LESSON LEARNED

Nova Scotia Assessment: Mathematics Grade 6

“For learners to succeed, teachers must assess students’ individual abilities and characteristics and choose appropriate and effective instructional strategies accordingly.”
– Helene J. Sherman

Contents

Purpose of this Document

This Lessons Learned document was developed based on an analysis of the Item Description Reports for the Nova Scotia Assessment: Mathematics in Grade 6 (NSA-M6). This document is intended to support all classroom teachers at grades 3 - 6, and administrators at the school, region, and provincial levels. The focus of the document is to help educators work through the process of taking in the information provided by the data analysis and see how it can inform lesson design and assessment in the classroom.

It is suggested that school teams make use of this resource in concert with their school's Item Description Report provided by the Department of Education and Early Childhood Development to all regional centres for education. These reports include student achievement data at the school, regional centre, and provincial level for all questions appearing on the Mathematics in Grade 6 Assessment. By analyzing their own performance on groupings of questions dealing with similar outcomes, schools can identify areas of strength and areas where changes in instruction and/or assessment might be made. This process is designed to foster continued discussions, explorations, and support for mathematics focus at the classroom, school, regional centre, and provincial levels that are all based on valid and reliable data.

This document specifically addresses some of the areas that students across the province found challenging based on provincial assessment data. It is essential that teachers consider assessment evidence from a variety of sources to inform the next steps most appropriate for their students. Effective classroom instruction and assessment strategies are responsive to the individual learners within a classroom.

This document highlights those outcomes where students seem to require additional support. It provides some information about student performance on the assessment in addition to suggested classroom instruction strategies. Sample assessment items are included for each topic explored.

Overview of the Nova Scotia Assessment: Mathematics in Grade 6

Nova Scotia Assessments are large-scale assessments that provide reliable data about how well all students in the province are learning the mathematics curricula. It is different from many standardized tests in that all questions are written by Nova Scotia teachers to align with curriculum outcomes and the results reflect a snapshot of how well students are learning these outcomes. These results can be counted on to provide a good picture of how well students are learning curriculum outcomes within schools, regions and in the province. Since the assessments are based on the Nova Scotia curriculum, and are developed by Nova Scotia teachers, results can be used to determine whether the curriculum, approaches to teaching and allocation of resources are effective. Furthermore, because individual student results are available, these, in conjunction with other classroom assessment evidence, help classroom teachers understand what each student has under control and identify next steps to inform instruction.

The assessment provides information about mathematics for each student and complements assessment data collected in the classroom. This assessment is administered at the beginning of grade 6. It is designed to provide detailed information for every student in the province regarding their progress in achieving a selection of mathematics curriculum outcomes at the end of Grade 5. Information from this assessment can be used by teachers to inform their instruction and next steps in providing support and intervention for their students.

Lessons Learned Overview

Provincial assessments and examinations generate information that teachers can use to help inform classroom instruction and assessment. Following the analysis of each assessment or examination, patterns and trends are identified. These include areas of strength and areas for growth. The Lessons Learned documents specifically highlight concepts where growth is still needed.

There are six areas that have been identified as the areas of focus for this Lessons Learned document.

They are:

- Solving whole number multiplication and division questions in context
- Representing decimals
- Relating fractions and decimal
- Generalizing to extend patterns
- Understanding the relationship between area and perimeter
- Identifying and describing the attributes of objects and shapes

Each section begins with an overview of the student errors and misconceptions identified through the provincial assessment. Strategies are then outlined that are designed to enhance student comprehension, drawing from researched best practices. The strategies emphasize the integration of essential models, tools, and interconnections to facilitate the transition between concrete, pictorial, and abstract representations of concepts, highlighted by the importance of deliberate planning and purposeful questioning. To support both assessment and instruction, sample lesson activities are presented alongside a series of cognitive-level questions, providing educators with ideas for addressing knowledge gaps and fostering strategic reasoning and problem-solving skills. Each section culminates with a selection of print and online resources, as well as recommended manipulatives to support professional learning and student understanding of that topic.

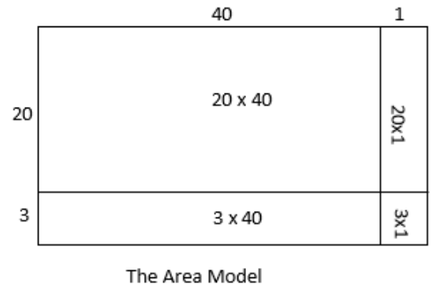
Solving Whole Number Multiplication and Division Questions in Context

Alignment to previous Outcomes		Related Outcome	Alignment to upcoming Outcomes
3N11: Students will be expected to demonstrate an understanding of multiplication to 5 × 5. **3N12:** Students will be expected to demonstrate an understanding of division (Limited to division related to multiplication facts up to 5 × 5.).	**4N06:** Students will be expected to demonstrate an understanding of multiplication (one-, two-, or three-digit by one-digit numerals) to solve problems. **4N07:** Students will be expected to demonstrate an understanding of division (one-digit divisor and up to two-digit dividend) to solve problems.	**5N05:** Students will be expected to demonstrate, with and without concrete materials, an understanding of multiplication (two-digit by two-digit) to solve problems. **5N06:** Students will be expected to demonstrate, with and without concrete materials, an understanding of division (three-digit by one-digit) and interpret remainders to solve problems.	**6N08:** Students will be expected to demonstrate an understanding of multiplication and division of decimals (one-digit whole number multipliers and one-digit natural number divisors)

What conclusions can be drawn from the NSA: Mathematics in Grade 6?

Students tend to perform better when explicitly given all the information needed to answer multiplication and division questions. For example, in questions that require students to use basic facts, skills, and symbolic procedures about two thirds of students were successful. And of those students, more were successful with multiplication questions than they were with division questions. When presented with application and analysis problem solving questions, students were not as able to apply higher order thinking skills. For example, roughly one third of the students were successful when answering questions that were presented in the context of a story problem where multiplication and/or division is used as a strategy. Understanding what to do with a remainder in contextual problems is an area of need.

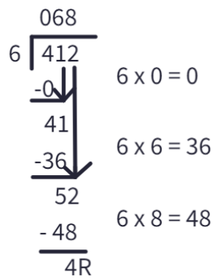
Why is this an area of need and how can we support students?

Place Value	
Misconceptions/Errors in Student Work	**Possible Next Steps in the Classroom**
Multiplication When given a multiplication question such as 23 x 41, students tend to read this and say, "4 times 2" when they should be saying, "4 tens times 2 tens" or 40 times 20. Students focus more so on the digits rather than the place value. For example, many students learn the method below by naming the digits rather than with the place value. 23 x 41 8 (4x2) 12 (4x3) 2 (1x2) + 3 (1x3) 25 23 x 41 23 (1x3; 1x2) 1 times 3 = 3; 1 times 2 = 2 +812 (4x3;4x2) 4 times 3 = 12; 4 times 2 = 8 835 The answer is 835 This results in students selecting the incorrect response when presented with the following question. Which expression represents 36 x 23? • (30 x 20) + (6 x 3); multiply only the tens together and the ones together • (6 x 3) + (3 x 3) + (6 x 2) + (3 x 2); multiply using digits • (30 x 20) + (30 x 3) + (6 x 20) + (6 x 3); correct answer	When learning about multiplication and division, focus on strategies and models rather than procedures. While tricks may work, they bypass the thinking behind the concepts. There is also always a way to explain the trick using conceptual understanding. Estimating will also help in catching possible errors when using various strategies for multiplication and division. ***Using Area Models and Partial Product*** One strategy to help students understand multidigit multiplication is to focus on place value by having students model the calculation with base-ten materials using an area model or draw out the area model as shown below. Students record partial products and then add them up at the end. When doing so, the teacher should draw attention away from the digits and focus more on the value of the number; use place value language. For example, rather than saying or having students say, "multiply the 4 by the 3, and the 4 by the 2," instead use "multiply the 40 by the 3, and the 40 by the 20." A focus on place value.  The Area Model 23 x 41 800 (40 x 20) 4 tens times 2 tens = 800 120 (40 x 3) 4 tens times 3 ones = 120 20 (1 x 20) 1 one times 2 tens = 20 + 3 (1 x 3) 1 one times 3 ones = 3 943 The answer is 943. ***Using Strategies that Promote Number Sense*** When students have had limited experiences thinking about the relationships between numbers, it is natural that they resort to traditional algorithms. They also come with varying dispositions to work on multiplication in different ways. It may help to just talk with students about this and keep asking if there could be another way of thinking about the multiplication or how else it could be solved. Many of the properties associated with multiplication can come to life through student generated strategies. Below are four strategies for multiplication. Some work in all cases, while others are better with composite or even numbers. Students should decide which is more effective and efficient for them. • Break a factor into two or more addends using the distributive property (23 x 41 is the same as 23 x (40 + 1) or 23 x (10 + 10 + 10 + 10 + 1)) • Factor a factor to support an understanding of the associative property (12 x 16 is the same as 12 x (4 x 2 x 2) • Round a factor to bring in the distributive and commutative properties (23 x 41 is the same as 23 x 40 plus another 23) • Halving and Doubling to help make problems simpler to solve (12 x 16 is the same as 24 x 8 or in this case also 6 x 32)

Division

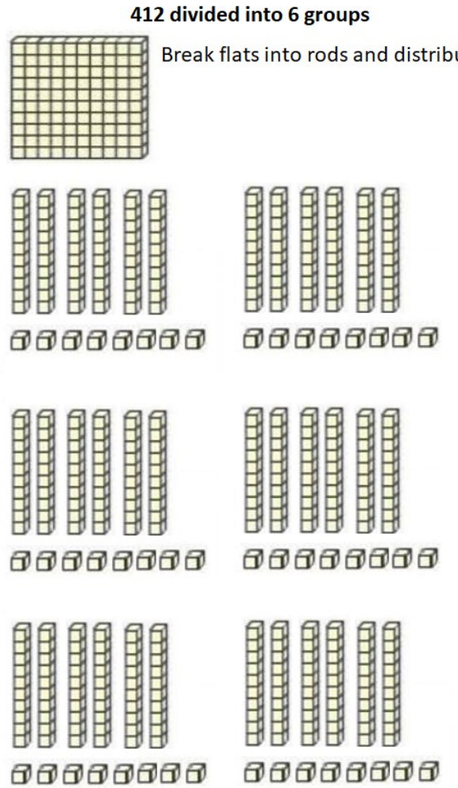
When given a division question such as 412 ÷ 6, students say, “6 into 4 doesn’t work,” they are mathematically incorrect. In the question, it is 400 not 4 that is being divided by 6. In addition, when the procedure below is presented without any context to why it supports solving a division problem, students can get caught up in the steps rather than the concept. Where there might be a remainder, students may not recognize when a remainder is significant or not involved in decision making for problem-solving contexts.

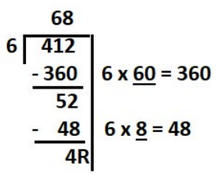
The process below highlights how some students focus on the digits rather than the values of the dividend and the divisor.

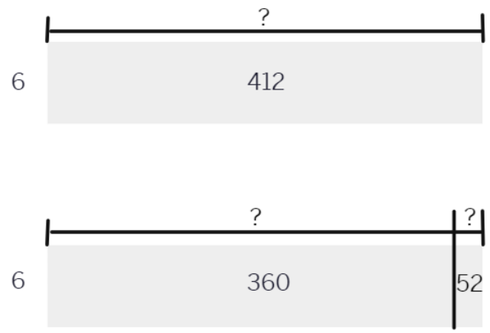


By providing students with base-ten materials, it allows them to solve problems and discuss grouping strategies, place value, and the concept of remainder. Teachers can work with students to demonstrate ways of documenting their thinking.

Examples of relating the tools / models and the strategy are highlighted below.







Further Strategies that Promote Number Sense with Division

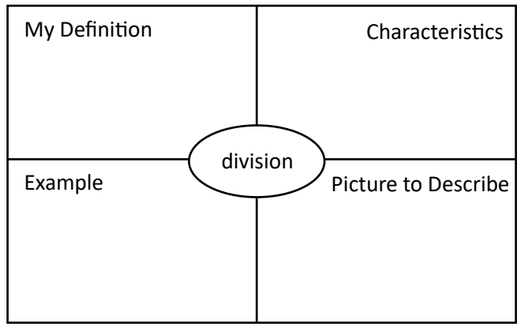
Like multiplication, the way that division is taught can lead students to have a negative disposition of it. There are several strategies that can be used other than the traditional algorithm to help students understand what it means to divide two numbers. In addition, they help to highlight the relationship between multiplication and division.

Below are strategies for division. Students should decide which is more effective and efficient for them.

- Multiply instead (17 ÷ 3 can be related to 3 x 5, plus two more)
- Chunk Out (643 ÷ 3 can be interpreted as 3 x 10 is 30 only 13 away from 43, so another 3 x 4 will get close; and 3 x 100 is 300, so 3 x 200 is 600, leaving 200 + 10 + 4 and a remainder)

Each jar holds 4 litres of liquid. How many jars will 37 L fill? The answer is 9R1. What does the remainder represent? • 1 jar; thinking in terms of whole jars • $\frac{1}{4}$ jar; correct answer – relation to the divisor • $\frac{1}{9}$ jar; relating it back to the quotient • $\frac{1}{37}$ jar; relating it back to the dividend	• Make a Tower of Factors (17 ÷ 3 can be represented as a column of multiples 3 x 1, 3 x 2, 3 x 3 and so on where students decide which multiple of the divisor to subtract each time.) • Halving and Halving to help make problems simpler to solve (102 ÷ 4 is the same as 51 ÷ 2) ***Meaning of the Remainder*** It is also important to help students to understand what the remainder means. Yes, it is the amount left over, but in relation to the question it represents a portion of a divisor. In the example above, the remainder is 4 or $\frac{4}{6}$. For story problems, this means helping students to relate back to the context to understanding the meaning of the remainder. The example on the left highlights the importance of making sense of the question and the result. Using concrete tools or pictures can help students to visualize the problem and solution. Decimals: examples of how to set up the models for multiplication and division using decimals can be found in the grade 6 curriculum guide (pp 261-262).

Reasoning to Solve Story Problems	
Misconceptions/Errors in Student Work	**Possible Next Steps in the Classroom**
While not an error or a misconception perse, one can infer that some students may not be checking for reasonableness in their answers. They could be stuck in incorrectly using standard procedures negating what they know about number sense. Students may also not be aware of other, more helpful strategies that support how to work with numbers to solve what is unknown in a story problem. Typically, students use learned strategies to look for numbers and key words in a story problem and use these to create a number sentence. When students consider that a problem is a mathematical problem, they believe, wrongly, that they should associate it simply to routine calculations with few concerns to the meaning of the context and to the credibility of the answers. Some students lack the skills to make inferences, generalize and verify their thinking. Here are a couple of examples of story problems and possible student answers. Each option is carefully constructed based on what is anticipated from student thinking. The first question requires students to determine the total quantity of the juice in mL and then in litres and identify the container whose capacity can contain this quantity of juice. Students can use repeated addition or multiplication to determine an answer. *Larry wants to make fruit juice in a container.* *To make the juice, Larry must follow the directions found on the frozen juice can. He must combine the 525 mL can of frozen juice with three full cans of 525 mL of water.* *Which litre container should Larry choose to hold all the juice?* • *1 L container (only considers the one can of juice, no water)* • *2 L container (considers the total amount of juice and water, but underestimates the size of the container)* • *3 L container (correct answer)*	***Using Mental Math*** Model and have students use mental math strategies and estimate before beginning a question. They will more than likely catch their mistakes. For example, they should know that 20 x 40 is 10 times bigger than 2 x 40. When they estimate, they realize that 80 would be an incorrect answer. This same misconception applies when dividing whole numbers containing dividends with zeros. This is the division of how many groups. Using place value vocabulary and modelling representations will support understanding and reasoning skills. Students should make use of place-value understandings in estimating or calculating and should be encouraged to talk about place-value concepts while explaining reasoning. This can include choosing friendly or benchmark numbers that are easier to work with and are near the given numbers. Further examples can be found in the guides. At the core of estimating is understanding number relationships, benchmark numbers, and compatible numbers. ***Number Talks*** Number talks foster the development of number sense by encouraging students to think flexibly about numbers and mathematical operations. Through engaging in discussions and exploring various strategies, students deepen their understanding of mathematical concepts. Moreover, number talks promote mathematical discourse, as students articulate their reasoning, listen to their peers' strategies, and engage in collaborative problem-solving. Hence, number talks support the use of multiple strategies for problem-solving, reinforcing the idea that there are often multiple paths to a solution. They also facilitate the development of mental math skills by providing opportunities for regular practice. Furthermore, number talks can be adapted to meet the needs of diverse learners, making them a versatile tool for differentiated instruction. They offer valuable insights into students' mathematical understanding, enabling teachers to assess student progress and plan future instruction accordingly. ***Retrieval Practice Strategies*** It is essential that students know their facts and understand how numbers relate to each other. Retrieval practice supports instruction and assessment by strengthening schemas that students have built, helping them commit the information to their long-term memory. Retrieval practice involves daily cumulative review and improves the efficiency of learning. Examples include making and using flashcards, concept maps, or grids, intentional games, choral response, and interleaved practice. All of which facilitate better learning than highlighting or re-reading questions. ***Teaching through Problem-Solving*** Provide guidance, not direct instruction on problem-solving strategies as students share their own solutions and findings. Focus on strategies and not procedures. Use students' methods to guide instruction. Be intentional when selecting student methods to share. Elaborate on the methods used by students to solve and justify and encourage students to comment and ask questions of their peers. Encourage students to think about the context of the question as a whole and show their thinking using a strategy of choice. Concepts and skills should be connected to everyday situations and other curricular areas. Encourage students to make connections to make mathematics come alive through math-to-world, math-to-math, and math-to-self connections. Develop students' mathematical vocabulary, initiate effective ways to navigate informational text, and encourage students to reflect on what they have learned.

