LESSON LEARNED

Focus: solving whole number multiplication and division questions in context

Nova Scotia Assessment: Mathematics Grade 6

“For learners to succeed, teachers must assess students’ individual abilities and characteristics and choose appropriate and effective instructional strategies accordingly.”
– Helene J. Sherman

NOVA SCOTIA

Purpose of this Document

This Lessons Learned document was developed based on an analysis of the Item Description Reports for the Nova Scotia Assessment: Mathematics in Grade 6 (NSA-M6). This document is intended to support all classroom teachers at grades 3 - 6, and administrators at the school, region, and provincial levels. The focus of the document is to help educators work through the process of taking in the information provided by the data analysis and see how it can inform lesson design and assessment in the classroom.

It is suggested that school teams make use of this resource in concert with their school's Item Description Report provided by the Department of Education and Early Childhood Development to all regional centres for education. These reports include student achievement data at the school, regional centre, and provincial level for all questions appearing on the Mathematics in Grade 6 Assessment. By analyzing their own performance on groupings of questions dealing with similar outcomes, schools can identify areas of strength and areas where changes in instruction and/or assessment might be made. This process is designed to foster continued discussions, explorations, and support for mathematics focus at the classroom, school, regional centre, and provincial levels that are all based on valid and reliable data.

This document specifically addresses some of the areas that students across the province found challenging based on provincial assessment data. It is essential that teachers consider assessment evidence from a variety of sources to inform the next steps most appropriate for their students. Effective classroom instruction and assessment strategies are responsive to the individual learners within a classroom.

This document highlights those outcomes where students seem to require additional support. It provides some information about student performance on the assessment in addition to suggested classroom instruction strategies. Sample assessment items are included for each topic explored.

Overview of the Nova Scotia Assessment: Mathematics in Grade 6

Nova Scotia Assessments are large-scale assessments that provide reliable data about how well all students in the province are learning the mathematics curricula. It is different from many standardized tests in that all questions are written by Nova Scotia teachers to align with curriculum outcomes and the results reflect a snapshot of how well students are learning these outcomes. These results can be counted on to provide a good picture of how well students are learning curriculum outcomes within schools, regions and in the province. Since the assessments are based on the Nova Scotia curriculum, and are developed by Nova Scotia teachers, results can be used to determine whether the curriculum, approaches to teaching and allocation of resources are effective. Furthermore, because individual student results are available, these, in conjunction with other classroom assessment evidence, help classroom teachers understand what each student has under control and identify next steps to inform instruction.

The assessment provides information about mathematics for each student and complements assessment data collected in the classroom. This assessment is administered at the beginning of grade 6. It is designed to provide detailed information for every student in the province regarding their progress in achieving a selection of mathematics curriculum outcomes at the end of Grade 5. Information from this assessment can be used by teachers to inform their instruction and next steps in providing support and intervention for their students.

Lessons Learned Overview

Provincial assessments and examinations generate information that teachers can use to help inform classroom instruction and assessment. Following the analysis of each assessment or examination, patterns and trends are identified. These include areas of strength and areas for growth. The Lessons Learned documents specifically highlight concepts where growth is still needed.

There are six areas that have been identified as the areas of focus for this Lessons Learned document.

They are:

- Solving whole number multiplication and division questions in context
- Representing decimals
- Relating fractions and decimal
- Generalizing to extend patterns
- Understanding the relationship between area and perimeter
- Identifying and describing the attributes of objects and shapes

This section specifically addresses solving whole number multiplication and division questions in context. It begins with an overview of the student errors and misconceptions identified through the provincial assessment. These include:

- Place Value
- Reasoning to solve story problems

Strategies are then outlined that are designed to enhance student comprehension, drawing from researched best practices. The strategies emphasize the integration of essential models, tools, and interconnections to facilitate the transition between concrete, pictorial, and abstract representations of concepts, highlighted by the importance of deliberate planning and purposeful questioning. To support both assessment and instruction, sample lesson activities are presented alongside a series of cognitive-level questions, providing educators with ideas for addressing knowledge gaps and fostering strategic reasoning and problem-solving skills. Each section culminates with a selection of print and online resources, as well as recommended manipulatives to support professional learning and student understanding of that topic.

