions, providing educators with ideas for addressing knowledge gaps and fostering strategic reasoning and problem-solving skills. Each section culminates with a selection of print and online resources, as well as recommended manipulatives to support professional learning and student understanding of that topic.

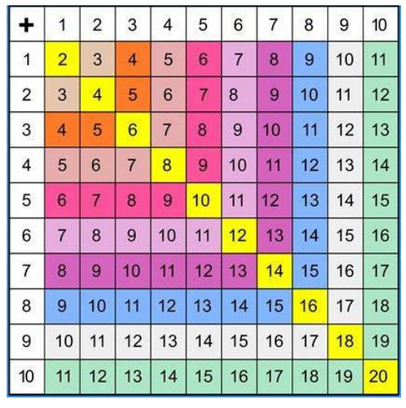
Generalizing to Extend Patterns.

Alignment to previous Outcomes		Related Outcome	Alignment to upcoming Outcomes
3PR01: Students will be expected to demonstrate an understanding of increasing patterns by describing, extending, comparing, and creating numerical patterns (numbers to 1000) and non-numerical patterns using manipulatives, diagrams, sounds, and actions. **3PR02:** Students will be expected to demonstrate an understanding of decreasing patterns by describing, extending, comparing, and creating numerical patterns (numbers to 1000) and non-numerical patterns using manipulatives, diagrams, sounds, and actions.	**4PR01:** Students will be expected to identify and describe patterns found in tables and charts, including a multiplication chart. **4PR02:** Students will be expected to translate among different representations of a pattern (a table, a chart, or concrete materials). **4PR03:** Students will be expected to represent, describe, and extend patterns and relationships, using charts and tables, to solve problems.	**5PR01:** Students will be expected to determine the pattern rule to make predictions about subsequent terms.	**6PR01:** Students will be expected to demonstrate an understanding of the relationship within tables of values to solve problems. **6PR02:** Students will be expected to represent and describe patterns and relationships, using graphs and tables.

What conclusions can be drawn from the NSA: Mathematics in Grade 6?

Many students still experience challenges when working with more complex patterns and relationships. They have difficulty when moving from the basic understanding of patterns to the generalization of a pattern rule to enable them to find any term. Students need to continue to work with representations of patterns, contextually, pictorially, symbolically, and verbally where they have to extend a pattern to predict a subsequent term that is not consecutive.

Why is this an area of need and how can we support students?

Extending a Pattern	
Misconceptions/Errors in Student Work	**Possible Next Steps in the Classroom**
Some students do not recognize that there are different ways to continue a pattern if a pattern rule is not described. They might believe that all patterns can be extended using a single rule or operation. While some patterns follow a straightforward rule (like adding a constant or multiplying by a factor), others may involve a combination of rules or irregular changes. Students might also extend a pattern based solely on the most recent data points without considering the overall trend or structure of the pattern. For example, if given 5, 15, 20 . . . in the beginning of the pattern, students may only see it as a pattern increasing by 10 or 5 and not by both amounts.	To address these misconceptions, educators can provide diverse examples of patterns that include the use of concrete materials and images. Students should be encouraged to explore and discuss complex numerical and visual patterns to help reinforce their understanding of the repeating elements. Teachers should spend time in helping students to make connections between the concrete models, numerical representations, and the abstract language used to describe patterns. For example, if students are working with colored cubes, relate the cube colors to the vocabulary of term, term number, core, and repetition. Encourage students to question assumptions and consider multiple possibilities when extending patterns. For example, there can be more than one way to extend a pattern: 5, 10, 15, 20, 25, 30,... or 5, 10, 15, 25, 35, 50, 65, ... (Small, 2009, p. 579). Teach strategies for analyzing patterns, such as looking for relationships between terms, identifying recurring elements, and testing predictions. Leverage opportunities to identify and explain the patterns in the addition and multiplication tables. These can include place value patterns, multiples, and factors, along with determining an unknown sum, difference, product, or quotient.  	+	1	2	3	4	5	6	7	8	9	10
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1	2	3	4	5	6	7	8	9	10	11
2	3	4	5	6	7	8	9	10	11	12
3	4	5	6	7	8	9	10	11	12	13
4	5	6	7	8	9	10	11	12	13	14
5	6	7	8	9	10	11	12	13	14	15
6	7	8	9	10	11	12	13	14	15	16
7	8	9	10	11	12	13	14	15	16	17
8	9	10	11	12	13	14	15	16	17	18
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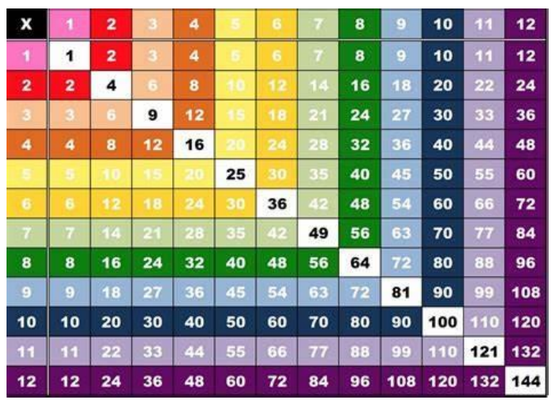
10	11	12	13	14	15	16	17	18	19	20 |  | X	1	2	3	4	5	6	7	8	9	10	11	12
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2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