In the second question, students need to demonstrate an understanding of division with whole numbers or use multiplication to support thinking. *You have 54 marbles.* *You put the marbles into 6 bags.* *There is the same number of marbles in each bag.* *How many marbles do 2 bags contain?* • *108 (2 bags with 54 marbles in each – multiplication)* • *18 (correct answer)* • *12 (student multiplies 6 bags by 2 bags)* • *11 (correct number of marbles in one bag added to 2 bags)*	Embed strategies/tools such as the Frayer Model (see below), Concept Circles, and Exit Cards to assess student learning.  ***Modelling with Think Alouds*** Model problem solving with students by sharing aloud their thinking as they read to understand a problem. Through the process, students learn how to verbalize to understand a question and to make sense of what is being asked and what is in their toolbox to answer the question. It is important that teachers model this process to encourage the thinking process, using various strategies in our toolbox and checking for reasonableness.

Activities to Support Lesson Planning

Number Talks and Number Strings

Use number talk routines to develop efficiency, flexibility, and accuracy with computations. They help to elicit specific strategies that focus on number relations and number theory rather than a series of steps found in traditional algorithms. Classroom conversations and discussions around purposefully crafted computation problems are at the very core of number talks.

A few examples are outlined below, while further examples of strategies are outlined in the curriculum guides and in the resources section.

Begin with examples that require less cognitive load to support the development of effective strategies. These can include activities that focus on number relationships, compatible numbers, and benchmark numbers. Encourage flexibility and talk about estimation. Each example helps to guide students in understanding the importance of place value.

Grade 3	Grade 4	Grade 5	Grade 6
Goal – Using partial products/quotients to support understanding of expanded notation and place value. Select either the list for multiplication or the list for division. Show each number sentence one at a time asking students the guiding questions in between. 2 x 2 8 ÷ 4 2 x 4 12 ÷ 4 4 x 2 20 ÷ 4 4 x 4 **Question Prompts:** Knowledge: Solve the number sentence. Explain your strategy or state your strategy. Application: How are these strategies related? How does it help you solve when using larger values? Analysis: Create your own number sentence that relates to these number sentences and would use the same or similar strategy. **Note:** Some grades 4 and 5 students may start here to support readiness.	Goal – Using partial products/quotients to support understanding of expanded notation and place value. Select either the list for multiplication or the list for division. Show each number sentence one at a time asking students the guiding questions in between. 2 x 45 40 ÷ 4 5 x 100 80 ÷ 4 5 x 40 16 ÷ 4 5 x 5 96 ÷ 4 5 x 245 **Question Prompts:** Knowledge: Solve the number sentence. Explain your strategy or state your strategy. Application: How are these strategies related? How does it help you solve when using larger values? Analysis: Create your own number sentence that relates to these number sentences and would use the same or similar strategy. **Note:** Some grade 5 students may start here to support readiness.	Goal – Using partial products/quotients to support understanding of expanded notation and place value. Select either the list for multiplication or the list for division. Show each number sentence one at a time asking students the guiding questions in between. 35 x 10 400 ÷ 4 35 x 2 80 ÷ 4 35 x 20 16 ÷ 4 35 x 24 496 ÷ 4 **Question prompts:** Knowledge: Solve the number sentence. Explain your strategy or state your strategy. Application: How are these strategies related? How does it help you solve when using larger values? Analysis: Create your own number sentence that relates to these number sentences and would use the same or similar strategy. **Note:** Some grade 6 students may start here to support readiness.	Goal – Using products/quotients to support understanding of expanded notation and place value. Select either the list for multiplication or the list for division. Show each number sentence one at a time asking students the guiding questions in between. 3 x 10 40 ÷ 4 3 x 1 4 ÷ 4 3 x 0.1 0.4 ÷ 4 3 x 0.01 0.04 ÷ 4 **Question prompts:** Knowledge: Solve the number sentence. Explain your strategy or state your strategy. Application: How are these strategies related? Will the value be larger or smaller than the previous? What do you notice about numbers? Why are the answers the way they are? Analysis: Create your own number sentence that relates to these number sentences and would use the same or similar strategy.

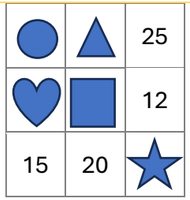
Open Questions and Games

Open questions can include supporting the understanding value and digit placement. Use questions that encourage the use of tools and different representations. An open task can be as simple as asking students to model multiplication and division sentences with base-ten blocks, counters, or a number line. They can also be questions where students explore numberless word problems that help them focus on the context and relationship between the quantities, rather than just apply operations. Math manipulatives and models encourage different strategies to build number sense and reinforce skills, while engaging students in the learning process. Remember to ask them to explain their models. How does it relate to the question being asked?

Games can also offer ways for students to apply and demonstrate operational sense in context and allow for multiple entry points. Cross-strand opportunities also help to reinforce and help students to see mathematics as holistic and not siloed into its strands. They can also be used across multiple grades and abilities. Games that include number cubes and cards also encourage students to determine count and quantity, and sums, differences, and products of numbers in a fun and interactive way.

The following activities can be set up to support a problem-solving lesson. They can be used to model a think aloud and engage in conversation with students. All values are based on grade level outcomes but can be modified to accommodate student need and learning progression. Grade 6 students for example can focus on whole numbers first then transition into decimal numbers.

Grade 3	Grade 4	Grade 5	Grade 6
Ask students to determine which digits should replace each letter. The same digit is used each time the letter appears. A x B = C and/or B x D = EA Ask students to place the digits 1, 3, and 5 to create the greatest and least quotients. __ __ ÷ __	Ask students to determine which digits should replace each letter. The same digit is used each time the letter appears. A x BC = DAE Ask students to place the digits 1, 3, 5, and 7 to create the greatest / least quotients. __ __ __ ÷ __	Ask students to determine which digits should replace each letter. The same digit is used each time the letter appears. A x BCB = DAEB Ask students to place the digits 1, 3, 5, and 7 to create the greatest / least quotients. __ __ __ ÷ __	Ask students to determine which digits should replace each letter. The same digit is used each time the letter appears. A x B.C = DA.E Ask students to place the digits 1, 2, and 6 to create the greatest and least quotients. __.__ ÷ __
Missing Values: In this activity, one of the numbers on the multiplication chart is missing. For example, ask students to come up with a way to multiply 3 x 5 if the 5 is missing on the chart. This requires students to use their number sense.	**Missing Values:** In this activity, one of the numbers on the multiplication chart is missing. For example, ask students to come up with a way to multiply 4 x 79 if the 9 is missing on the chart. This requires students to use their number sense.	**Broken calculator:** In this activity, one of the number keys is broken. For example, ask students to come up with a way to multiply 7 x 59 if the 9 key on the calculator is not working. This requires students to use their number sense.	**Broken calculator:** In this activity, one of the number keys is broken. For example, ask students to come up with a way to multiply 4 x 0.25 if the 4 key on the calculator is not working. This requires students to use their number sense.
Further examples using leveled questions: **Multiplication** Knowledge: What two numbers can be multiplied to get 24? Application: The boxes at the end of each row and the bottom of each column give the result of multiplying the two numbers in that row and column. What are the missing values?	**Further examples using leveled questions:** **Multiplication** Knowledge: What two numbers can be multiplied to get 108? Application: The boxes at the end of each row and the bottom of each column give the result of multiplying the two numbers in that row and column. What are the missing values?	**Further examples using leveled questions:** **Multiplication** Knowledge: What three numbers can be multiplied to get a value of 108? Application: The boxes at the end of each row and the bottom of each column give the result of multiplying the three numbers in that row or column. What are the missing values?	**Further examples using leveled questions:** **Multiplication** Knowledge: What two numbers can be multiplied to get a value of 0.75? Application: The boxes at the end of each row and the bottom of each column give the result of multiplying the three numbers in that row or column. What are the missing values?



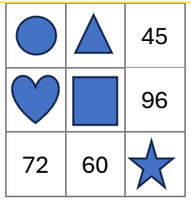
Analysis: Create your own square. Share with a partner.

Division
This challenge is about dividing a two-digit number (up to 25) by a single-digit number. Have students decide on which number they are going to be dividing by. This is the divisor. The challenge is going to be to relate division to multiplication through repeated subtraction or determining equal grouping for this divisor. Now students generate a two-digit number. This is your dividend. You could use the spinners or number cubes to generate the digits or use their imagination.

Knowledge: Have students divide their dividend by their divisor. Record the quotient. Create other dividends and divide them by the same divisor. Record the quotients. Students can use counters to support equal grouping.

Application: Look carefully at the quotients. When do you have equal groups? What types of numbers can you not have equal groups?

Analysis: Can you spot any patterns? Can you come up with any ‘rules’ to help you in solving for division questions?



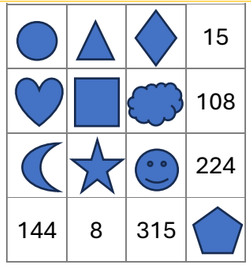
Analysis: Create your own square. Share with a partner.

Division
This challenge is about dividing a two-digit number by a single-digit number. Have students decide on which number they are going to be dividing by. This is the divisor. The challenge is going to be to come up with some rules (e.g. what happens when you divide by 1) for this divisor. Now generate a two-digit number. This is your dividend. You could use the spinners or number cubes to generate the digits or use their imagination.

Knowledge: Have students divide their dividend by their divisor. Record the quotient. Create other dividends and divide them by the same divisor. Record the quotients. Students can use base-ten materials of other manipulatives to support the process.

Application: Look carefully at the quotients. When is the quotient a whole number? When is there a remainder? What is the remainder?

Analysis: Can you spot any patterns? Can you come up with any ‘rules’ to help you in solving for division questions?



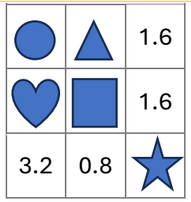
Analysis: Create your own square. Share with a partner.

Division
This challenge is about dividing a three-digit number by a single-digit number. Have students decide on which number they are going to be dividing by. This is the divisor. The challenge is going to be to come up with some rules (e.g. what happens when you divide by 0) for this divisor. Now generate a three-digit number. This is your dividend. You could use the spinners or number cubes to generate the digits or use their imagination.

Knowledge: Have students divide their dividend by their divisor. Record the quotient. Create other dividends and divide them by the same divisor. Record the quotient. Students can use base-ten materials of other manipulatives to support the process.

Application: Look carefully at the quotients. When is the quotient a whole number? When is there a remainder? What is the remainder?

Analysis: Can you spot any patterns? Can you come up with any ‘rules’ to help you in solving for division questions?



Analysis: Create your own square. Share with a partner.

Division
This challenge is about dividing a decimal number by a single-digit whole number. Have students decide on which number they are going to be dividing by. This is the divisor. The challenge is going to be to come up with some rules (e.g. even/odd, relation to whole numbers) for this divisor. Now generate a decimal number. This is your dividend. You could use the spinners or number cubes to generate the digits or use their imagination.

Knowledge: Have students divide their dividend by their divisor. Record the quotient. Create other dividends and divide them by the same divisor. Record the quotient. Students can use base-ten materials of other manipulatives to support the process.

Application: Look carefully at the quotients. What happens with the decimal place? When is there a remainder? What is the remainder? What is happening with the place value?

Analysis: Can you spot any patterns? Can you come up with any ‘rules’ to help you in solving for division questions using decimals?

What are some sample questions to help support assessment?

Cognitive Level	Grade 3	Grade 4	Grade 5	Grade 6
Knowledge	Rewrite the following number sentences using multiplication or division. 8 - 2 - 2 - 2 - 2 = 0 12 - 3 - 3 - 3 - 3 = 0 2 + 2 + 2 + 2 = 8 3 + 3 + 3 + 3 = 12 What number is four times larger than three? What number is three times smaller than fifteen?	Calculate: 8 x 365 = ___ 68 ÷ 3 = ___ 123 x 9 = ___ 96 ÷ 4 = ___ Can 3 people share 18 marbles fairly? How about 60? Describe the solution procedure for determining the product of a single digit and a 3-digit number. What numbers can be multiplied to give you a product of 24? What numbers are divisible by 4 and 5?	Calculate: 36 x 42 = ___ 123 ÷ 6 = ___ 18 x 9 = ___ 645 ÷ 5 = ___ Describe the solution procedure for determining the product of two 2-digt numbers. What number is double 8? What number is divisible by 18 and 3? Can 3 people share 183 marbles fairly?	Calculate: 5 x 0.27 = ___ 6.05 ÷ 5 = ___ 3 x 5.67 = ___ 14.50 ÷ 8 = ___ Estimate the following: 4 x 57.9 82.2 ÷ 9 1.62 x 5 1.16 ÷ 6 What happens when you divide by 0 or 1? Describe a solution procedure for determining the product of a whole number and a decimal. Describe a solution procedure for 18 x 5 using mental math.

Application

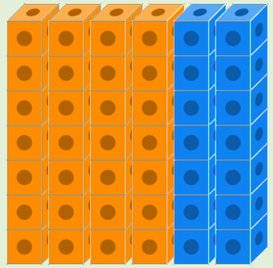
Write a multiplication and/or division sentence to describe this picture.

*Adapted from *Eyes on Math* M. Small g3-5

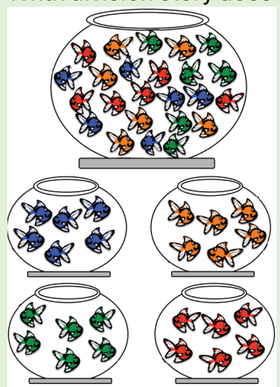
What arrays does this picture represent?

Three friends want to share 15 apples. How many apples will each friend get? Show how you solved it.

What multiplication or division (or both) does this picture represent?



What division story does this image show?



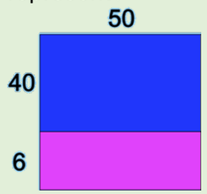
*Adapted from Eyes on Math M. Small g3-5

Dani has 145 cards.
Dani keeps 5 cards and shares the rest with three friends Arden, Tatum, and Onyx.
Arden receives 80 cards.
Tatum receives double the number of cards that are given to Onyx.
How many cards does Onyx get?

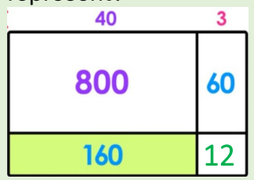
JJ has $400 to buy 8 games. Each game costs $37. Without calculating, does JJ have enough money? How do you know?

To multiply 12 X 5, Chris thinks “6 X 10”. Explain his thinking.

What multiplication sentence does this image represent?



What division sentence does this image represent?



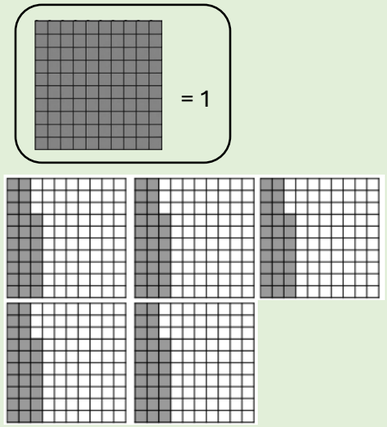
Pens come in packages of 3, 5, and 8. Mrs. O’Regan bought 26 pens for her class. How many packages of each type might she have bought?

The local arena is hosting an event. It can hold up to a maximum of 800 people. If 86 people can fit on a bus, and the organizers are expecting 9 full buses, will there be enough room in the arena for everyone? Why or why not?

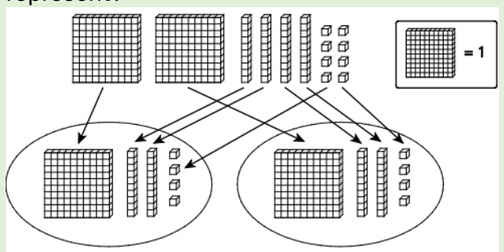
The soccer team collected 680 recyclable containers during the recycling drive. That was almost 3 times more than the hockey team collected. About how many recyclables did the hockey team collect?

To multiply 12 X 15, Chris thinks “6 X 30”. Explain this thinking.

What multiplication sentence does this image represent?



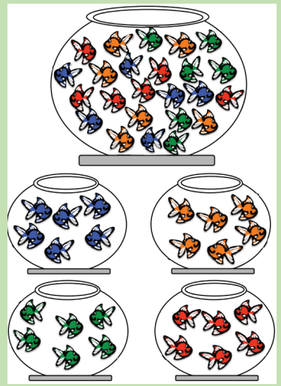
What division sentence does this image represent?



You can buy a 2 kg bag of apples for $8.46 or apples of your choice for $4.38 for every kilogram. Which option will you choose if you need 4 kg of apples?

The length of a killer whale on average is 9.3 m. The length of a cow is about 3.1 m. How many times bigger is the whale compared to the cow?