Solving Whole Number Multiplication and Division Questions in Context

Alignment to previous Outcomes		Related Outcome	Alignment to upcoming Outcomes
3N11: Students will be expected to demonstrate an understanding of multiplication to 5 × 5. **3N12:** Students will be expected to demonstrate an understanding of division (Limited to division related to multiplication facts up to 5 × 5.).	**4N06:** Students will be expected to demonstrate an understanding of multiplication (one-, two-, or three-digit by one-digit numerals) to solve problems. **4N07:** Students will be expected to demonstrate an understanding of division (one-digit divisor and up to two-digit dividend) to solve problems.	**5N05:** Students will be expected to demonstrate, with and without concrete materials, an understanding of multiplication (two-digit by two-digit) to solve problems. **5N06:** Students will be expected to demonstrate, with and without concrete materials, an understanding of division (three-digit by one-digit) and interpret remainders to solve problems.	**6N08:** Students will be expected to demonstrate an understanding of multiplication and division of decimals (one-digit whole number multipliers and one-digit natural number divisors)

What conclusions can be drawn from the NSA: Mathematics in Grade 6?

Students tend to perform better when explicitly given all the information needed to answer multiplication and division questions. For example, in questions that require students to use basic facts, skills, and symbolic procedures about two thirds of students were successful. And of those students, more were successful with multiplication questions than they were with division questions. When presented with application and analysis problem solving questions, students were not as able to apply higher order thinking skills. For example, roughly one third of the students were successful when answering questions that were presented in the context of a story problem where multiplication and/or division is used as a strategy. Understanding what to do with a remainder in contextual problems is an area of need.

Why is this an area of need and how can we support students?

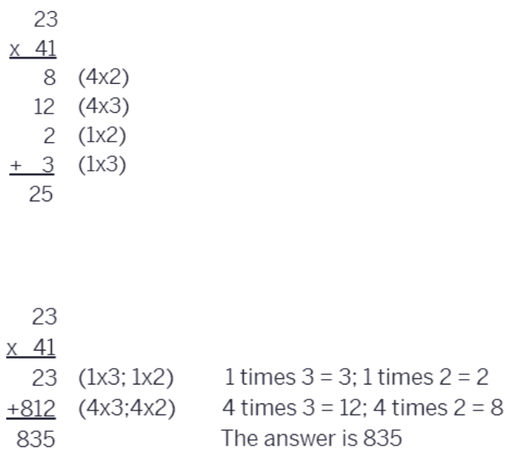
<table>
<tr><th colspan="2">Place Value</th></tr>
<tr><th>Misconceptions/Errors in Student Work</th><th>Possible Next Steps in the Classroom</th></tr>
<tr><td>

Multiplication

When given a multiplication question such as 23 x 41, students tend to read this and say, “4 times 2” when they should be saying, “4 tens times 2 tens” or 40 times 20.

Students focus more so on the digits rather than the place value.

For example, many students learn the method below by naming the digits rather than with the place value.



This results in students selecting the incorrect response when presented with the following question.

Which expression represents 36 x 23?

- (30 x 20) + (6 x 3); multiply only the tens together and the ones together
- (6 x 3) + (3 x 3) + (6 x 2) + (3 x 2); multiply using digits
- (30 x 20) + (30 x 3) + (6 x 20) + (6 x 3); correct answer

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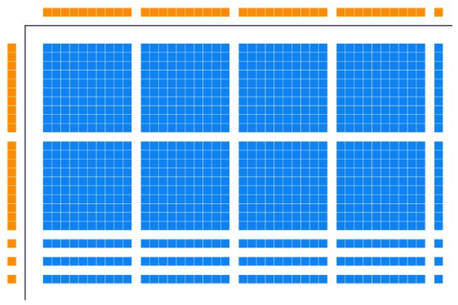
When learning about multiplication and division, focus on strategies and models rather than procedures. While tricks may work, they bypass the thinking behind the concepts. There is also always a way to explain the trick using conceptual understanding.