Sample Activities for Lesson Planning			
Grade 3	**Grade 4**	**Grade 5**	**Grade 6**
Warm-up: Students can explore an addition chart to find as many patterns as they can in the table. Ask students to create a growing pattern where the 10th term is 25. OR Ask students to create a shrinking pattern where the 4th term is 16. Knowledge: What is your pattern? Application: How does your pattern compare to your partner's pattern? How is it the same and/or different? What did you do to come up with your pattern? Analysis: Why can everyone have different patterns, but all end up with the same 10th / 4th term?	Warm-up: Students can explore an addition chart or multiplication chart to find as many patterns as they can in the table. Ask students to create a growing pattern where the 10th term is 56. OR Ask students to create a shrinking pattern where the 4th term is 24. Knowledge: What is your pattern? C Application: How does your pattern compare to your partner's pattern? How is it the same and/or different? What did you do to come up with your pattern? Analysis: Why can everyone have different patterns, but all end up with the same 10th / 4th term?	Warm-up: Students can explore a multiplication chart to find as many patterns as they can in the table. Ask students to create a growing pattern where the 10th term is 84. OR Ask students to create a shrinking pattern where the 4th term is 24. Knowledge: What is your pattern? Application: How does your pattern compare to your partner's pattern? How is it the same and/or different? What did you do to come up with your pattern? Analysis: Why can everyone have different patterns, but all end up with the same 10th / 4th term?	Warm-up: Students can explore a multiplication or division chart to find as many patterns as they can in the table. Ask students to create a growing pattern where the 10th term is 96. Record the values in a table or graph it. OR Ask students to create a shrinking pattern where the 4th term is 24. Record the values in a table or graph it. Knowledge: What is your pattern? Application: How does your pattern compare to your partner's pattern? How is it the same and/or different? What did you do to come up with your pattern? Analysis: Why can everyone have different patterns, but all end up with the same 10th / 4th term?

Generalizing a Pattern

Misconceptions/Errors in Student Work

Some students have difficulty predicting the value of an unknown term using the relationship in a table of values and verifying the prediction. This involves formulating a rule to describe the relationship between two columns of numbers in a table of values. In the example below, the student only sees that the term value increasing by 2 and is not relating the term value to the term to accurately predict a term out of reach. The student is not developing an equation to generalize what is happening.

Term	Term Value
1	3
2	5
3	7
...	...
8	?

Add 2 each time.