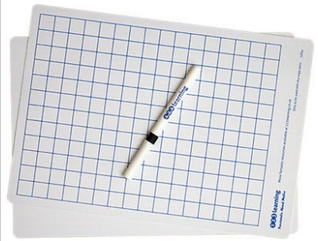
To multiply 12 X 1.5, Chris thinks “6 X 3”. Explain this thinking.

Analysis	You are planning a game day party. You decide to prepare mini sandwiches and fresh lemonade for you and your 4 friends. If you make 3 mini sandwiches each for everyone, how many sandwiches do you make? If you need 2 lemons for every cup of lemonade and you only have 8 lemons, how many cups of lemonade can you make? What numbers can be used to multiply to get the value of 12? 24? Justify your answer.	Suppose there are four more fish added to this picture. Would it still show a division story? How?  *Eyes on Math M. Small g3-5 Sophie builds a tower with green and blue blocks. There are 9 levels in the tower. If less than half of the blocks are green, how many could be green blocks and how many could be blue blocks? How do you know? Create a story problem and solve it using the values of 6 and 9. How many people could share 18 marbles fairly?	You decide to set up a stand over the summer on weekends to sell lemonade and giant cookies. You sell the lemonade for $1 a glass and the cookies for $2 each. After one day you make $62 and on the second day you make $56. If you made 4 dozen cookies and have none left, how many glasses of lemonade did you sell? Name a three-digit number where the last two digits is a number divisible by 4. Repeat for three more numbers. What do you notice? You divide a 3-digit number by a 1-digit number and get a whole number quotient. How many digits can the quotient have? You multiply two numbers, and the product is close to 2600. Both numbers are greater than 10. What could they be? How many people could share 60 marbles fairly?	You decide to set up a stand over the summer on weekends to sell lemonade and giant cookies. You sell the lemonade for $1.25 a glass and the cookies for $1.75 each. After one day you make $62.50 and on the second day you make $56.25. If you made 4 dozen cookies and have none left, how many glasses of lemonade did you sell? List lots of numbers greater than 50 that are multiples of 3. Add the digits of each number. What do you notice? Does the same thing happen for multiples of numbers other than 3? You multiply two numbers, and the product is close to 24.55. What could they be? You divide two numbers, and the quotient is smaller than 1. What could they be? Create a story problem and solve it using the values 3.46 and 7.

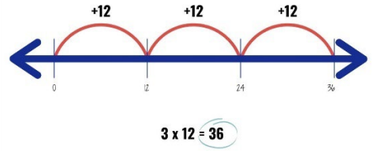
Supporting Resources

Manipulatives and Models to Support Learning

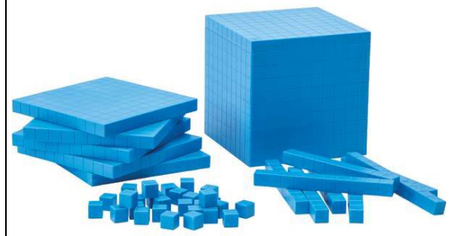
Grids



Open Number Line



Base-ten blocks

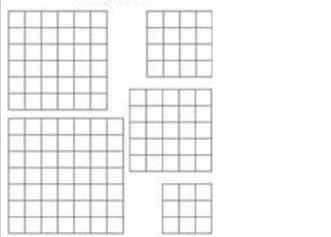


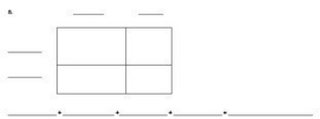
Place Value Chart

Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths

Play money

Arrays or Area Models





Hundreds Charts

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Multiplication Charts

X	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

Number Cubes and Cards

Print and Electronic Resources

Bay-Williams, Jennifer M, et al. (2021). *Figuring out Fluency in Mathematics Teaching and Learning, Grades K-8: Moving beyond Basic Facts and Memorization*. Thousand Oaks, California, Corwin.

Cameron, Antonia. (2020). *Early Childhood Math Routines: Empowering Young Minds to Think*. Portsmouth, New Hampshire, Stenhouse Publishers.

Costello, D. (2021), *Making Math Stick: Classroom strategies that support the long-term understanding of math concepts.* Markham, ON: Pembroke Publishers.

Department of Education and Early Childhood Development (EECD), Province of Nova Scotia (2013). Mathematics 3 Curriculum Guide. Halifax, NS: Author.

Department of Education and Early Childhood Development (EECD), Province of Nova Scotia (2014a). Mathematics 4 Curriculum Guide. Halifax, NS: Author.

Department of Education and Early Childhood Development (EECD), Province of Nova Scotia (2014b). Mathematics 5 Curriculum Guide. Halifax, NS: Author.

Department of Education and Early Childhood Development (EECD), Province of Nova Scotia (2014c). Mathematics 6 Curriculum Guide. Halifax, NS: Author.

Fiore, Mary, and Maria Luisa Lebar. (2016). *The Four Roles of the Numerate Learner*. Pembroke Publishers Limited.

Marks Krpan, C., (2017), *Teaching Math with Meaning Cultivating Self-Efficacy Through Learning competencies, Grades K - 8*. Toronto, ON: Pearson Education Canada. (Chapters 5 and 6 – Communication and Thinking)

Newton, Nicki. (2021). *Guided Math in Action: Building Each Student's Mathematical Proficiency with Small-Group Instruction*. London, Routledge.

Parrish, S., (2010), Number Talks Helping Children Build Mental Math and Computation Strategies. Portsmouth, NH: Heinemann.

SanGiovanni, John. (2018). *Mine the Gap for Mathematical Understanding, Grades 3-5*. Corwin Press.

SanGiovanni, John, and Novak, Jennifer. (2018). *Mine the Gap for Mathematical Understanding Common Holes and Misconceptions and What to Do about Them*. Thousand Oaks, California, Corwin, a SAGE Company.

SanGiovanni, John, Bay-Williams, Jennifer, and McFadden, Rosalba. (2021). *Figuring Out Fluency – Multiplication and Division with Whole Numbers.* Corwin Press.

Small, M. (2009). *Making mathematics meaningful to Canadian students, K–8*. Toronto, ON: Nelson Education Ltd.

Small, M (2012). *Eyes on Math: A Visual Approach to Teaching Math Concepts*. Toronto, ON: Nelson Education Ltd.

Van De Walle, J.A. (2001). *Elementary and middle school mathematics teaching developmentally fourth edition*. New York, NY: Addison Wesley Longman.

Van de Walle, J.A. and Lovin, L. (2006). *Teaching student-centered mathematics grades 3–5*. Boston: Pearson Allyn & Bacon

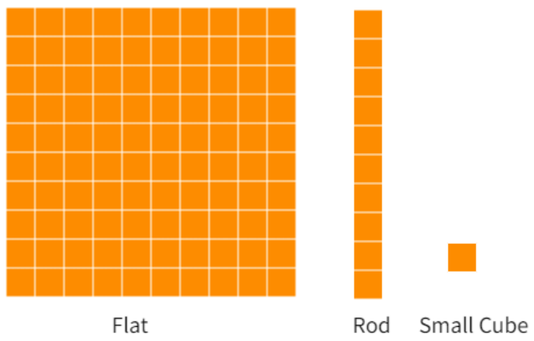
Representing Decimal Numbers

Alignment to previous Outcomes	Related Outcome	Alignment to upcoming Outcomes
4N09: Students will be expected to describe and represent decimals (tenths and hundredths), concretely, pictorially, and symbolically.	**5N08:** Students will be expected to describe and represent decimals (tenths, hundredths, and thousandths) concretely, pictorially, and symbolically. **5N10:** Students will be expected to compare and order decimals (to thousandths) by using benchmarks, place value, and equivalent decimals.	**6N01** Students will be expected to demonstrate an understanding of place value for numbers greater than one million and less than one thousandth.

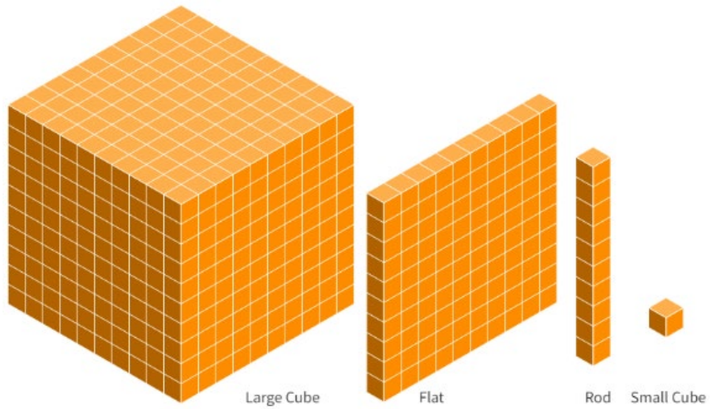
What conclusions can be drawn from the NSA: Mathematics in Grade 6?

When presented with base-ten blocks to represent decimal numbers, almost half of students were unable to identify the decimal number being represented. This was evident especially when the flat was not used to represent one. Students had difficulty thinking proportionally between the blocks to identify the decimal number being represented. Students also had challenges going between representing decimal numbers in words and symbolic form. This resulted in place value errors where students identify digits in the hundreds position rather than the hundredths and likewise in the thousands position rather than the thousandths.

Why is this an area of need and how can we support students?

Models for Decimal Values	
Misconceptions/Errors in Student Work	**Possible Next Steps in the Classroom**
Some students struggle with using the same models for whole numbers as for decimals. If the flat represents 100 with whole numbers, students have difficulty switching to a flat representing the whole, or 1, for decimals.  Decimals: If the flat is 1, the rod is 0.1, and the small cube is 0.01.	One important strategy to remember when transitioning to decimals using base ten materials is helping students to understand that the same proportional relationship still exists when working with decimal numbers. Whatever block represents the whole, proportionally we are still thinking in terms of tens or tenths. Like with whole numbers, students will need many opportunities to explore this concept and explain the relationship between the same digits in different numbers. When working with base ten materials, make sure that students do not refer to the flat as 100 but as 1 whole. Relate the flat to everyday items such as one whole cake. In this case, the rod becomes a slice that is one tenth of the cake, and the small cube becomes a piece that is one tenth of the slice and one hundredth of the whole cake.  Another analogy may also include thinking about the large cube as a big piece of cheese, the flat as a cheese slice, the rod as a “cheese string”, and the small cube as a bite or a small piece.

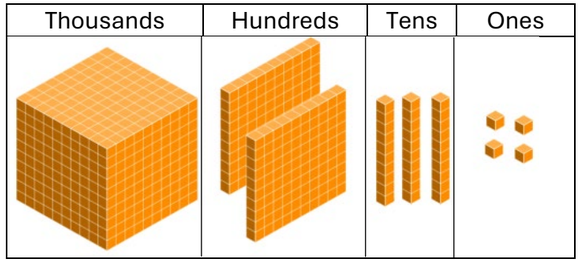
As such, students also struggle when another block is used to represent 1, such as a large cube or the rod. Essentially, students are unable to think proportionally or move flexibly between representations in knowing how many parts make up the whole.



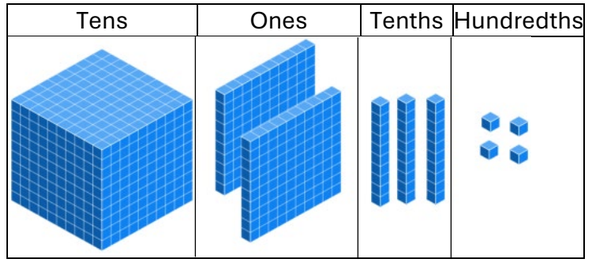
For thousandths: If the large cube is 1, the flat is 0.1, the rod is 0.01, and the small cube is 0.001.

Using a place value chart along with base-ten blocks may help to support student understanding like whole numbers. If available, using a set of base-ten blocks of a different colour to differentiate decimal blocks from whole number blocks.

Example (whole numbers)



Example (decimals)

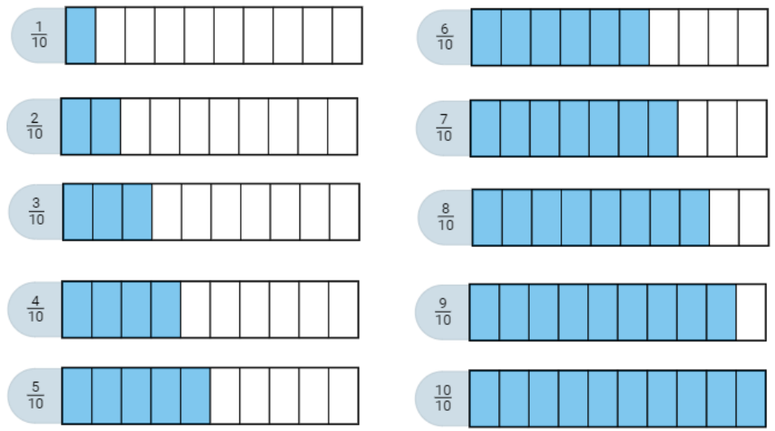


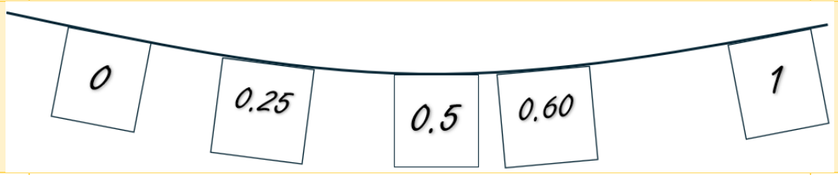
If students struggle with this model for decimals, they might feel more comfortable working with hundredths grids instead. Students can relate how many small squares (hundredths) fill the grid (one whole).

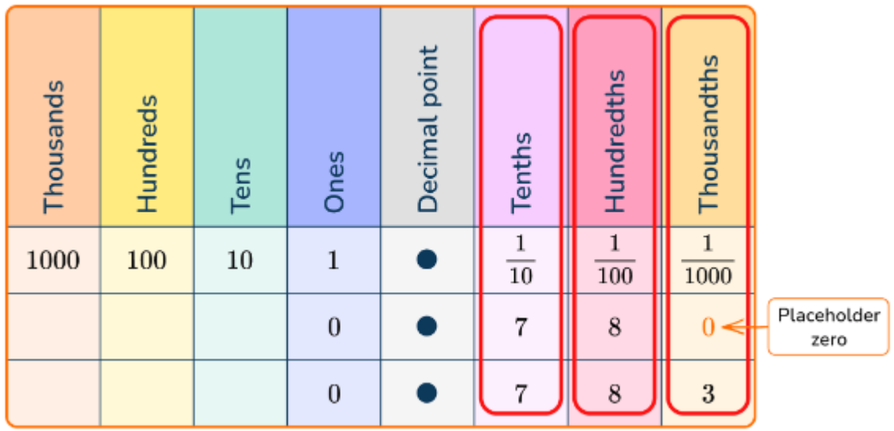
When working with hundredths, another strategy is to use coins. A loonie can represent 1, a dime for 0.1, and a penny for 0.01. Students can build an understanding of tenths and hundredths by relating the number of coins to the decimal value.

Read decimals and encourage students to read decimals as tenths, hundredths, thousandths, and so on and not as digits. This can include activities where students count by tenths or hundredths and relate to their fractional representations.

Sample Activities to Support Lesson Planning		
Grade 4	**Grade 5**	**Grade 6**
Ask students to draw a picture on a decimal grid. Each colour or figure used represents a decimal value including tenths and hundredths. The activity could also include the use or a discussion on money where students relate pennies, nickels, dimes and quarters to a dollar or loonie; e.g. “My figure is worth 30 cents or $0.30.” Knowledge: Draw two different shapes on your grid worth the same decimal amount. Draw two shapes on your grid worth different decimal amounts. Application: What decimals are represented in your drawing? What are ways to write and say this decimal using the words tenths and hundredths? Analysis: Which part of your drawing contains the largest decimal and the smallest decimal? How do you know? Other possibilities: Application: Draw something to match the value _____. Analysis: How do you know you are representing _____? How does it compare to your partners drawing?	Ask students to draw a picture on a thousandths grid. Each colour or figure used represents a decimal value. OR Ask students to create a picture using pattern blocks or tangrams. Each shape/colour/size represents a decimal value. Knowledge: Draw two different shapes on your grid worth the same decimal amount. Draw two shapes on your grid worth different decimal amounts. Application: What decimals are represented in your drawing? What are ways to write and say this decimal using the words tenths, hundredths, or thousandths? Analysis: Which part of your drawing contains the largest decimal and the smallest decimal? How do you know? Other possibilities: Application: Draw something to match the value _____. Analysis: How do you know you are representing _____? How does it compare to your partners drawing?	Ask students to draw a picture on a thousandths grid. Each colour or figure used represents a decimal value. OR Ask students to create a picture using pattern blocks or tangrams. Each shape/colour/size represents a decimal value. Knowledge: Draw two different shapes on your grid worth the same decimal amount. Draw two shapes on your grid worth different decimal amounts. Application: What decimals are represented in your drawing? What are ways to write and say this decimal using the words tenths, hundredths, or thousandths? Analysis: Which part of your drawing contains the largest decimal and the smallest decimal? How do you know? Other possibilities: Application: Draw something to match the value _____. Analysis: How do you know you are representing _____? How does it compare to your partners drawing?