Estimating will also help in catching possible errors when using various strategies for multiplication and division.

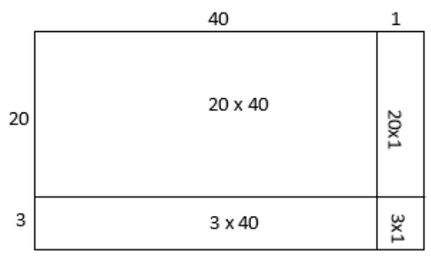
Using Area Models and Partial Product

One strategy to help students understand multidigit multiplication is to focus on place value by having students model the calculation with base-ten materials using an area model or draw out the area model as shown below. Students record partial products and then add them up at the end. When doing so, the teacher should draw attention away from the digits and focus more on the value of the number; use place value language.

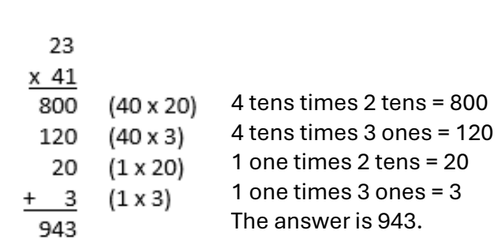
For example, rather than saying or having students say, “multiply the 4 by the 3, and the 4 by the 2,” instead use “multiply the 40 by the 3, and the 40 by the 20.” A focus on place value.



OR



The Area Model



Using Strategies that Promote Number Sense

When students have had limited experiences thinking about the relationships between numbers, it is natural that they resort to traditional algorithms. They also come with varying dispositions to work on multiplication in different ways. It may help to just talk with students about this and keep asking if there could be another way of thinking about the multiplication or how else it could be solved. Many of the properties associated with multiplication can come to life through student generated strategies.

Below are four strategies for multiplication. Some work in all cases, while others are better with composite or even numbers. Students should decide which is more effective and efficient for them.

- Break a factor into two or more addends using the distributive property (23 x 41 is the same as 23 x (40 + 1) or 23 x (10 + 10 + 10 + 10 + 1))
- Factor a factor to support an understanding of the associative property (12 x 16 is the same as 12 x (4 x 2 x 2)
- Round a factor to bring in the distributive and commutative properties (23 x 41 is the same as 23 x 40 plus another 23)
- Halving and Doubling to help make problems simpler to solve (12 x 16 is the same as 24 x 8 or in this case also 6 x 32)

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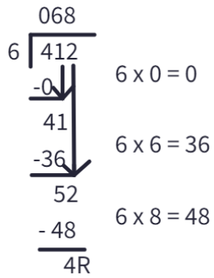
Division