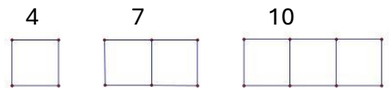
Possible Next Steps in the Classroom

Students should begin by representing a pattern with concrete materials and/or pictures. Then, they should represent the same pattern in a table or chart. Once a table or chart is developed, students have two representations of a pattern: the one created with the drawing or materials and the numeric version that is in the table or chart. They can then explain how these patterns are mathematically alike, that is, why the same relationship exists between the pattern in a table and its concrete representation.

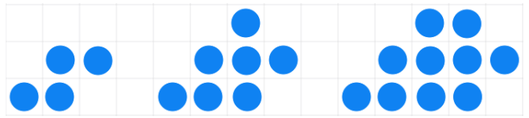
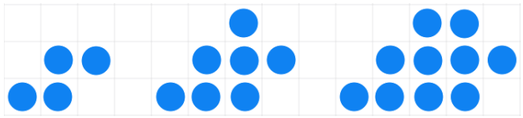
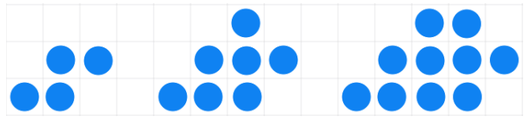
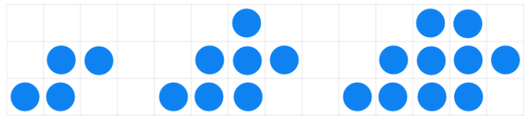
Students should also be given opportunities to reproduce a pattern using concrete materials when presented with a pattern displayed in a table or chart. Students should also be asked to describe what is happening as the pattern increases (or decreases) and how the next step is related to the previous one. It is helpful for students to think of a pattern rule and apply it when analyzing tables or charts for errors.

Using multiple representations helps students to visualize the pattern to generalize what is happening.

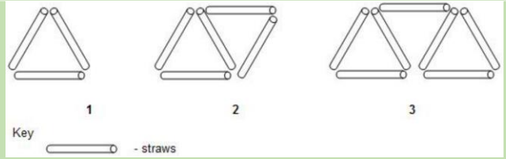
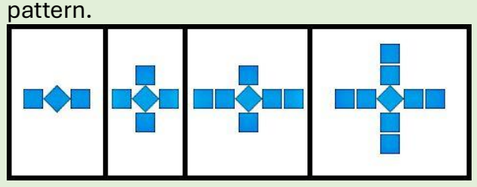
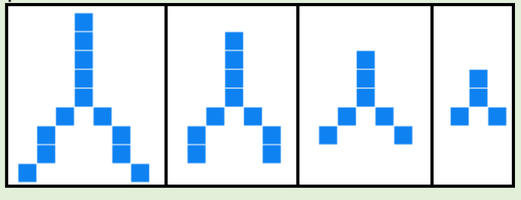
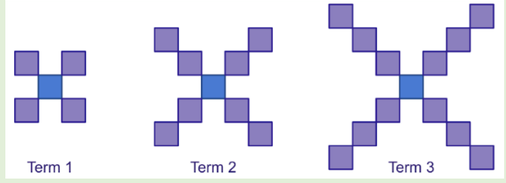
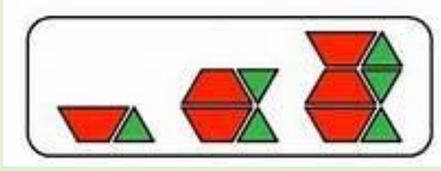
The example below illustrates the same pattern using a picture and a table.