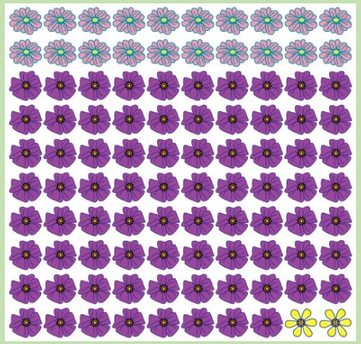
Counting in Tenths	
Misconceptions/Errors in Student Work	**Possible Next Steps in the Classroom**
Some students have difficulty counting in tenths as they bridge whole numbers. For example, … 0.8, 0.9, 0.10, 0.11, 0.12 …instead of 0.8, 0.9, 1.0, 1.1, 1.2. While students may know through counting that they are 8 tenths, 9 tens, 10 tenths, 11 tenths, 12 tenths, representing them in decimal form is difficult.	If students are reading 0.10 as 10 tenths, present them with a 10 by 10 grid, and ask them to show you 10 hundredths and to write the decimal. Have them look at the decimal that they read as 10 tenths to see and hear the contradiction. When students are reading decimals, encourage them to use proper decimal terminology. When instructing, use the decimal name rather than what is typically used in society. 3.04 would be read as “three and 4 hundredths” and not “three decimal zero four.” 0.56 would be read as “fifty-six hundredths” and not “decimal fifty-six.” Have students practice counting in decimals in a similar way students learn how to count in early years. Match the count with the visual and/or symbolic representation of the decimal. Making a connection to how you count in unit fractions supports and strengthens this relationship as well.  Think about what students will say when they get to ten tenths. Will they see this as one? What will happen with eleven tenths or one and one tenth? Reinforce further with images illustrating decimals that go beyond one whole.

Sample Activities for Lesson Planning		
Grade 4	**Grade 5**	**Grade 6**
Have students create a concrete number line or use a clothesline. Provide each student with a decimal value and have them place or stand on the number line. Each student must say their number as they place it. When the number line is complete students count up or count back. Initially, use values less than one, then have students work with values greater than one as well.	Have students create a concrete number line or use a clothesline. Provide each student with a decimal value and have them place or stand on the number line. Each student must say their number as they place it. When the number line is complete students count up or count back. Include numbers that represent the same value (e.g. 0.5 and 0.50) and are greater than and less than one.	Have students create a concrete number line or use a clothesline. Provide each student with a decimal value and have them place or stand on the number line. Each student must say their number as they place it. When the number line is complete students count up or count back. Include numbers that represent the same value (e.g. 0.55 and 0.550 or fractions) and are greater than and less than one.
		
Knowledge: Write a decimal and explain what each digit means. Where could you see your decimal outside of school? Application: Represent your decimal value using base-ten blocks or hundredths grid. How does your value compare to your partner's value? What decimal values are between your two numbers? Name them and represent them. Analysis: What are some decimal values that are less than/greater than yours and your partners? How can you prove these decimals are less or greater? Place the numbers in order and label them using numbers and words.	Knowledge: Write a decimal and explain what each digit means. Where could you see your decimal outside of school? Application: Represent your decimal value using a thousandths grid. How does your value compare to your partner's value? What decimal values are between your two numbers? Name them and represent them. Analysis: What are some decimal values that are less than/greater than yours and your partners? How can you prove these decimals are less or greater? Place the numbers in order and label them using numbers and words.	Knowledge: Write two decimals that are equivalent and greater than 1. Explain how the values are equivalent. Application: Represent your decimal value using a thousandths grid. How does your value compare to your partner's value? What decimal values are between your two numbers? Name them and represent them. Analysis: What are some decimal values that are less than/greater than yours and your partners? How can you prove these decimals are less or greater? Place the numbers in order and label them using numbers and words.

Interpreting and Comparing Decimal Digits	
Misconceptions/Errors in Student Work	**Possible Next Steps in the Classroom**
Some students think that decimal numbers with a greater number of digits or with digits that are greater are “bigger” than they really are. For example, some students believe that 0.248 is larger because it has more digits. 0.248 > 0.79 2.45 < 2.358 Likewise, if the whole numbers of both decimal numbers being compared are the same, students read the values after the decimal as whole numbers (e.g., 45.380 as forty-five and three hundred eighty).	Place value language and estimation is helpful. When working with decimals, the teacher should model and use the decimal name and encourage the students to do the same. Instead of saying, “zero decimal two four eight”, say, “two hundred forty-eight thousandths”. Images and concrete tools that can be used to represent decimal values should be continually used to help reinforce quantity and comparison. Another strategy is to allow students to record zeros to align the numbers along the decimal. 0.248 can be compared to 0.79 by placing a zero to the right of the digit 9. 0.248 0.79**0** Similarly, zeros can be placed to the right of the decimal with a whole number. 4.000 0.591 

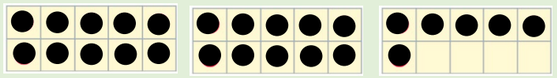
Sample Activities for Lesson Planning		
Grade 4	**Grade 5**	**Grade 6**
Ask students to build the number 0.35 using base-ten materials. Have them show it in more than one way. Do the same again but use the value of 0.4 for example where there are fewer digits, but a larger value. Discuss the various representations using place value language. Knowledge: Place the following decimals on a number line: 0.5, 0.9, 0.12, 0.35, 0.44, 0.76. Application: Represent the decimal values using base-ten blocks. Label each one. How many base-ten blocks did you use in each representation? Are your values the same/different? How do you know? Prove to me/partner which of your two values is greater. Analysis: Represent a different value of your choice. Prove to me/partner which value is greater. Order your values and your partner's values from smallest to greatest. How do you know you are correct? Note: The number line activity noted above in the "counting" section is also helpful at comparing the sequence and size of decimal values.	Ask students to build the number 0.358 using base-ten materials. Have them show it in more than one way. Do the same again but use the value of 0.45 for example where there are fewer digits, but a larger value. Discuss the various representations using place value language. You may also want to include a whole number as well, e.g. 1.358. Knowledge: Place the following decimals on a number line: 1.12, 0.35, 2.4, 0.545, 0.76, 0.09. Application: Represent the decimal values using base-ten blocks. How many base-ten blocks did you use in each representation? Are your values the same/different? How do you know? Prove to me/partner which of your two values is greater. Analysis: Represent a different value of your choice. Prove to me/partner which value is greater. Order your values and your partner's values from smallest to greatest. How do you know you are correct? Note: The number line activity noted above in the "counting" section is also helpful at comparing the sequence and size of decimal values.	Ask students to build the number 0.358 using base-ten materials. Have them show it in more than one way. Do the same again but use the value of 0.45 for example where there are fewer digits, but a larger value. Discuss the various representations using place value language. You may also want to include a whole number as well, e.g. 1.358. Knowledge: Place the following decimals on a number line: 1.12, 0.358, 2.4, 0.545, 0.76, 1.09. Application: Represent the decimal values using base-ten blocks. Label each one. How many base-ten blocks did you use in each representation? Are your values the same/different? How do you know? Prove to me/partner which of your two values is greater. Analysis: Represent a different value of your choice. Prove to me/partner which value is greater. Order your values and your partner's values from smallest to greatest. How do you know you are correct? Note: The number line activity noted above in the "counting" section is also helpful at comparing the sequence and size of decimal values.

What are some sample questions to help support assessment?

Cognitive Level	Grade 4	Grade 5	Grade 6
Knowledge	Why does the arrangement of flowers make it easy to describe 0.2 and 0.02 of the flowers?  *Eyes on Math g3–5 Identify the place value of each digit in the following number: 4.56 What is the number 6.87 in words?	What is the number 6.803 in words? Identify the place value of each digit of 14.352? Choose the appropriate symbol >, =, or < to compare 36.09 to 36.090.	What is the number 6.8039 in words? Identify the place value of each digit of 14.35279? Choose the appropriate symbol >, =, or < to compare: 36.09 to 36.090 123.456 to 123.4567

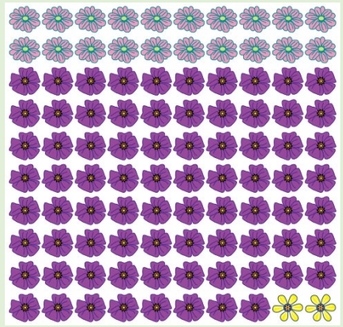
Application

If a ten-frame represents one whole or 1, which decimal number is represented by the following illustration?



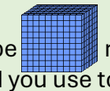
Represent the value 0.27 on a hundredths grid or with base-ten blocks.

What other decimals of the flowers are easy to describe?

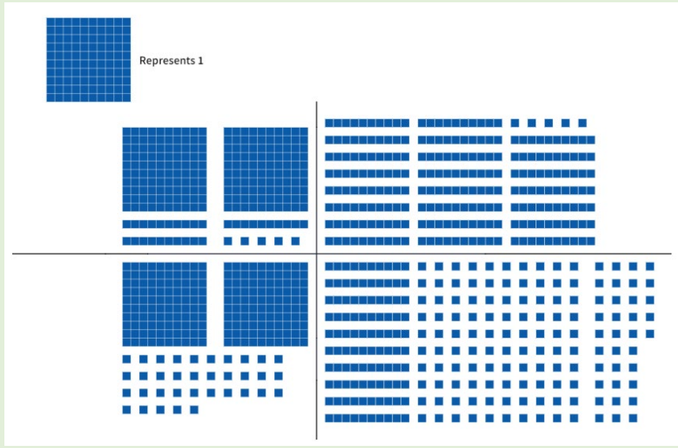


*Eyes on Math g3–5

Represent two values using base-ten blocks; one that is smaller than 1.34 and one that is larger. Prove it to your partner. Record all three values in increasing order.

The large cube  represents 1. Which set of base-ten blocks would you use to represent the decimal number 0.233?

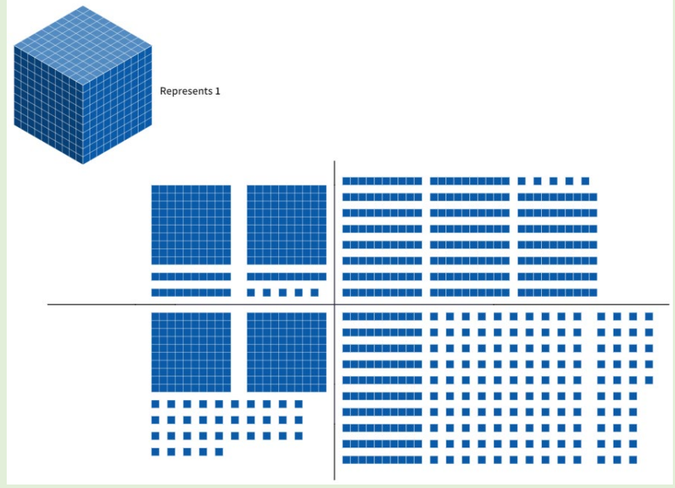
How are these representations the same and how are they different? Which representation is easiest to know the value and which is most difficult?

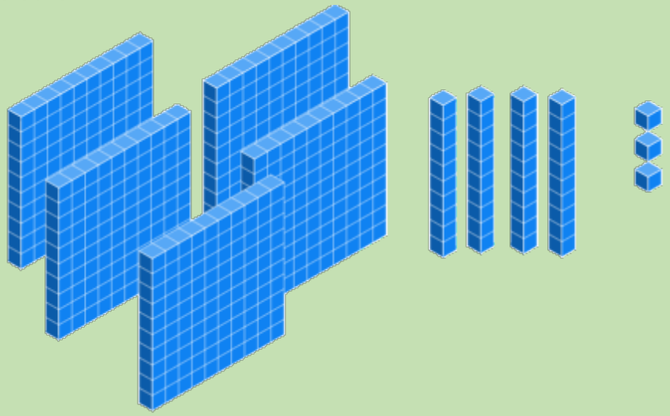
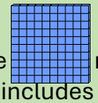
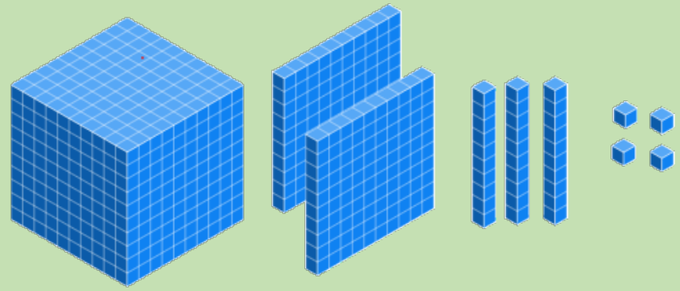


Represent two values using base-ten blocks; one that is smaller than 1.346 and one that is larger. Prove it to your partner. Record all three values in increasing order.

Place the following values on a number line:
3.8 2.1399 4.855 2.14 0.2358 0.98 1.25

How are these representations the same and how are they different? Which representation is easiest to know the value and which is most difficult?

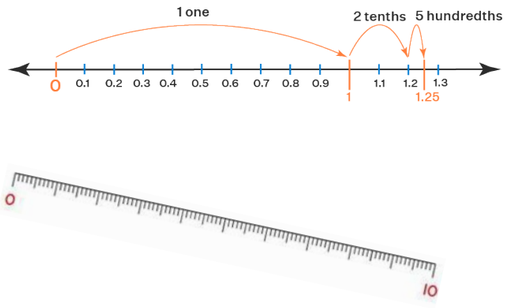


Analysis	Represent two values using base-ten blocks; one that is smaller than 1.34 and one that is larger. Prove it to your partner. Record all three values in increasing order. What number is 2 hundredths more than 4.89?	You represent a decimal number using all the following base-ten blocks.  Give two possible decimal numbers that can be represented. Can you think of a third number that can be represented? How are they the same and how are they different? If the  represents 1 whole, represent a decimal number that includes the use of the large cube, rod, and small cube. What number is it? How does it compare to your partner's number/representation? Which value is larger? How do you know?	You represent a decimal number using all the following base-ten blocks.  Give two possible decimal numbers that can be represented. Can you think of a third number that can be represented? How are they the same and how are they different? If the  represents 1 whole, represent a decimal number that includes the use of the large cube, rod, and small cube. What number is it? How does it compare to your partner's number/representation? Which value is larger? How do you know?

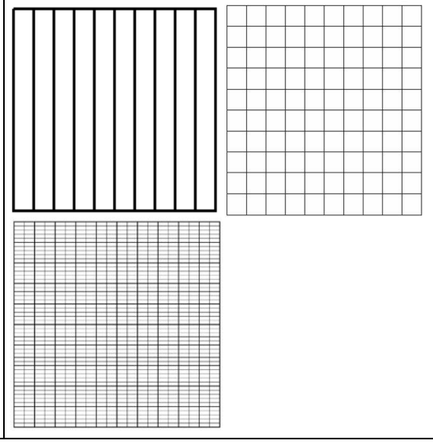
Supporting Resources

Manipulatives and Models to Support Learning

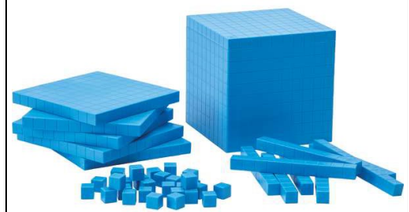
Open number line or a Ruler



Tenths, hundredths, thousandths grids



Base-ten blocks



Place value chart

Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths

Play money

Print and Electronic Resources

Bay-Williams, Jennifer M, et al. (2021). *Figuring out Fluency in Mathematics Teaching and Learning, Grades K-8: Moving beyond Basic Facts and Memorization*. Thousand Oaks, California, Corwin.

Cameron, Antonia. (2020). *Early Childhood Math Routines: Empowering Young Minds to Think*. Portsmouth, New Hampshire, Stenhouse Publishers.

Costello, D. (2021), *Making Math Stick: Classroom strategies that support the long-term understanding of math concepts.* Markham, ON: Pembroke Publishers.

Department of Education and Early Childhood Development (EECD), Province of Nova Scotia (2014a). Mathematics 4 Curriculum Guide. Halifax, NS: Author.

Department of Education and Early Childhood Development (EECD), Province of Nova Scotia (2014b). Mathematics 5 Curriculum Guide. Halifax, NS: Author.

Department of Education and Early Childhood Development (EECD), Province of Nova Scotia (2014c). Mathematics 6 Curriculum Guide. Halifax, NS: Author.

Fiore, Mary, and Maria Luisa Lebar. (2016). *The Four Roles of the Numerate Learner*. Pembroke Publishers Limited.

Marks Krpan, C., (2017), *Teaching Math with Meaning Cultivating Self-Efficacy Through Learning competencies, Grades K - 8*. Toronto, ON: Pearson Education Canada. (Chapters 5 and 6 – Communication and Thinking)

Newton, Nicki. (2021). *Guided Math in Action: Building Each Student's Mathematical Proficiency with Small-Group Instruction*. London, Routledge.

Parrish, S., (2010), Number Talks Helping Children Build Mental Math and Computation Strategies. Portsmouth, NH: Heinemann.

SanGiovanni, John. (2018). *Mine the Gap for Mathematical Understanding, Grades 3-5*. Corwin Press.

SanGiovanni, John, and Jennifer Rose Novak. (2018). *Mine the Gap for Mathematical Understanding Common Holes and Misconceptions and What to Do about Them*. Thousand Oaks, California, Corwin, a SAGE Company.

Small, M. (2009). *Making mathematics meaningful to Canadian students, K–8*. Toronto, ON: Nelson Education Ltd.

Small, M (2012). *Eyes on Math: A Visual Approach to Teaching Math Concepts*. Toronto, ON: Nelson Education Ltd.

Van De Walle, J.A. (2001). *Elementary and middle school mathematics teaching developmentally fourth edition*. New York, NY: Addison Wesley Longman.