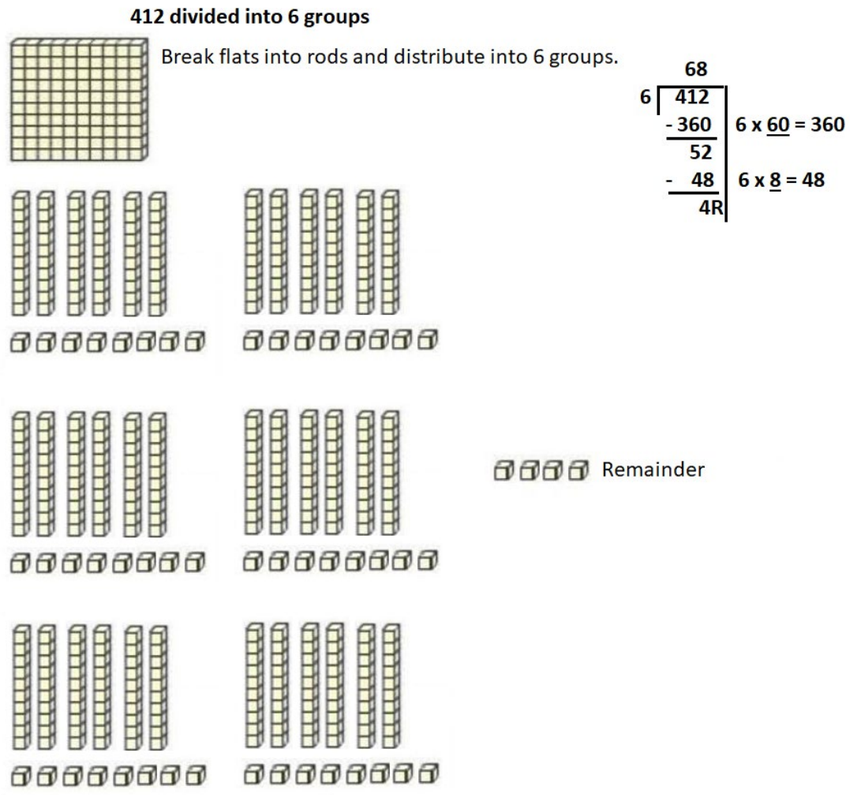
When given a division question such as 412 ÷ 6, students say, “6 into 4 doesn’t work,” they are mathematically incorrect. In the question, it is 400 not 4 that is being divided by 6. In addition, when the procedure below is presented without any context to why it supports solving a division problem, students can get caught up in the steps rather than the concept. Where there might be a remainder, students may not recognize when a remainder is significant or not involved in decision making for problem-solving contexts.

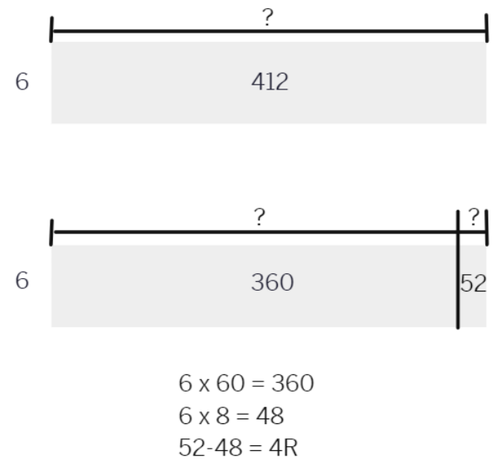
The process below highlights how some students focus on the digits rather than the values of the dividend and the divisor.



By providing students with base-ten materials, it allows them to solve problems and discuss grouping strategies, place value, and the concept of remainder. Teachers can work with students to demonstrate ways of documenting their thinking.

Examples of relating the tools / models and the strategy are highlighted below.





Further Strategies that Promote Number Sense with Division

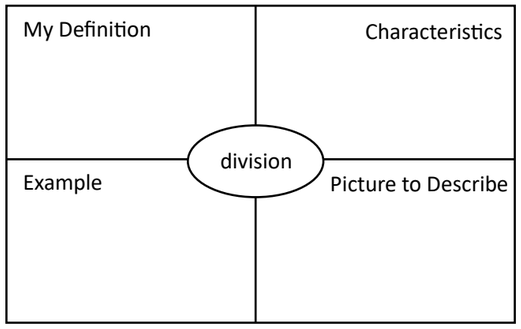
Like multiplication, the way that division is taught can lead students to have a negative disposition of it. There are several strategies that can be used other than the traditional algorithm to help students understand what it means to divide two numbers. In addition, they help to highlight the relationship between multiplication and division.

Below are strategies for division. Students should decide which is more effective and efficient for them.