Term Number	Number of Toothpicks
1	4
2	7
3	10

Sample Activities for Lesson Planning			
Grade 3	**Grade 4**	**Grade 5**	**Grade 6**
Show students the following image:  Knowledge: Is this a growing or shrinking pattern? Application: Draw what you think the shapes before and after the ones on the image would look like. What is changing between the shapes? Analysis: How does knowing how the shapes change help in knowing what the next term is? How can you represent this pattern in a table? How can you use this information to predict the next term value?	Show students the following image:  Knowledge: Is this a growing or shrinking pattern? Application: Draw what you think the shapes before and after the ones on the image would look like. What is changing between the shapes? Analysis: How does knowing how the shapes change help in knowing what the next term is? How can you represent this pattern in a table? How can you use this information to predict the 6^{th} term?	Show students the following image:  Knowledge: Is this a growing or shrinking pattern? Application: Draw what you think the shapes before and after the ones on the image would look like. What is changing between the shapes? Analysis: How does knowing how the shapes change help in knowing what the next term is? How can you represent this pattern in a table? How can you use this information to predict the 10^{th} term?	Show students the following image:  Knowledge: Is this a growing or shrinking pattern? Application: Draw what you think the shapes before and after the ones on the image would look like. What is changing between the shapes? Analysis: How does knowing how the shapes change help in knowing what the next term is? How can you represent this pattern in a table? How can you use this information to predict the 20^{th}? What general expression represents the pattern?

What are some sample questions to help support assessment?

Cognitive Level	Grade 3	Grade 4	Grade 5	Grade 6
Knowledge	Write the first 5 terms of a number pattern with the following rule: Starts at 8 and add 3 each time. Write the first 5 terms of a number pattern with the following rule: Starts at 92, subtract 6 each time. What is a growing pattern? What is a shrinking pattern?	Identify two patterns on the hundreds chart. Identify two patterns on the multiplication chart. Write the first 5 terms of a number pattern with the following rule: Starts at 8, add 2, then add 1 alternating each time.	 Using the pattern above, fill in the following table indicating the term number and term value. Term Number | Term Value | | | | Write the first 5 terms of a pattern that has term values decreasing by 3 each time.	 Using the pattern above, fill in the following table indicating the term number and term value. Term Number | Term Value | | | |
Application	What is the pattern rule for the following pattern.  What is the pattern rule for the following pattern. 	What is the pattern rule?  Make a table of values that shows the pattern up to term 6. Which pattern would reach 40 first? 120, 100, 80, ... 8, 16, 24, ... A pattern begins like this: 2, 6, ... How might it continue?	Look at the picture below.  The first term is made up of two pattern blocks. The second term is made up of four pattern blocks, and the third term is made up of six pattern blocks. Predict the number of pattern blocks in the eighth term. Fill in the missing values for each pattern: 4, ___, 12, ___, 20, ... 18, 16, 14, ___, ___, ... 2.4, 2.7, ___, ___, 3.6, ...	Use the table and image to determine the pattern rule. Predict the number of chairs for 10 tables. Number of tables | Number of chairs 1 | 4 2 | 6 3 | 8 4 | 10 5 | 12  Sheila works in a computer repair shop. She gets paid $75 a day plus $5 for every computer she fixes. (a) Create a table to display the total amount of money Sheila could make in a day for any number of computers she might fix.

Examine the following pattern of Figure 3, Figure 4, and Figure 5 created using small squares:

How many small squares are there in Figure 1?

A pattern begins like this: 1, 3, ...
How might it continue?

A pattern begins like this: 12, 10, ...
How might it continue?

A pattern begins like this: 23, 19, ...
How might it continue?

Identify where the pattern has errors. Explain your thinking.

Figure Number	Number of Tiles
1	4
2	8
3	12
4	18
5	20
6	22
7	28
8	32

Examine the following pattern. How many squares would be in the 7th term?

Show your work using a table of values.

Identify where the pattern has errors. Explain your thinking.

Figure Number	Number of Tiles
1	4
2	8
3	12
4	18
5	22
6	24
7	26
8	30

(b) Write a pattern rule that you could use to find the total amount of money Sheila could make in a day for any number of computers she might fix.
(c) Use your rule to determine how much money Sheila would make if she fixed 12 computers in one day

Graph the information in the table. Use the graph to help you fill in the missing information.