Van de Walle, J.A. and Lovin, L. (2006). *Teaching student-centered mathematics grades 3–5*. Boston: Pearson Allyn & Bacon

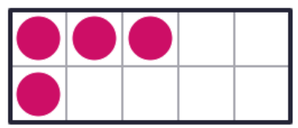
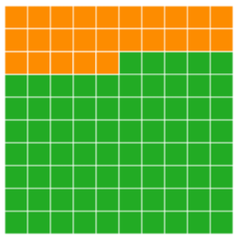
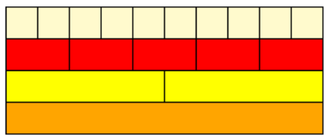
Relating Fractions and Decimal Numbers

Alignment to previous Outcomes	Related Outcome	Alignment to upcoming Outcomes
4N10: Students will be expected to relate decimals to fractions and fractions to decimals (to hundredths).	**5N09:** Students will be expected to relate decimals to fractions and fractions to decimals (to thousandths).	**6N06**: Students will be expected to demonstrate an understanding of percent (limited to whole numbers) concretely, pictorially, and symbolically.

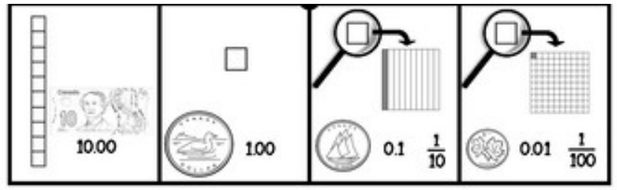
What conclusions can be drawn from the NSA: Mathematics in Grade 6?

Students had some difficulty identifying decimals that are equivalent to a given fraction. This included fractions represented as tenths and thousandths. Students commonly identified incorrect decimal values related to misconceptions in place value. In these questions, no images were provided to students to support their thinking.

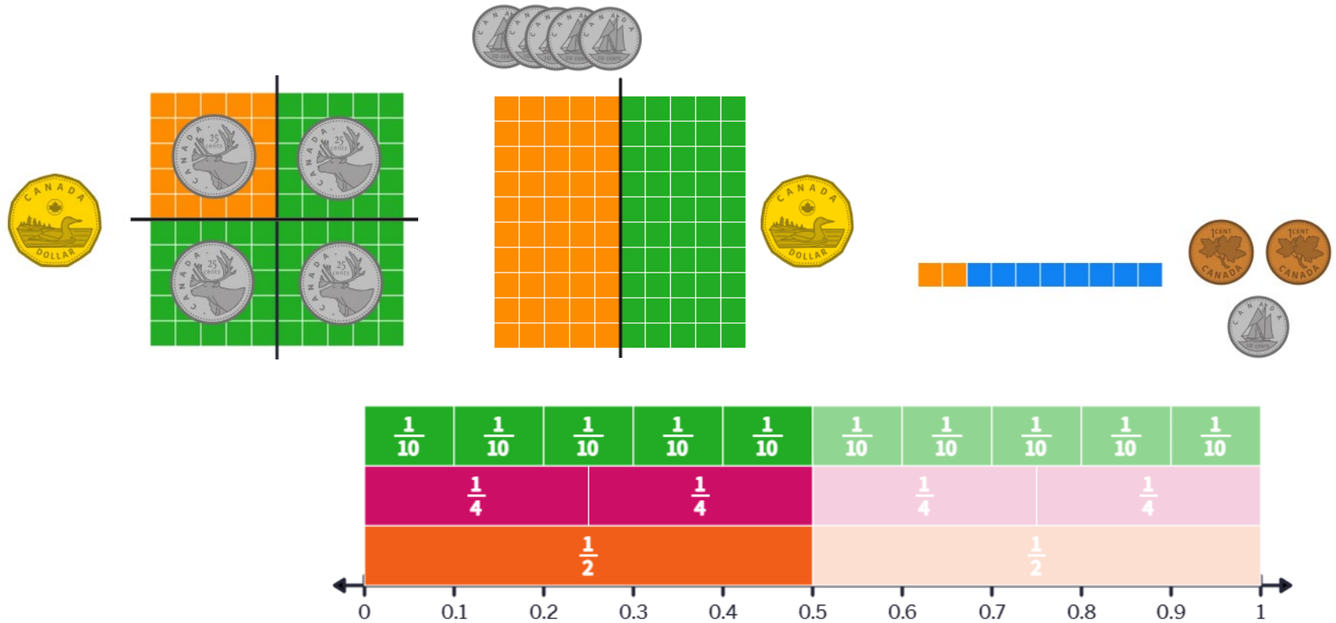
Why is this an area of need and how can we support students?

Fractions and Decimals are Different Types of Numbers	
Misconceptions/Errors in Student Work	**Possible Next Steps in the Classroom**
Some students think that fractions and decimals are entirely different and unrelated concepts. They have yet to understand that decimals and fractions are two different ways of representing the same concept: parts of a whole. Some errors can also stem from a lack of understanding of fractions and when decimals are introduced, requires familiarity with the concept of fractional tenths. This works the other way as well, where students may misunderstand the relationship between place value and the size of decimals in relation to a whole. Understanding equivalence in terms of base ten is essential for moving between fractions and decimals. Errors and misconceptions may be a result of not having enough exposure to the language of place value and decimals and the various representations to help define the meaning of parts of a whole.	Before diving into exploring the relationship between fractions and decimal numbers, students should have a good understanding of each in so far as they both represent parts of whole. A decimal number is expressed using the place value system but can be expressed in an equivalent fractional form. To start, it is natural to begin with items that come in tens to begin renaming fraction tenths as decimal tenths. Using ten frames, Cuisenaire rods, hundred grids and base-ten blocks are strategies to connect fractions to decimals and decimals to fractions concretely and pictorially. Concrete tools and images are great ways to help students visualize how fractions and decimals are connected. Frequent use of visuals and tools will support how student begin to internalize or imagine fractions and decimals (internalize) in their minds. Note how the values are presented, ensuring students use and practice with conventional and non-conventional representations. Examples:   

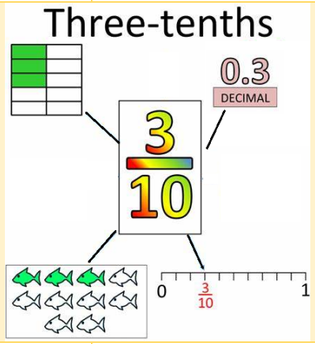
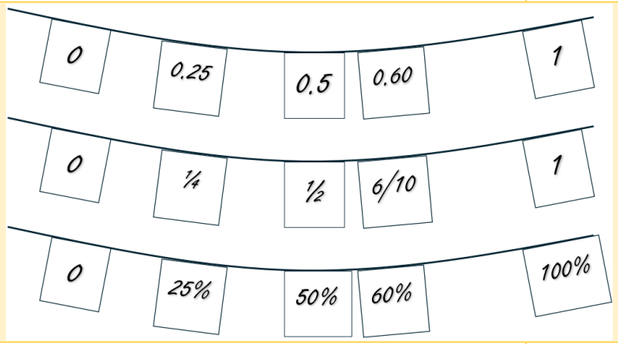
Another strategy is to use money models. Since 100 cents is analogous to 1 dollar, students can begin to show money amounts where the whole is 100 and the equivalent decimal form which many may already be familiar with through real-life experiences.



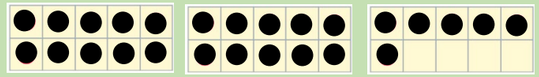
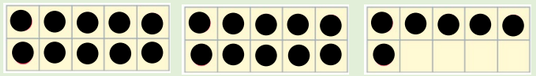
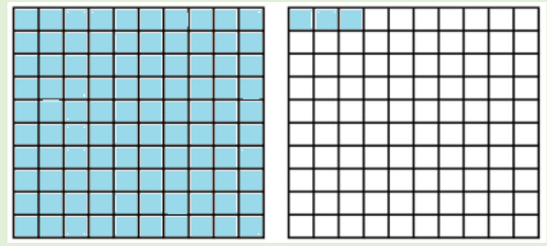
Do not begin dividing the digits in the fractions or multiplying to find equivalencies too quickly. Students need to develop the understanding of how the two types of numbers are related using images and concrete materials first. Common fractions like fourths, halves, tenths, or hundredths can be used to illustrate their equivalence.

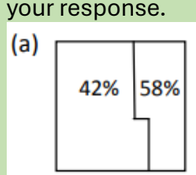
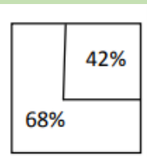


When working with fractions and decimals with students, make sure to consistently use place value language. This involves saying values in tenths, hundredths, and thousandths. For example, instead of saying, “zero decimal four five” or “zero decimal forty-five”, use place value language, “forty-five hundredths”. This is very important in supporting the understanding of the relationship between these types of numbers.

Sample Activities for Lesson Planning		
Grade 4	**Grade 5**	**Grade 6**
Provide students in the class with a card. There are three types of cards: a decimal value, a fraction, and a picture of a representation of a decimal value. Students are to find their equivalent form and end up in groups of three. When they get into their group, they need to prove to themselves that all representations of the value are equivalent and be prepared to share with others. Once all the groups have been formed, you can also have the students place their values on a number line (clothesline). They must then reflect to see if they notice any patterns with the representations. Examples of decimals can include, 0.1, 0.2, 0.5, 0.01, 0.02, 0.9, 0.05, 0.25, 0.75.	Provide students with a similar activity for that in grade 4. Except students can be challenged with cards that also have values into the thousandths. Similarly, students are to find their equivalent form and end up in groups of three. When they get into their group, they need to prove to themselves that all representations of the value are equivalent and be prepared to share with others. Once all the groups have been formed, you can also have the students place their values on a number line (clothesline). They must then reflect to see if they notice any patterns with the representations. Examples of decimals can include, 0.250, 0.5, 0.01, 0.020, 0.900, 1.05, 0.75, 1.45.	Provide students with a similar activity for that in grade 4 or 5. Except students can be challenged with cards that also have percent values. Similarly, students are to find their equivalent form and end up in groups of three or four. When they get into their group, they need to prove to themselves that all representations of the value are equivalent and be prepared to share with others. Once all the groups have been formed, you can also have the students place their values on a number line (clothesline). They must then reflect to see if they notice any patterns with the representations. Examples of decimals can include, 0.250, 0.5, 0.01, 0.020, 0.900, 0.75, 1.000, 1.5.
		
Knowledge: Find an equivalent representation of your number. Place your value on the number line in relation to other values. Application: How does your value compare to your group's value? How do you know they are equivalent? Which card(s) supported your understanding the most. How does your value compare to the others on the number line. What patterns/relationships do you notice between the values on the number line? Analysis: Provide another card with a decimal value to all students. Complexity may differ based on readiness. Have students create their own matching pictorial representation of that decimal as well as fraction. They can then share and prove to their partner of the equivalence. Students can also compare their values and then place their new values on the number line. What further patterns/relationships do you notice between the values on the number line? What has supported your understanding the most?	Knowledge: Find the equivalent representation of your number. Place your value on the number line in relation to other values. Application: How does your value compare to your group's value? How do you know they are equivalent? Which card(s) supported your understanding the most. How does your value compare to the others on the number line. What patterns/relationships do you notice between the values on the number line? Analysis: Provide another card with a decimal value to all students. Complexity may differ based on readiness. Have students create their own matching pictorial representation of that decimal as well as fraction. They can then share and prove to their partner of the equivalence. Students can also compare their values and then place their new values on the number line. What further patterns/relationships do you notice between the values on the number line? What has supported your understanding the most?	Knowledge: Find the equivalent representation of your number. Place your value on the number line in relation to other values. Application: How does your value compare to your group's value? How do you know they are equivalent? Which card(s) supported your understanding the most. How does your value compare to the others on the number line. What patterns/relationships do you notice between the values on the number line? Analysis: Provide another card with a decimal value to all students. Complexity may differ based on readiness. Have students create their own matching pictorial representation of that decimal as well as fraction. They can then share and prove to their partner of the equivalence. Students can also compare their values and then place their new values on the number line. What further patterns/relationships do you notice between the values on the number line? What has supported your understanding the most?

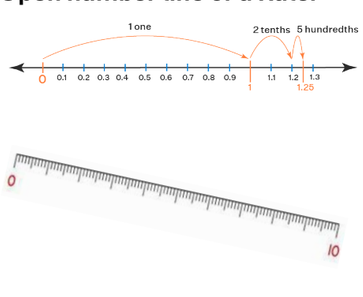
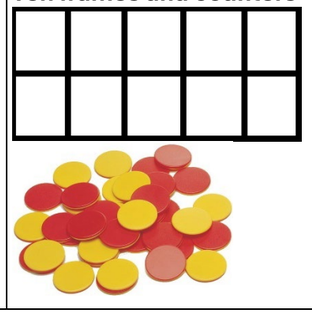
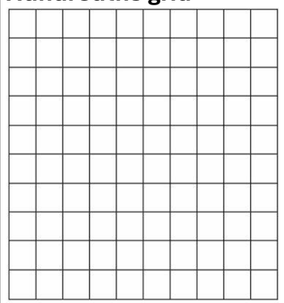
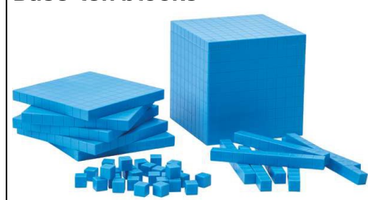
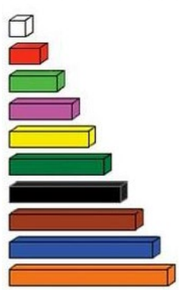
What are some sample questions to help support assessment?

Cognitive Level	Grade 4	Grade 5	Grade 6
Knowledge	Give an example of a fraction with a value less than 0.5. Give an example of a fraction with a value greater than 0.5. What fraction is equivalent to 0.50?	Determine the fraction that is equivalent to the decimal number 0.004? If a ten-frame represents one whole or 1, which decimal number will be represented by the following illustration? 	Which is the least? The greatest? Explain your answer. $\frac{1}{20}$ 20% 0.02 Which one does not belong? Explain your choice. $\frac{3}{4}$ 0.75 0.34 75%
Application	In the diagram below, a ten-frame represents one whole or 1. What decimal number is represented?  Show how 0.43 is the same as 43/100?	How can you show that $\frac{3}{4}$ is the same as 0.75? If a hundredths chart represents one whole or 1, what decimal number is represented by the following illustration? 	60 new floor tiles are being installed in one room. The tiles used must be the following colours: 25% must be blue; half must be red; 0.20 must be green; the rest are yellow. How many would there be of each colour? Draw a picture to support your answer. Changing to newer, more energy-efficient light bulbs can save up to 50% on your electric bill for lighting. If a person's electric bill was $30 before changing light bulbs, what would the bill be with the newer light bulbs? Use a number line to help model. Which of the following representations are equivalent to 20%?  2/10 0.020

Analysis	Pick a fraction and a decimal you find easier to compare. What makes them easy for you to compare? Pick a fraction and a decimal you find harder to compare. What makes them hard for you to compare? What would make it easier to compare them? Create a story problem involving a decimal value and a fraction. Solve your problem.	How many ways can you represent the value 0.25? How do you know all your representations are correct? What patterns exist between the decimal representations of the following fractions? 1/4, 2/4, 3/4, 4/4? How does this help you understand equivalent fractions and decimals? What tools support your understanding? Create a story problem involving a decimal value and a fraction. Solve your problem.	Pick a fraction and a percent that are easy/hard for you to compare. What makes it easy/hard for you to compare? What is incorrect about each of the following diagrams? Justify your response. (a)  (b)  When could 45% be greater than 90%?

Supporting Resources

Manipulatives and Models to Support Learning

Open number line or a Ruler 	**Ten frames and counters** 	**Hundredths grid** 	**Base-ten blocks** 	**Cuisenaire rods** 	**Play money** 

Print and Electronic Resources

Bay-Williams, Jennifer M, et al. (2021). *Figuring out Fluency in Mathematics Teaching and Learning, Grades K-8: Moving beyond Basic Facts and Memorization*. Thousand Oaks, California, Corwin.

Cameron, Antonia. (2020). *Early Childhood Math Routines: Empowering Young Minds to Think*. Portsmouth, New Hampshire, Stenhouse Publishers.

Costello, D. (2021), *Making Math Stick: Classroom strategies that support the long-term understanding of math concepts.* Markham, ON: Pembroke Publishers.

Department of Education and Early Childhood Development (EECD), Province of Nova Scotia (2014a). Mathematics 4 Curriculum Guide. Halifax, NS: Author.

Department of Education and Early Childhood Development (EECD), Province of Nova Scotia (2014b). Mathematics 5 Curriculum Guide. Halifax, NS: Author.

Department of Education and Early Childhood Development (EECD), Province of Nova Scotia (2014c). Mathematics 6 Curriculum Guide. Halifax, NS: Author.

Fiore, Mary, and Maria Luisa Lebar. (2016). *The Four Roles of the Numerate Learner*. Pembroke Publishers Limited.

Marks Krpan, C., (2017), *Teaching Math with Meaning Cultivating Self-Efficacy Through Learning competencies, Grades K - 8*. Toronto, ON: Pearson Education Canada. (Chapters 5 and 6 – Communication and Thinking)

Newton, Nicki. (2021). *Guided Math in Action: Building Each Student's Mathematical Proficiency with Small-Group Instruction*. London, Routledge.