- Multiply instead (17 ÷ 3 can be related to 3 x 5, plus two more)
- Chunk Out (643 ÷ 3 can be interpreted as 3 x 10 is 30 only 13 away from 43, so another 3 x 4 will get close; and 3 x 100 is 300, so 3 x 200 is 600, leaving 200 + 10 + 4 and a remainder)

Each jar holds 4 litres of liquid. How many jars will 37 L fill? The answer is 9R1. What does the remainder represent? • 1 jar; thinking in terms of whole jars • $\frac{1}{4}$ jar; correct answer – relation to the divisor • $\frac{1}{9}$ jar; relating it back to the quotient • $\frac{1}{37}$ jar; relating it back to the dividend	• Make a Tower of Factors (17 ÷ 3 can be represented as a column of multiples 3 x 1, 3 x 2, 3 x 3 and so on where students decide which multiple of the divisor to subtract each time.) • Halving and Halving to help make problems simpler to solve (102 ÷ 4 is the same as 51 ÷ 2) ***Meaning of the Remainder*** It is also important to help students to understand what the remainder means. Yes, it is the amount left over, but in relation to the question it represents a portion of a divisor. In the example above, the remainder is 4 or $\frac{4}{6}$. For story problems, this means helping students to relate back to the context to understanding the meaning of the remainder. The example on the left highlights the importance of making sense of the question and the result. Using concrete tools or pictures can help students to visualize the problem and solution. **Decimals:** examples of how to set up the models for multiplication and division using decimals can be found in the grade 6 curriculum guide (pp 261-262).

Reasoning to Solve Story Problems	
Misconceptions/Errors in Student Work	**Possible Next Steps in the Classroom**
While not an error or a misconception perse, one can infer that some students may not be checking for reasonableness in their answers. They could be stuck in incorrectly using standard procedures negating what they know about number sense. Students may also not be aware of other, more helpful strategies that support how to work with numbers to solve what is unknown in a story problem. Typically, students use learned strategies to look for numbers and key words in a story problem and use these to create a number sentence. When students consider that a problem is a mathematical problem, they believe, wrongly, that they should associate it simply to routine calculations with few concerns to the meaning of the context and to the credibility of the answers. Some students lack the skills to make inferences, generalize and verify their thinking. Here are a couple of examples of story problems and possible student answers. Each option is carefully constructed based on what is anticipated from student thinking. The first question requires students to determine the total quantity of the juice in mL and then in litres and identify the container whose capacity can contain this quantity of juice. Students can use repeated addition or multiplication to determine an answer. *Larry wants to make fruit juice in a container.* *To make the juice, Larry must follow the directions found on the frozen juice can. He must combine the 525 mL can of frozen juice with three full cans of 525 mL of water.* *Which litre container should Larry choose to hold all the juice?* • *1 L container (only considers the one can of juice, no water)* • *2 L container (considers the total amount of juice and water, but underestimates the size of the container)* • *3 L container (correct answer)*	***Using Mental Math*** Model and have students use mental math strategies and estimate before beginning a question. They will more than likely catch their mistakes. For example, they should know that 20 x 40 is 10 times bigger than 2 x 40. When they estimate, they realize that 80 would be an incorrect answer. This same misconception applies when dividing whole numbers containing dividends with zeros. This is the division of how many groups. Using place value vocabulary and modelling representations will support understanding and reasoning skills. Students should make use of place-value understandings in estimating or calculating and should be encouraged to talk about place-value concepts while explaining reasoning. This can include choosing friendly or benchmark numbers that are easier to work with and are near the given numbers. Further examples can be found in the guides. At the core of estimating is understanding number relationships, benchmark numbers, and compatible numbers. ***Number Talks*** Number talks foster the development of number sense by encouraging students to think flexibly about numbers and mathematical operations. Through engaging in discussions and exploring various strategies, students deepen their understanding of mathematical concepts. Moreover, number talks promote mathematical discourse, as students articulate their reasoning, listen to their peers' strategies, and engage in collaborative problem-solving. Hence, number talks support the use of multiple strategies for problem-solving, reinforcing the idea that there are often multiple paths to a solution. They also facilitate the development of mental math skills by providing opportunities for regular practice. Furthermore, number talks can be adapted to meet the needs of diverse learners, making them a versatile tool for differentiated instruction. They offer valuable insights into students' mathematical understanding, enabling teachers to assess student progress and plan future instruction accordingly. ***Retrieval Practice Strategies*** It is essential that students know their facts and understand how numbers relate to each other. Retrieval practice supports instruction and assessment by strengthening schemas that students have built, helping them commit the information to their long-term memory. Retrieval practice involves daily cumulative review and improves the efficiency of learning. Examples include making and using flashcards, concept maps, or grids, intentional games, choral response, and interleaved practice. All of which facilitate better learning than highlighting or re-reading questions. ***Teaching through Problem-Solving*** Provide guidance, not direct instruction on problem-solving strategies as students share their own solutions and findings. Focus on strategies and not procedures. Use students' methods to guide instruction. Be intentional when selecting student methods to share. Elaborate on the methods used by students to solve and justify and encourage students to comment and ask questions of their peers. Encourage students to think about the context of the question as a whole and show their thinking using a strategy of choice. Concepts and skills should be connected to everyday situations and other curricular areas. Encourage students to make connections to make mathematics come alive through math-to-world, math-to-math, and math-to-self connections. Develop students' mathematical vocabulary, initiate effective ways to navigate informational text, and encourage students to reflect on what they have learned.