Side Length (cm)	1	2	3	4	5	6	?
Perimeter (cm)	6	12	18	?	30	?	48

What is the pattern rule?

Input	Output
1	2
2	3
3	4
4	5
5	6

Fill in the missing values in the table.

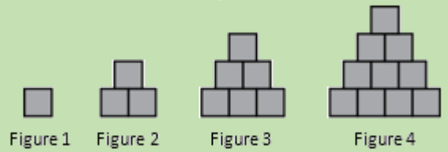
Term number	1	2	3	?
Term value	4	8	?	16

Analysis

What are the next three terms in this counting pattern?

5, 8, 7, 10, 9, 12, 11, ____, ____, ____, ...

Observe the following pattern:



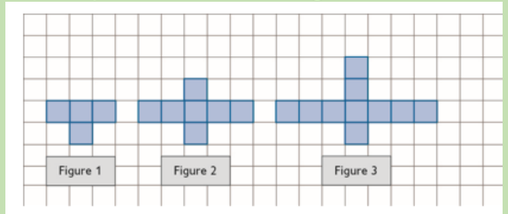
How do you see the pattern growing?

A pattern starts with 2, and the fourth number is 8. What could the in-between numbers be? Create the same pattern in two different ways. Explain how you know your patterns are the same.

Examine the following pattern.

How do you see the pattern growing?

Examine the following pattern consisting of small squares found in the figures.



How many small squares would be found in Figure 5?

A pattern starts with 3, and the fifth number is 11. What could the in-between numbers be? Create the same pattern in two different ways. Explain how you know your patterns are the same.

Examine the following pattern.

How do you see the pattern growing?

Plants grow every day.
The following table represents the height, *h* in cm, of a plant in terms of the number, *n*, of days.

Number of days	Height of the Plants (cm)
1	4
2	5
3	6
4	7
5	8

Write an expression to describe the relationship between the height of the plant and the number of days?

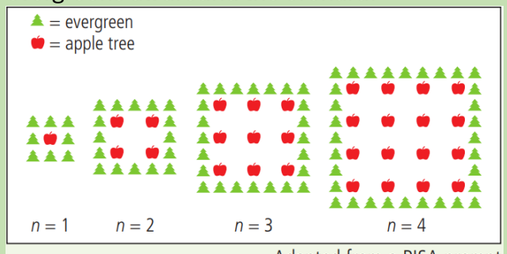
Look at these patterns:

How many white squares would be needed for an image with 7 dark squares?
If an image has 30 squares in total. How many of them are dark? White?

You have a weekly paper route and get paid $30 a week. The following table of values shows your earnings over a five-week period.
Identify the value that does not fit the pattern. Explain why the value is incorrect. Justify your answer.

Number of Weeks	Earnings
1	$30
2	$60
3	$90
4	$100
5	$130

I noticed something interesting about my neighbour's apple orchards. They plant their apple trees in square patterns in each orchard. To protect the trees from the wind, they plant evergreens all around the orchard.

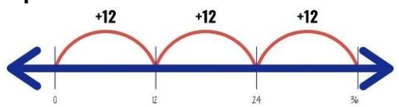
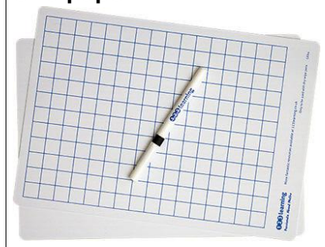


Adapted from a PISA prompt

When does the number of apple trees equal the number of evergreens? Justify your response.
How does the growth of the number of apple trees compare with the growth of the number of evergreens?

Supporting Resources

Manipulatives and Models to Support Learning

Counters	Linking cubes	Square tiles	Pattern Blocks	Open number line	Grid paper
					

Print and Electronic Resources

(n.d). *Tasks*. Tasks Archive - YouCubed (retrieved Sept. 2023).

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