Parrish, S., (2010), Number Talks Helping Children Build Mental Math and Computation Strategies. Portsmouth, NH: Heinemann.

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SanGiovanni, John, and Jennifer Rose Novak. (2018). *Mine the Gap for Mathematical Understanding Common Holes and Misconceptions and What to Do about Them*. Thousand Oaks, California, Corwin, a SAGE Company.

Small, M. (2009). *Making mathematics meaningful to Canadian students, K–8*. Toronto, ON: Nelson Education Ltd.

Small, M (2012). *Eyes on Math: A Visual Approach to Teaching Math Concepts*. Toronto, ON: Nelson Education Ltd.

Van De Walle, J.A. (2001). *Elementary and middle school mathematics teaching developmentally fourth edition*. New York, NY: Addison Wesley Longman.

Van de Walle, J.A. and Lovin, L. (2006). *Teaching student-centered mathematics grades 3–5*. Boston: Pearson Allyn & Bacon

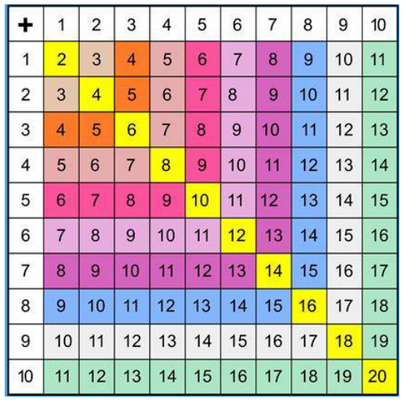
Generalizing to Extend Patterns.

Alignment to previous Outcomes		Related Outcome	Alignment to upcoming Outcomes
3PR01: Students will be expected to demonstrate an understanding of increasing patterns by describing, extending, comparing, and creating numerical patterns (numbers to 1000) and non-numerical patterns using manipulatives, diagrams, sounds, and actions. **3PR02:** Students will be expected to demonstrate an understanding of decreasing patterns by describing, extending, comparing, and creating numerical patterns (numbers to 1000) and non-numerical patterns using manipulatives, diagrams, sounds, and actions.	**4PR01:** Students will be expected to identify and describe patterns found in tables and charts, including a multiplication chart. **4PR02:** Students will be expected to translate among different representations of a pattern (a table, a chart, or concrete materials). **4PR03:** Students will be expected to represent, describe, and extend patterns and relationships, using charts and tables, to solve problems.	**5PR01:** Students will be expected to determine the pattern rule to make predictions about subsequent terms.	**6PR01:** Students will be expected to demonstrate an understanding of the relationship within tables of values to solve problems. **6PR02:** Students will be expected to represent and describe patterns and relationships, using graphs and tables.

What conclusions can be drawn from the NSA: Mathematics in Grade 6?

Many students still experience challenges when working with more complex patterns and relationships. They have difficulty when moving from the basic understanding of patterns to the generalization of a pattern rule to enable them to find any term. Students need to continue to work with representations of patterns, contextually, pictorially, symbolically, and verbally where they have to extend a pattern to predict a subsequent term that is not consecutive.

Why is this an area of need and how can we support students?

Extending a Pattern	
Misconceptions/Errors in Student Work	**Possible Next Steps in the Classroom**
Some students do not recognize that there are different ways to continue a pattern if a pattern rule is not described. They might believe that all patterns can be extended using a single rule or operation. While some patterns follow a straightforward rule (like adding a constant or multiplying by a factor), others may involve a combination of rules or irregular changes. Students might also extend a pattern based solely on the most recent data points without considering the overall trend or structure of the pattern. For example, if given 5, 15, 20 . . . in the beginning of the pattern, students may only see it as a pattern increasing by 10 or 5 and not by both amounts.	To address these misconceptions, educators can provide diverse examples of patterns that include the use of concrete materials and images. Students should be encouraged to explore and discuss complex numerical and visual patterns to help reinforce their understanding of the repeating elements. Teachers should spend time in helping students to make connections between the concrete models, numerical representations, and the abstract language used to describe patterns. For example, if students are working with colored cubes, relate the cube colors to the vocabulary of term, term number, core, and repetition. Encourage students to question assumptions and consider multiple possibilities when extending patterns. For example, there can be more than one way to extend a pattern: 5, 10, 15, 20, 25, 30,... or 5, 10, 15, 25, 35, 50, 65, ... (Small, 2009, p. 579). Teach strategies for analyzing patterns, such as looking for relationships between terms, identifying recurring elements, and testing predictions. Leverage opportunities to identify and explain the patterns in the addition and multiplication tables. These can include place value patterns, multiples, and factors, along with determining an unknown sum, difference, product, or quotient.  	+	1	2	3	4	5	6	7	8	9	10
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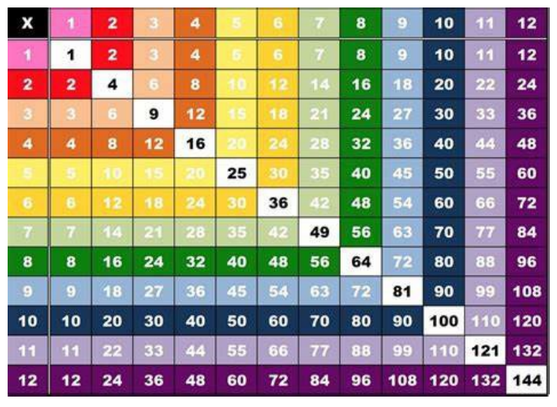
1	2	3	4	5	6	7	8	9	10	11
2	3	4	5	6	7	8	9	10	11	12
3	4	5	6	7	8	9	10	11	12	13
4	5	6	7	8	9	10	11	12	13	14
5	6	7	8	9	10	11	12	13	14	15
6	7	8	9	10	11	12	13	14	15	16
7	8	9	10	11	12	13	14	15	16	17
8	9	10	11	12	13	14	15	16	17	18
9	10	11	12	13	14	15	16	17	18	19
10	11	12	13	14	15	16	17	18	19	20 |  | X	1	2	3	4	5	6	7	8	9	10	11	12
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1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

Sample Activities for Lesson Planning			
Grade 3	**Grade 4**	**Grade 5**	**Grade 6**
Warm-up: Students can explore an addition chart to find as many patterns as they can in the table. Ask students to create a growing pattern where the 10^{th} term is 25. OR Ask students to create a shrinking pattern where the 4^{th} term is 16. Knowledge: What is your pattern? Application: How does your pattern compare to your partner's pattern? How is it the same and/or different? What did you do to come up with your pattern? Analysis: Why can everyone have different patterns, but all end up with the same 10^{th} / 4^{th} term?	Warm-up: Students can explore an addition chart or multiplication chart to find as many patterns as they can in the table. Ask students to create a growing pattern where the 10^{th} term is 56. OR Ask students to create a shrinking pattern where the 4^{th} term is 24. Knowledge: What is your pattern? C Application: How does your pattern compare to your partner's pattern? How is it the same and/or different? What did you do to come up with your pattern? Analysis: Why can everyone have different patterns, but all end up with the same 10^{th} / 4^{th} term?	Warm-up: Students can explore a multiplication chart to find as many patterns as they can in the table. Ask students to create a growing pattern where the 10^{th} term is 84. OR Ask students to create a shrinking pattern where the 4^{th} term is 24. Knowledge: What is your pattern? Application: How does your pattern compare to your partner's pattern? How is it the same and/or different? What did you do to come up with your pattern? Analysis: Why can everyone have different patterns, but all end up with the same 10^{th} / 4^{th} term?	Warm-up: Students can explore a multiplication or division chart to find as many patterns as they can in the table. Ask students to create a growing pattern where the 10^{th} term is 96. Record the values in a table or graph it. OR Ask students to create a shrinking pattern where the 4^{th} term is 24. Record the values in a table or graph it. Knowledge: What is your pattern? Application: How does your pattern compare to your partner's pattern? How is it the same and/or different? What did you do to come up with your pattern? Analysis: Why can everyone have different patterns, but all end up with the same 10^{th} / 4^{th} term?

Generalizing a Pattern

Misconceptions/Errors in Student Work

Some students have difficulty predicting the value of an unknown term using the relationship in a table of values and verifying the prediction. This involves formulating a rule to describe the relationship between two columns of numbers in a table of values. In the example below, the student only sees that the term value increasing by 2 and is not relating the term value to the term to accurately predict a term out of reach. The student is not developing an equation to generalize what is happening.

Term	Term Value
1	3
2	5
3	7
...	...
8	?

Add 2 each time.

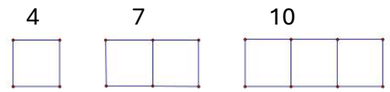
Possible Next Steps in the Classroom

Students should begin by representing a pattern with concrete materials and/or pictures. Then, they should represent the same pattern in a table or chart. Once a table or chart is developed, students have two representations of a pattern: the one created with the drawing or materials and the numeric version that is in the table or chart. They can then explain how these patterns are mathematically alike, that is, why the same relationship exists between the pattern in a table and its concrete representation.

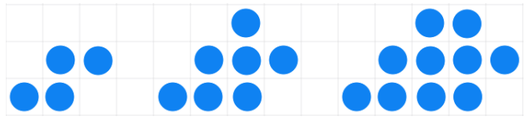
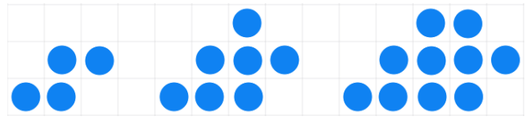
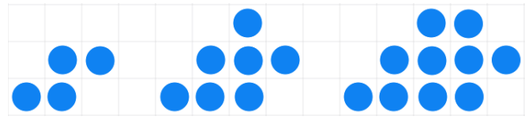
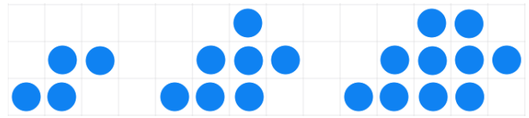
Students should also be given opportunities to reproduce a pattern using concrete materials when presented with a pattern displayed in a table or chart. Students should also be asked to describe what is happening as the pattern increases (or decreases) and how the next step is related to the previous one. It is helpful for students to think of a pattern rule and apply it when analyzing tables or charts for errors.

Using multiple representations helps students to visualize the pattern to generalize what is happening.

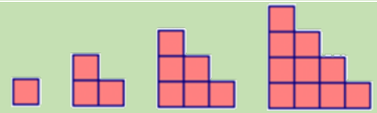
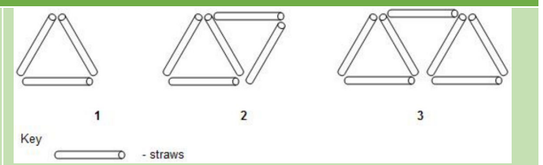
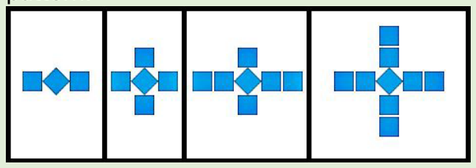
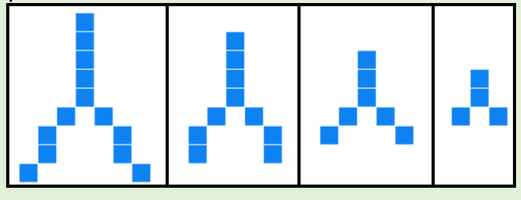
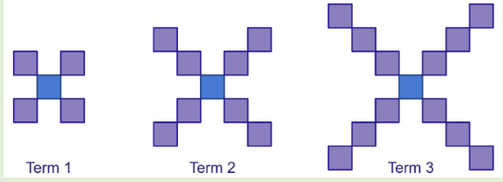
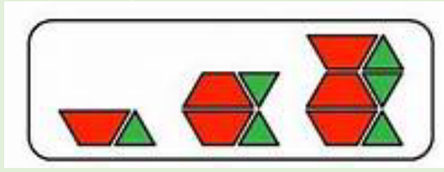
The example below illustrates the same pattern using a picture and a table.



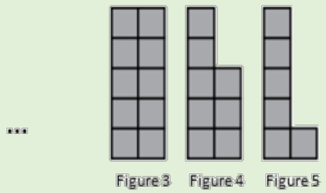
Term Number	Number of Toothpicks
1	4
2	7
3	10

Sample Activities for Lesson Planning			
Grade 3	**Grade 4**	**Grade 5**	**Grade 6**
Show students the following image:  Knowledge: Is this a growing or shrinking pattern? Application: Draw what you think the shapes before and after the ones on the image would look like. What is changing between the shapes? Analysis: How does knowing how the shapes change help in knowing what the next term is? How can you represent this pattern in a table? How can you use this information to predict the next term value?	Show students the following image:  Knowledge: Is this a growing or shrinking pattern? Application: Draw what you think the shapes before and after the ones on the image would look like. What is changing between the shapes? Analysis: How does knowing how the shapes change help in knowing what the next term is? How can you represent this pattern in a table? How can you use this information to predict the 6^{th} term?	Show students the following image:  Knowledge: Is this a growing or shrinking pattern? Application: Draw what you think the shapes before and after the ones on the image would look like. What is changing between the shapes? Analysis: How does knowing how the shapes change help in knowing what the next term is? How can you represent this pattern in a table? How can you use this information to predict the 10^{th} term?	Show students the following image:  Knowledge: Is this a growing or shrinking pattern? Application: Draw what you think the shapes before and after the ones on the image would look like. What is changing between the shapes? Analysis: How does knowing how the shapes change help in knowing what the next term is? How can you represent this pattern in a table? How can you use this information to predict the 20^{th}? What general expression represents the pattern?

What are some sample questions to help support assessment?

Cognitive Level	Grade 3	Grade 4	Grade 5	Grade 6
Knowledge	Write the first 5 terms of a number pattern with the following rule: Starts at 8 and add 3 each time. Write the first 5 terms of a number pattern with the following rule: Starts at 92, subtract 6 each time. What is a growing pattern? What is a shrinking pattern?	Identify two patterns on the hundreds chart. Identify two patterns on the multiplication chart. Write the first 5 terms of a number pattern with the following rule: Starts at 8, add 2, then add 1 alternating each time.	 Using the pattern above, fill in the following table indicating the term number and term value. | Term Number | Term Value | | | | | | | | | | | | | Write the first 5 terms of a pattern that has term values decreasing by 3 each time.	 Using the pattern above, fill in the following table indicating the term number and term value. | Term Number | Term Value | | | | | | | | | | | | |
Application	What is the pattern rule for the following pattern.  What is the pattern rule for the following pattern. 	What is the pattern rule?  Make a table of values that shows the pattern up to term 6. Which pattern would reach 40 first? 120, 100, 80, ... 8, 16, 24, ... A pattern begins like this: 2, 6, ... How might it continue?	Look at the picture below.  The first term is made up of two pattern blocks. The second term is made up of four pattern blocks, and the third term is made up of six pattern blocks. Predict the number of pattern blocks in the eighth term. Fill in the missing values for each pattern: 4, ___, 12, ___, 20, ... 18, 16, 14, ___, ___, ... 2.4, 2.7, ___, ___, 3.6, ...	Use the table and image to determine the pattern rule. Predict the number of chairs for 10 tables. | Number of tables | Number of chairs | | 1 | 4 | | 2 | 6 | | 3 | 8 | | 4 | 10 | | 5 | 12 |  Sheila works in a computer repair shop. She gets paid $75 a day plus $5 for every computer she fixes. (a) Create a table to display the total amount of money Sheila could make in a day for any number of computers she might fix.

Examine the following pattern of Figure 3, Figure 4, and Figure 5 created using small squares:



How many small squares are there in Figure 1?

A pattern begins like this: 1, 3, ...
How might it continue?

A pattern begins like this: 12, 10, ...
How might it continue?

A pattern begins like this: 23, 19, ...
How might it continue?

Identify where the pattern has errors. Explain your thinking.

Figure Number	Number of Tiles
1	4
2	8
3	12
4	18
5	20
6	22
7	28
8	32

Examine the following pattern. How many squares would be in the 7th term?

Show your work using a table of values.

Identify where the pattern has errors. Explain your thinking.

Figure Number	Number of Tiles
1	4
2	8
3	12
4	18
5	22
6	24
7	26
8	30

(b) Write a pattern rule that you could use to find the total amount of money Sheila could make in a day for any number of computers she might fix.
(c) Use your rule to determine how much money Sheila would make if she fixed 12 computers in one day

Graph the information in the table. Use the graph to help you fill in the missing information.

Side Length (cm)	1	2	3	4	5	6	?
Perimeter (cm)	6	12	18	?	30	?	48

What is the pattern rule?

Input	Output
1	2
2	3
3	4
4	5
5	6

Fill in the missing values in the table.

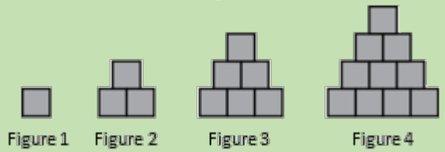
Term number	1	2	3	?
Term value	4	8	?	16

Analysis

What are the next three terms in this counting pattern?

5, 8, 7, 10, 9, 12, 11, ____, ____, ____, ...

Observe the following pattern:



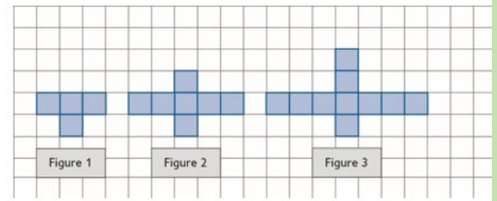
How do you see the pattern growing?