In the second question, students need to demonstrate an understanding of division with whole numbers or use multiplication to support thinking. *You have 54 marbles.* *You put the marbles into 6 bags.* *There is the same number of marbles in each bag.* *How many marbles do 2 bags contain?* • *108 (2 bags with 54 marbles in each – multiplication)* • *18 (correct answer)* • *12 (student multiplies 6 bags by 2 bags)* • *11 (correct number of marbles in one bag added to 2 bags)*	Embed strategies/tools such as the Frayer Model (see below), Concept Circles, and Exit Cards to assess student learning.  ***Modelling with Think Alouds*** Model problem solving with students by sharing aloud their thinking as they read to understand a problem. Through the process, students learn how to verbalize to understand a question and to make sense of what is being asked and what is in their toolbox to answer the question. It is important that teachers model this process to encourage the thinking process, using various strategies in our toolbox and checking for reasonableness.

Activities to Support Lesson Planning

Number Talks and Number Strings

Use number talk routines to develop efficiency, flexibility, and accuracy with computations. They help to elicit specific strategies that focus on number relations and number theory rather than a series of steps found in traditional algorithms. Classroom conversations and discussions around purposefully crafted computation problems are at the very core of number talks.

A few examples are outlined below, while further examples of strategies are outlined in the curriculum guides and in the resources section.

Begin with examples that require less cognitive load to support the development of effective strategies. These can include activities that focus on number relationships, compatible numbers, and benchmark numbers. Encourage flexibility and talk about estimation. Each example helps to guide students in understanding the importance of place value.

Grade 3	Grade 4	Grade 5	Grade 6
Goal – Using partial products/quotients to support understanding of expanded notation and place value. Select either the list for multiplication or the list for division. Show each number sentence one at a time asking students the guiding questions in between. 2 x 2 8 ÷ 4 2 x 4 12 ÷ 4 4 x 2 20 ÷ 4 4 x 4 **Question Prompts:** Knowledge: Solve the number sentence. Explain your strategy or state your strategy. Application: How are these strategies related? How does it help you solve when using larger values? Analysis: Create your own number sentence that relates to these number sentences and would use the same or similar strategy. **Note:** Some grades 4 and 5 students may start here to support readiness.	Goal – Using partial products/quotients to support understanding of expanded notation and place value. Select either the list for multiplication or the list for division. Show each number sentence one at a time asking students the guiding questions in between. 2 x 45 40 ÷ 4 5 x 100 80 ÷ 4 5 x 40 16 ÷ 4 5 x 5 96 ÷ 4 5 x 245 **Question Prompts:** Knowledge: Solve the number sentence. Explain your strategy or state your strategy. Application: How are these strategies related? How does it help you solve when using larger values? Analysis: Create your own number sentence that relates to these number sentences and would use the same or similar strategy. **Note:** Some grade 5 students may start here to support readiness.	Goal – Using partial products/quotients to support understanding of expanded notation and place value. Select either the list for multiplication or the list for division. Show each number sentence one at a time asking students the guiding questions in between. 35 x 10 400 ÷ 4 35 x 2 80 ÷ 4 35 x 20 16 ÷ 4 35 x 24 496 ÷ 4 **Question prompts:** Knowledge: Solve the number sentence. Explain your strategy or state your strategy. Application: How are these strategies related? How does it help you solve when using larger values? Analysis: Create your own number sentence that relates to these number sentences and would use the same or similar strategy. **Note:** Some grade 6 students may start here to support readiness.	Goal – Using products/quotients to support understanding of expanded notation and place value. Select either the list for multiplication or the list for division. Show each number sentence one at a time asking students the guiding questions in between. 3 x 10 40 ÷ 4 3 x 1 4 ÷ 4 3 x 0.1 0.4 ÷ 4 3 x 0.01 0.04 ÷ 4 **Question prompts:** Knowledge: Solve the number sentence. Explain your strategy or state your strategy. Application: How are these strategies related? Will the value be larger or smaller than the previous? What do you notice about numbers? Why are the answers the way they are? Analysis: Create your own number sentence that relates to these number sentences and would use the same or similar strategy.

Open Questions and Games

Open questions can include supporting the understanding value and digit placement. Use questions that encourage the use of tools and different representations. An open task can be as simple as asking students to model multiplication and division sentences with base-ten blocks, counters, or a number line. They can also be questions where students explore numberless word problems that help them focus on the context and relationship between the quantities, rather than just apply operations. Math manipulatives and models encourage different strategies to build number sense and reinforce skills, while engaging students in the learning process. Remember to ask them to explain their models. How does it relate to the question being asked?

Games can also offer ways for students to apply and demonstrate operational sense in context and allow for multiple entry points. Cross-strand opportunities also help to reinforce and help students to see mathematics as holistic and not siloed into its strands. They can also be used across multiple grades and abilities. Games that include number cubes and cards also encourage students to determine count and quantity, and sums, differences, and products of numbers in a fun and interactive way.

The following activities can be set up to support a problem-solving lesson. They can be used to model a think aloud and engage in conversation with students. All values are based on grade level outcomes but can be modified to accommodate student need and learning progression. Grade 6 students for example can focus on whole numbers first then transition into decimal numbers.

Grade 3	Grade 4	Grade 5	Grade 6
Ask students to determine which digits should replace each letter. The same digit is used each time the letter appears. A x B = C and/or B x D = EA	Ask students to determine which digits should replace each letter. The same digit is used each time the letter appears. A x BC = DAE	Ask students to determine which digits should replace each letter. The same digit is used each time the letter appears. A x BCB = DAEB	Ask students to determine which digits should replace each letter. The same digit is used each time the letter appears. A x B.C = DA.E
Ask students to place the digits 1, 3, and 5 to create the greatest and least quotients. __ __ ÷ __	Ask students to place the digits 1, 3, 5, and 7 to create the greatest / least quotients. __ __ __ ÷ __	Ask students to place the digits 1, 3, 5, and 7 to create the greatest / least quotients. __ __ __ ÷ __	Ask students to place the digits 1, 2, and 6 to create the greatest and least quotients. __.__ ÷ __
Missing Values: In this activity, one of the numbers on the multiplication chart is missing. For example, ask students to come up with a way to multiply 3 x 5 if the 5 is missing on the chart. This requires students to use their number sense.	**Missing Values:** In this activity, one of the numbers on the multiplication chart is missing. For example, ask students to come up with a way to multiply 4 x 79 if the 9 is missing on the chart. This requires students to use their number sense.	**Broken calculator:** In this activity, one of the number keys is broken. For example, ask students to come up with a way to multiply 7 x 59 if the 9 key on the calculator is not working. This requires students to use their number sense.	**Broken calculator:** In this activity, one of the number keys is broken. For example, ask students to come up with a way to multiply 4 x 0.25 if the 4 key on the calculator is not working. This requires students to use their number sense.
Further examples using leveled questions: **Multiplication** Knowledge: What two numbers can be multiplied to get 24? Application: The boxes at the end of each row and the bottom of each column give the result of multiplying the two numbers in that row and column. What are the missing values?	**Further examples using leveled questions:** **Multiplication** Knowledge: What two numbers can be multiplied to get 108? Application: The boxes at the end of each row and the bottom of each column give the result of multiplying the two numbers in that row and column. What are the missing values?	**Further examples using leveled questions:** **Multiplication** Knowledge: What three numbers can be multiplied to get a value of 108? Application: The boxes at the end of each row and the bottom of each column give the result of multiplying the three numbers in that row or column. What are the missing values?	**Further examples using leveled questions:** **Multiplication** Knowledge: What two numbers can be multiplied to get a value of 0.75? Application: The boxes at the end of each row and the bottom of each column give the result of multiplying the three numbers in that row or column. What are the missing values?