A pattern starts with 2, and the fourth number is 8. What could the in-between numbers be? Create the same pattern in two different ways. Explain how you know your patterns are the same.

Examine the following pattern.

How do you see the pattern growing?

Examine the following pattern consisting of small squares found in the figures.



How many small squares would be found in Figure 5?

A pattern starts with 3, and the fifth number is 11. What could the in-between numbers be? Create the same pattern in two different ways. Explain how you know your patterns are the same.

Examine the following pattern.

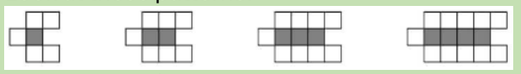
How do you see the pattern growing?

Plants grow every day.
The following table represents the height, *h* in cm, of a plant in terms of the number, *n*, of days.

Number of days	Height of the Plants (cm)
1	4
2	5
3	6
4	7
5	8

Write an expression to describe the relationship between the height of the plant and the number of days?

Look at these patterns:

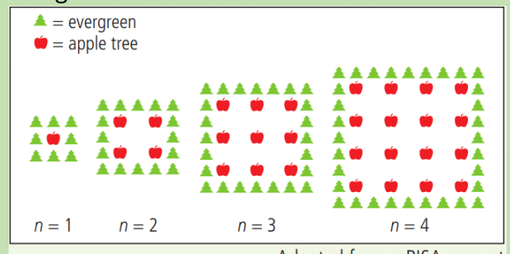


How many white squares would be needed for an image with 7 dark squares?
If an image has 30 squares in total. How many of them are dark? White?

You have a weekly paper route and get paid $30 a week. The following table of values shows your earnings over a five-week period.
Identify the value that does not fit the pattern. Explain why the value is incorrect. Justify your answer.

Number of Weeks	Earnings
1	$30
2	$60
3	$90
4	$100
5	$130

I noticed something interesting about my neighbour's apple orchards. They plant their apple trees in square patterns in each orchard. To protect the trees from the wind, they plant evergreens all around the orchard.

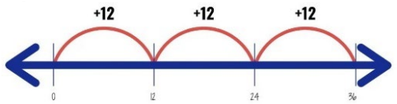
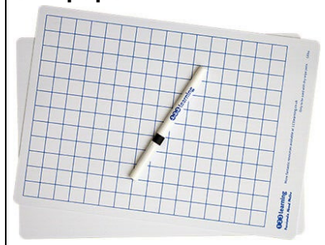


Adapted from a PISA prompt

When does the number of apple trees equal the number of evergreens? Justify your response.
How does the growth of the number of apple trees compare with the growth of the number of evergreens?

Supporting Resources

Manipulatives and Models to Support Learning

Counters	Linking cubes	Square tiles	Pattern Blocks	Open number line	Grid paper
					

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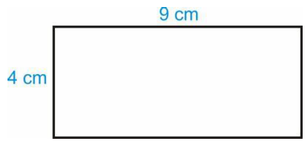
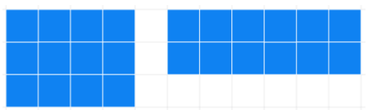
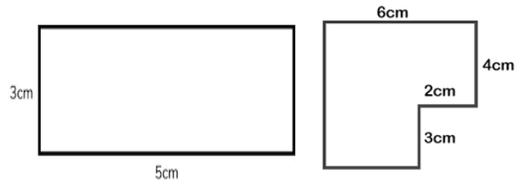
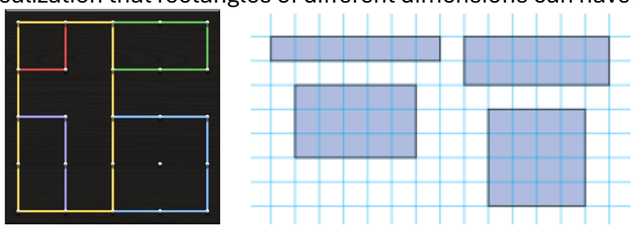
Understanding the Relationship Between Area and Perimeter

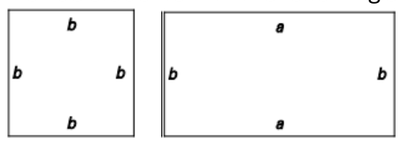
Alignment to previous Outcomes		Related Outcome	Alignment to upcoming Outcomes
3M05: Students will be expected to demonstrate an understanding of perimeter of regular, irregular, and composite shapes by estimating perimeter using referents for centimetre or metre (cm, m) measuring and recording perimeter (cm, m) create different shapes for a given perimeter (cm, m) to demonstrate that many shapes are possible for a perimeter	**4M03:** Students will be expected to demonstrate an understanding of area of regular and irregular 2-D shapes by: • recognizing that area is measured in square units • selecting and justifying referents for the units square centimetre (cm^2) or square metre (m^2) • estimating area using referents for cm^2 or m^2 • determining and recording area (cm^2 or m^2) constructing different rectangles for a given area (cm^2 or m^2) in order to demonstrate that many different rectangles may have the same area	**5M01:** Students will be expected to design and construct different rectangles, given a perimeter or an area or both (whole numbers), and make generalizations.	**6M03**: Students will be expected to develop and apply a formula for determining the • perimeter of polygons • area of rectangles • volume of right rectangular prism

What conclusions can be drawn from the NSA: Mathematics in Grade 6?

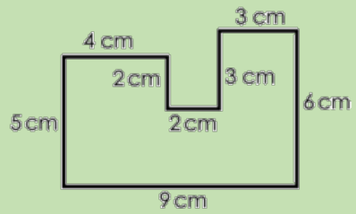
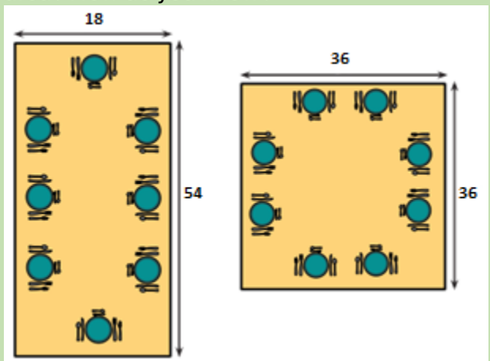
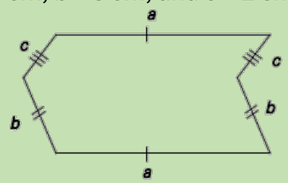
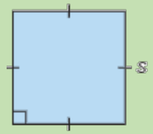
Two big ideas in measurement are perimeter and area. Students were able to determine the perimeter and area of regular geometric shapes, especially when all the information was provided. When asked to find a perimeter or area when not all the measurements in a diagram are provided, students sometimes forgot to include the measures of the unlabelled sides or found it difficult to calculate a missing side length. Students also struggled when asked to work with perimeter and area together in application and analysis questions. This included problems that involved the relationship between perimeter and area. Students were unable to predict the impact on the perimeter or on the area of a two-dimensional figure when the shape of this figure changes, while maintaining the same area or the same perimeter.

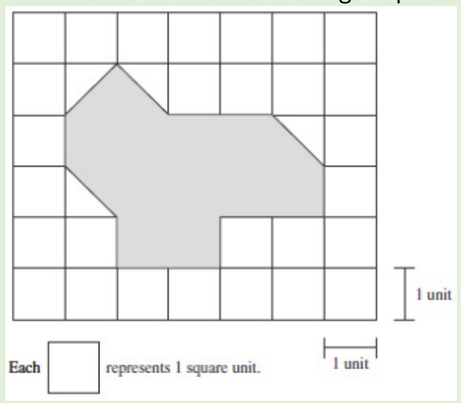
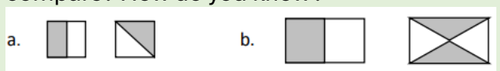
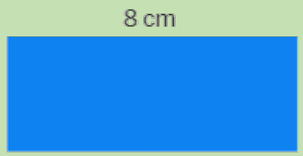
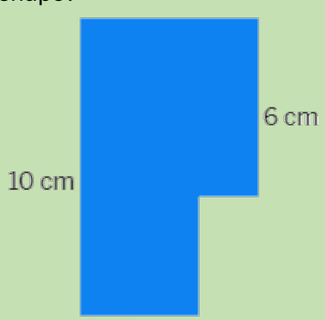
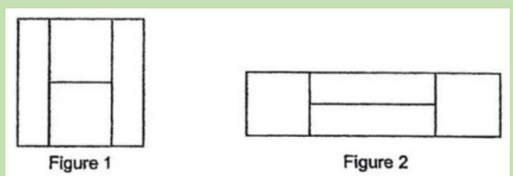
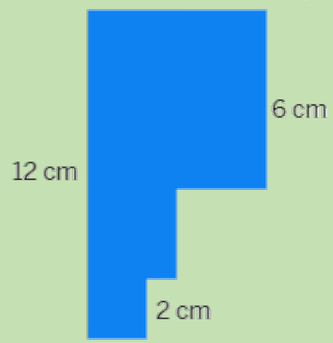
Why is this an area of need and how can we support students?

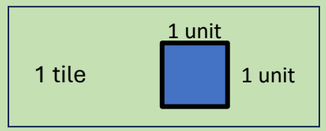
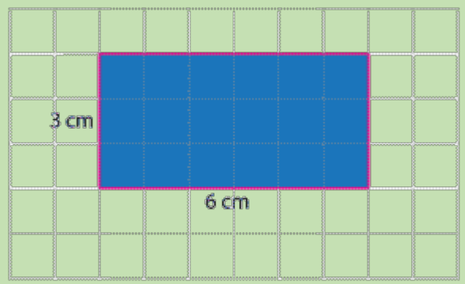
Properties of Perimeter and Area	
Misconceptions/Errors in Student Work	**Possible Next Steps in the Classroom**
Firstly, a common error is that sometimes students forget to include the measures of unlabeled sides. In the example to the left, the student has only added one of the lengths and one of the widths when determining the perimeter of the object. The student may have limited experience with missing information in a diagram.  P = 4 + 9 = 13 cm Some students may continue to focus on linear dimensions of an object to decide which has a greater area. This leads to them having a limited understanding that the area of a shape does not change if it is cut up and rearranged to make a different shape. In the example below, the student thinks the rectangles have different areas since their perimeters are different. Rectangles of the same area can have different perimeters. The opposite is true for rectangles with the same perimeter; they will also have different areas.  Larger area Smaller area	Provide opportunities for students to practice their understanding of perimeter by giving them examples with unlabeled side lengths. This will also support problem solving with composite shapes. Before students begin to calculate the perimeter, encourage them to label any side lengths that do not already have labels. Discuss why these labels were not originally provided on the diagram. Emphasize that the perimeter is the entire distance around the shape. Examples of possible rectangles and composite shapes include the following:  It is important that students have many opportunities to construct rectangles of different areas and perimeters concretely and pictorially. Students should learn about area and perimeter together. Focus on: • the perimeter and area are two different but related concepts. • it is possible for rectangles of a certain area to have different perimeters. • it is possible for rectangles with the same perimeter to have different areas. • the closer the shape is to a square, the larger the area will be. • for any given perimeter, the rectangle with the smallest possible width will result in the least area. Have students develop a chart to observe these patterns between the areas and perimeters. Geo-boards or grid paper can be used to create various rectangles all with the same perimeter. Students should be working toward the realization that rectangles of different dimensions can have the same perimeter.  Students should also determine the area of each of these rectangles to understand that though each of these rectangles has the same perimeter, the area of each of the rectangles is different.

Sample Activities for Lesson Planning			
Grade 3	**Grade 4**	**Grade 5**	**Grade 6**
Draw 3 different rectangles with a perimeter of 12 cm. OR Draw 3 different shapes with the perimeter of 12 cm. OR Draw 3 different shapes with a perimeter of 12 cm with at least one being a composite and/or irregular shape. Knowledge: What shapes did you draw? What are the side lengths? Application: What do you notice about the shapes you drew? How are they the same/different? How do they compare to others in the class? Analysis: How can you have different shapes with the same perimeter? Find different shapes in the classroom that have a perimeter of about 12 cm. How can you make sure they are about 12 cm? How do they compare?	Draw 3 different rectangles with an area of 12 cm^2. OR Draw 3 different shapes with an area of 12 cm^2. OR Draw 3 different shapes with at least one being a composite shape with an area of 12 cm^2. Knowledge: What shapes did you draw? What are the side lengths/dimensions? Application: What do you notice about the shapes you drew? How are they the same/different? How do they compare to others in the class? Analysis: How can you have different shapes with the same area? Find or build different shapes in the class with an area of about 12 cm^2. How can you make sure they are about 12 cm^2? How do they compare?	Ask students to draw a rectangle with not much area but a lot of perimeter. Knowledge: What are your dimensions? Why did you choose those dimensions? Application: How does your rectangle compare to those of your classmates? How do your dimensions differ? Analysis: Draw as many different rectangles as possible with an area of 12cm^2. What do you notice? When do you have the largest and smallest area/perimeter? Can you generalize your findings to your partner/class? Prove your conjecture (i.e., use a different area). Is this true for all rectangles?	Provide students with the following shapes.  Arrange the shapes to create various combinations of different composite shapes. Knowledge: In terms of the variables, what are the dimensions? Perimeters? Areas? Application: How do your dimensions, perimeters and areas compare to your classmates? What do you notice about the variables and operations used? Analysis: Generalize your findings to your partner/class? What is an efficient strategy to determine the perimeter of a polygon? Area of a polygon? How could you use this information to help you determine the volume of prisms?

What are some sample questions to help support assessment?

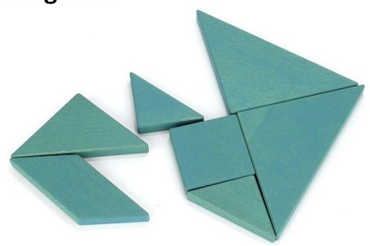
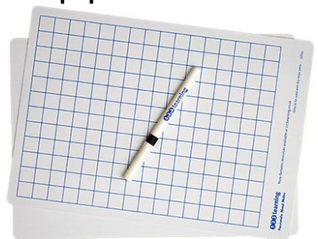
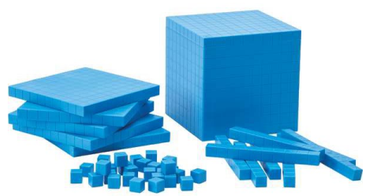
Cognitive Level	Grade 3	Grade 4	Grade 5	Grade 6
Knowledge	What is the perimeter of this shape?  Draw a rectangle with a perimeter of 12 cm. Draw a shape with a perimeter of 12 cm.	What is the area of this shape?  Draw a shape with an area of 12cm^2? Draw a rectangle with an area of 12cm^2?	Tom drew two rectangles, A and B. The dimensions of A are 3 cm by 4 cm. The dimensions of B are 2 cm by 5 cm. What can you say about their areas? Without solving, which table has the larger area? How do you know?  *Adapted from *Eyes on Math* Solve for the perimeter and area of the tables. What do you notice about the perimeters and areas of the tables?	What is the expression for the perimeter of the shape below? Solve for the perimeter if a = 6 cm, b = 3 cm, and c = 2 cm.  A square has a side length *s*. Write a formula for the area of the square. 

Application	Draw 3 different shapes with a perimeter of 12 cm. What do you notice? How can you estimate the perimeter of a playing card? What tools or personal referents could you use? Estimate and then measure. Compare your estimate and the measure. Is there something you could do differently next time?	What is the area of the following shape?  How do the areas of the shaded sections compare? How do you know?  	The perimeter of a rectangular garden is 120 m. What are the possible measurements (length and width) of this garden? How do they compare? Which is the largest? Smallest? The area of a rectangle is 24 cm^2. What are the possible measurements of the dimensions of this rectangle? How do they compare? Which is the largest? Smallest?	A rectangular prism has a volume of 192 cm^3. The prism is 16 cm high. What is the area of its base? What are possible dimensions of the area of its base? The top of your desk measures 68 cm long and 50 cm wide. What is the area of your desk? If you are working on a puzzle that measures 2500 cm^2 when complete, will it fit? What are the dimensions of the puzzle?
Analysis	What are the missing side lengths if the perimeter is 26 cm?  The figure below is made up of two squares. What is the perimeter of the composite shape?  	You cut a square piece of paper measuring 12 cm in length into 2 square pieces and 2 rectangular pieces as shown in figure 1. You arrange the pieces to form a large rectangle as shown in figure 2. What is the perimeter of the large rectangle in figure 2?  The area of a rectangular carpet is 18 m^2. What is the largest perimeter of the rectangular carpet?	Mr. MacDonald wants to construct an enclosed rectangular kennel for his dog. The perimeter of the kennel is 20 m. One side of the kennel will be the wall of the house. The kennel must have the biggest possible area. What is the area of the kennel? The perimeter of a rectangular wall is 12 m. Its width is 2 m. What is the area of the wall?	A teen mowed two lawns. One lawn was 10 m × 12 m, and the other was 15 m × 10 m. The teen charges $3 for each 10 m^2. How much did she charge to mow the two lawns? The figure below is made up of three squares. What is the area of the composite shape? 

			A rectangle is made up of 20 square unit tiles.  is Which perimeter is **not** possible? a) 42 units b) 24 units c) 20 units d) 18 units	A rectangle with a length of 6 cm and a width of 3 cm is shown below.  If you double the length and the width, what happens to the area?

Supporting Resources

Manipulatives and Models to Support Learning

Geoboards	Square tiles	Pattern Blocks	Tangrams	Grid paper	Base-ten blocks
					

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Identifying and Describing the Attributes of Objects and Shapes

Alignment to previous Outcomes		Related Outcome	Alignment to upcoming Outcomes
3G01: Students will be expected to describe 3-D objects according to the shape of the faces and the number of edges and vertices. **3G02:** Students will be expected to name, describe, compare, create, and sort regular and irregular polygons, including triangles, quadrilaterals, pentagons, hexagons, and octagons according to the number of sides.	**4G01:** Students will be expected to describe and construct rectangular and triangular prisms.	**5G01:** Students will be expected to describe and provide examples of edges and faces of 3-D objects, and sides of 2-D shapes that are parallel, intersecting, perpendicular, vertical, and horizontal.	**6G01:** Students will be expected to construct and compare triangles, including scalene, isosceles, equilateral, right, obtuse, or acute in different orientations.

What conclusions can be drawn from the NSA: Mathematics in Grade 6?

Students had difficulty when asked to draw upon their previous knowledge of shapes to assist them in their identification and descriptions of prisms and pyramids. Students require more experiences to identify and name common attributes between prisms such as parallel and perpendicular sides. Likewise, know what attributes make pyramids different from prisms. In doing so, be able to sort a given set of objects. Students need to be able to identify examples of shapes and objects in their environment. While students are more successful with knowledge type questions, they do require more exposure and experience with varied types of cognitive levels of questioning (knowledge, application, and analysis) to apply their higher order thinking skills when working with all geometry concepts.

Why is this an area of need and how can we support students?

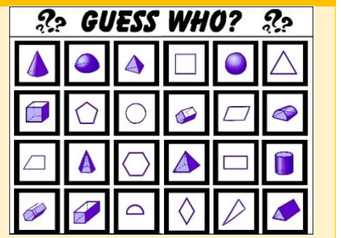
Aligning Vocabulary with Attributes	
Misconceptions/Errors in Student Work	**Possible Next Steps in the Classroom**
A common error or misconception that students encountered was identifying parallel and perpendicular faces of given objects. Students seem to be unfamiliar with the meaning of the vocabulary related to parallel and perpendicular and potentially experience working with different types of objects. This has resulted in students misidentifying objects that have perpendicular faces or parallel faces. For example, indicating that a pyramid has perpendicular and/or parallel faces or that a cylinder does not have parallel faces. In addition, a common error is relating the shapes they see in 2D like circles, rectangles, triangles, and squares to that of a prism rather than other objects like a cylinder or pyramids. This may be because students have not had enough experience working with different objects and talking about the various attributes of objects to be able to internalize what it looks like from different perspectives. This is evident when students associate faces with a shape rather than an object. For example, an object with rectangular faces is identified as a rectangle rather than a rectangular prism. ***Please note that while students investigate attributes of polygons, such as side lengths and vertices in P – 5, they will build on these experiences in Mathematics 6 as angles and other properties are investigated in greater depth. Students should continue to deepen their understanding and strengthen their skills in identifying, describing and sorting as they explore triangles.***	While the language of geometry is important, the teaching of mathematically correct geometric language should be done in the context of physical models rather than as definitions. It is important to note that students' ability to conceptualize shapes and objects develops through different stages, and that this development is fostered through many opportunities to visualize and analyse various shapes and objects. **Shapes /Polygons** Students should be provided with opportunities to draw upon their previous knowledge of shapes/polygons to assist them developing more detailed ways to describe objects. This can involve students comparing the number of sides as the key attribute for classifying polygons. Students should be able to name the specific polygons—triangle, quadrilateral, pentagon, hexagon, and octagon. Provide students with various sizes of a polygon. Have students count the number of sides and identify the polygon. Having a variety of these experiences with different polygons, students should begin to realize that a polygon, regardless of its dimensions, remains the same shape. Use geoboards to create irregular polygons. Students should start to generalize the characteristics that distinguish between various polygons. **Objects** There is a developmental sequence associated with how students think and reason geometrically. As levels of geometric thinking develop, students will notice more attributes of objects. Opportunities where students can investigate the faces and edges of objects will help students to construct their own understanding and make connections that make prisms and pyramids. Another way to have students explore the edges and faces of objects is to have them work in small groups construct various shapes and put them together to form prisms and pyramids. This can be done by stacking pattern blocks or using materials that lend themselves to skeletal models. Challenge students to build shapes with a certain number of edges, faces, and vertices. Determine what objects are possible or not possible. After students have constructed the objects, discuss the following questions: • Which solid has the most parallel faces? • Which solid has the least number of edges? • Which solids has only two parallel faces? • Which solids have eight intersecting edges? • Which solid has four sets of parallel faces?

Sample Activities for Lesson Planning

Lead a discussion that will have students describe shapes and/or objects using grade appropriate vocabulary and shapes/objects.
Then, have students work in pairs.

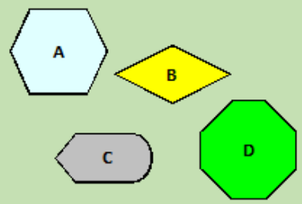
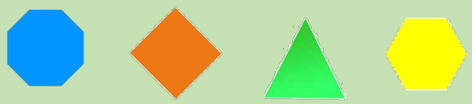
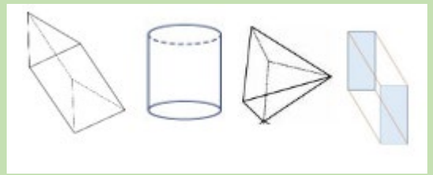
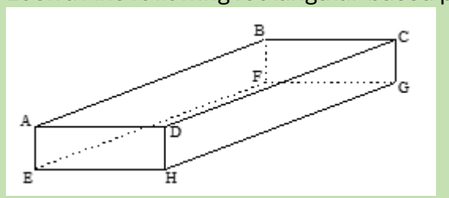
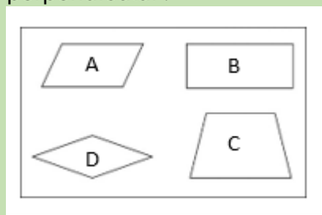
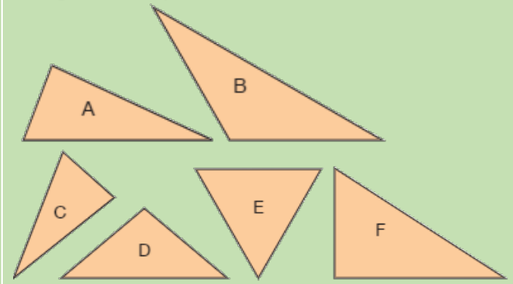
Without showing it to their partner, one student chooses a geometric shape/solid and describes it according to its attributes. The second student then tries to identify the shape/solid. Once the shape/solid is identified, students switch roles.

For grade 6 regarding triangles, discuss the vocabulary after students have investigated side lengths and angles using a sorting exercise.



Grade 3	Grade 4	Grade 5	Grade 6
Knowledge: Provide images of a series of regular and irregular shapes and/or objects from the real world. Have students identify the shapes and/or objects and list the common attributes. Application: Create/Draw an image using the following shapes: quadrilaterals, triangles, and circles. The image must also include at least three shapes that have 5 or more sides. OR Sort a series of objects. Have a partner identify the sorting rule. Switch. Have a partner sort a series of objects and you need to identify the sorting rule. What is similar / different about each sorting rule? What objects were included together or apart in both and why? Analysis: Build a tower that includes 9 vertices, 16 edges and 9 faces. What shapes and objects have you used for the tower? How would the tower change if you had 10 vertices? What shapes and objects would be used for this tower?	Knowledge: Provide images of a series of triangular and rectangular prisms from the real world. Have students identify the objects and list the common attributes. Application: Sort a series of objects that include triangular and rectangular prisms. Have a partner identify the sorting rule. Switch. Have a partner sort a series of objects and you need to identify the sorting rule. What is similar / different about each sorting rule? What objects were included together or apart in both and why? Analysis: Build a tower that includes at least one triangular prism and one rectangular prism. What shapes have you used for the tower? How would the tower change if you had more rectangles and triangles or vice versa? What shapes and objects would be used for this tower? Create your own tower and compare it to your partner's tower.	Knowledge: Provide images of a series of regular and irregular shapes and/or objects from the real world. Have students identify the shapes and/or objects and list the common attributes. Application: Sort a series of objects based on the edges and faces in terms of parallel, intersecting, perpendicular, vertical, and horizontal. Have a partner identify the sorting rule. Switch. Have a partner sort a series of objects and you need to identify the sorting rule. What is similar / different about each sorting rule? What objects were included together or apart in both and why? Analysis: Build a tower that includes 3 objects. One object has parallel faces, another has perpendicular edges and the third only has triangular faces. What objects and shapes have you used for the tower? How would the tower change if you built the tower with objects that had different faces or edges? What shapes and objects would be used for this tower?	Knowledge: Provide images of a series of triangles either drawn on a piece of paper or of images from the real world. Have students sort the triangles first according to the length of the sides and explain their sorting rule. Repeat having them sort the triangles according to the measures of the angles and explain their sorting rule. What do you notice about your sorting? Are the triangles sorted in the same way? Application: Draw an image using the following shapes: quadrilaterals, triangles, and circles. The following triangles must be used: scalene, isosceles, equilateral, right, obtuse, or acute. Analysis: Create a Mandala art on a piece of paper. The Mandala art must include at least one of each triangle: scalene, isosceles, equilateral, right, obtuse, and acute in different orientations. You must also include up to 4 different shapes that have sides that are parallel and/or perpendicular. The final design must have at least two lines of symmetry. How does your design differ from a partner? What decisions did you have to make with your design to meet all of the criteria?

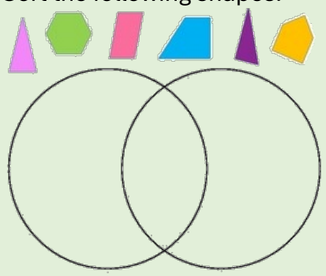
What are some sample questions to help support assessment?

Cognitive Level	Grade 3	Grade 4	Grade 5	Grade 6
Knowledge	Which figure is **not** a polygon?  Which shape is a quadrilateral? Pentagon? Hexagon?  What faces do you have on a cylinder, a cone, and a cube? What shapes do you see in this image? 	Name the prisms that best represents the following items: - Book - Chocolate bar - Number cubes - Roof - Slice of cake - Playing card - Tent Circle all the rectangular prisms.  	Which of these objects **does not** have parallel faces?  Look at the following rectangular-based prism.  Identify the parallel and perpendicular faces and edges. Which quadrilateral has two sides that are perpendicular?  	Identify the following triangles as acute, obtuse, or right and equilateral, isosceles or scalene.  What are the different ways you can sort triangles? What makes triangles congruent?

Application

What makes a cone different from a cylinder and different from a cube?

Sort the following shapes.



What is your sorting rule?

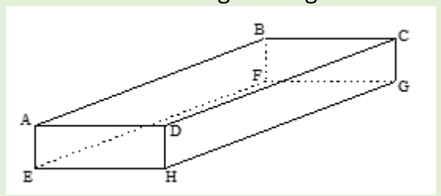
Sort the following objects in two different ways. Which objects have you included in the same groups both times and which ones are different? What makes both sorting rules the same or different?

What makes triangular prisms different from rectangular prisms? Can a triangular prism be a pyramid?

Sort the following objects in two different ways (below are examples of real-life objects that can be used for triangular and rectangular prisms). Which objects have you included in the same groups both times and which ones are different? What makes both sorting rules the same or different?

Construct the net of triangular prism and a rectangular prism. What makes them the same? Different? Use the correct vocabulary.

Look at the following rectangular-based prism.



Write a statement that is true about the faces and the edges. Write a statement that is false.

Construct a pyramid and a prism using toothpicks and plasticine. Compare the objects and describe the attributes shared by and that differ between both objects. Share your results with a partner. Do your findings differ?

What is the common attribute shared by the following objects?

What is a common attribute shared by the following shapes?

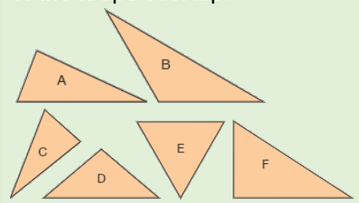
Construct the following triangles:

- an obtuse triangle with an angle of 130°
- a triangle with 3 cm and 4 cm sides that form a right angle
- an equilateral triangle with 10 cm sides
- an obtuse triangle with a 110° angle and one 5 cm side

Could you draw another one of each of the triangles with the same characteristics?

An isosceles triangle has one angle that measures 12°. Find the measures of the unknown angles.

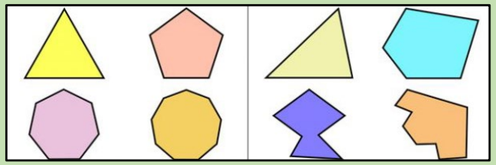
Sort the following triangles using a Venn diagram with 2 loops. Do any of the loops overlap? Sort them again using 3 loops. Do any of the loops overlap?



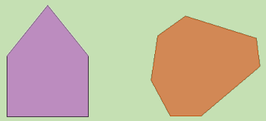
Can an obtuse triangle be an equilateral triangle? Explain.
Can a right triangle be an isosceles triangle? Explain.

Analysis

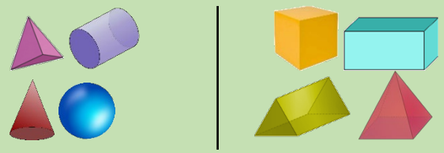
These shapes have been sorted. What is the sorting rule?



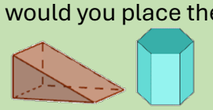
Where would you place the following shapes for sorting.



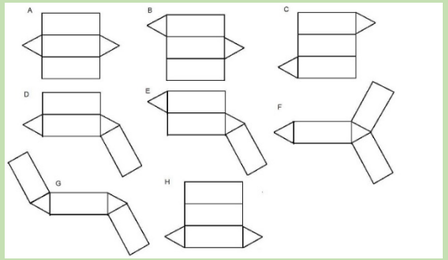
These objects have been sorted. What is the sorting rule?



If you had to add the objects below, where would you place them.

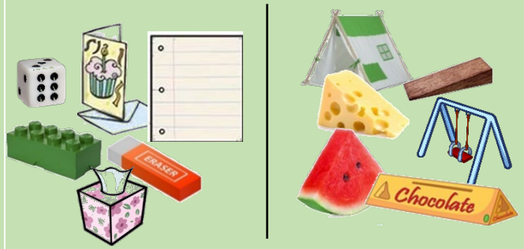


Which of the nets below could you use to create a triangular prism? Justify your selection.



This diagram is part of a net for a rectangular prism. Complete the net.

These objects have been sorted. What is the sorting rule?



If you had to add the objects below, where would you place them? Why?

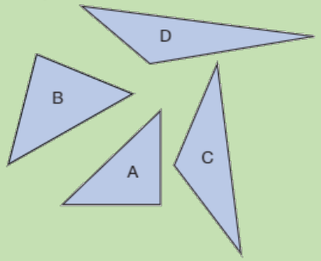
Do all pyramids share the same common attributes? Do all prisms share the same common attributes?

Construct two different objects that have parallel faces and intersecting edges. What other attributes does this object have? Compare the similarities and differences between the objects.

Can a student draw a triangle ABC that has a side length AB of 4.2 cm, angle A = 90° and angle B = 95°? How do you know?

One side of an obtuse triangle is 20 cm. What could be the values of the lengths of the other two sides? What could they not be?

A student drew the following triangles. Can the following conclusion be made: “All triangles must have at least two acute angles.” Why or why not?



Matina was organizing an activity with triangles. She sorted triangles, then placed them in 3 envelopes labelled A, B, and C. Each envelope contains one type of triangle: equilateral, isosceles, or scalene. Which type of triangle is in each envelope.

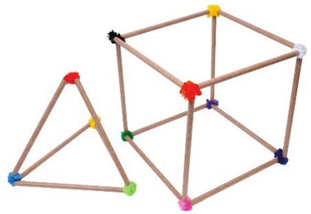
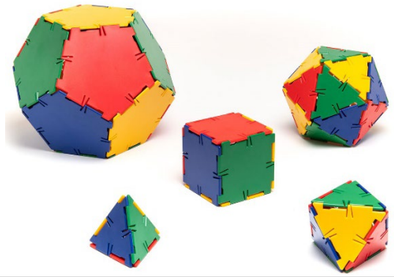
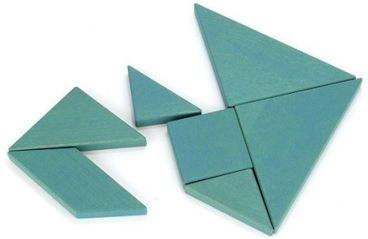
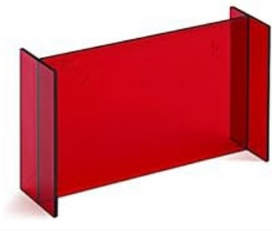
Clues:

- Envelope B does not contain any regular polygons.
- Envelope A has some right triangles.

All the triangles in envelopes A and C have a line of symmetry.

Supporting Resources

Manipulatives and Models to Support Learning

Pattern Blocks 	**Straws and Pipe Cleaners** 	**Objects/Blocks** 	**Polydrons** 	**Tangrams** 
Geoboards 	**Miras** 	**Linking Cubes** 	**Ruler** 	**Measuring Tape** 

Print and Electronic Resources

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