		25
		12
15	20	

Analysis: Create your own square. Share with a partner.

Division
This challenge is about dividing a two-digit number (up to 25) by a single-digit number. Have students decide on which number they are going to be dividing by. This is the divisor. The challenge is going to be to relate division to multiplication through repeated subtraction or determining equal grouping for this divisor. Now students generate a two-digit number. This is your dividend. You could use the spinners or number cubes to generate the digits or use their imagination.

Knowledge: Have students divide their dividend by their divisor. Record the quotient. Create other dividends and divide them by the same divisor. Record the quotients. Students can use counters to support equal grouping.

Application: Look carefully at the quotients. When do you have equal groups? What types of numbers can you not have equal groups?

Analysis: Can you spot any patterns? Can you come up with any 'rules' to help you in solving for division questions?

		45
		96
72	60	

Analysis: Create your own square. Share with a partner.

Division
This challenge is about dividing a two-digit number by a single-digit number. Have students decide on which number they are going to be dividing by. This is the divisor. The challenge is going to be to come up with some rules (e.g. what happens when you divide by 1) for this divisor. Now generate a two-digit number. This is your dividend. You could use the spinners or number cubes to generate the digits or use their imagination.

Knowledge: Have students divide their dividend by their divisor. Record the quotient. Create other dividends and divide them by the same divisor. Record the quotients. Students can use base-ten materials of other manipulatives to support the process.

Application: Look carefully at the quotients. When is the quotient a whole number? When is there a remainder? What is the remainder?

Analysis: Can you spot any patterns? Can you come up with any 'rules' to help you in solving for division questions?

			15
			108
			224
144	8	315	

Analysis: Create your own square. Share with a partner.

Division
This challenge is about dividing a three-digit number by a single-digit number. Have students decide on which number they are going to be dividing by. This is the divisor. The challenge is going to be to come up with some rules (e.g. what happens when you divide by 0) for this divisor. Now generate a three-digit number. This is your dividend. You could use the spinners or number cubes to generate the digits or use their imagination.

Knowledge: Have students divide their dividend by their divisor. Record the quotient. Create other dividends and divide them by the same divisor. Record the quotient. Students can use base-ten materials of other manipulatives to support the process.

Application: Look carefully at the quotients. When is the quotient a whole number? When is there a remainder? What is the remainder?

Analysis: Can you spot any patterns? Can you come up with any 'rules' to help you in solving for division questions?

		1.6
		1.6
3.2	0.8	

Analysis: Create your own square. Share with a partner.

Division
This challenge is about dividing a decimal number by a single-digit whole number. Have students decide on which number they are going to be dividing by. This is the divisor. The challenge is going to be to come up with some rules (e.g. even/odd, relation to whole numbers) for this divisor. Now generate a decimal number. This is your dividend. You could use the spinners or number cubes to generate the digits or use their imagination.

Knowledge: Have students divide their dividend by their divisor. Record the quotient. Create other dividends and divide them by the same divisor. Record the quotient. Students can use base-ten materials of other manipulatives to support the process.

Application: Look carefully at the quotients. What happens with the decimal place? When is there a remainder? What is the remainder? What is happening with the place value?

Analysis: Can you spot any patterns? Can you come up with any 'rules' to help you in solving for division questions using decimals?

What are some sample questions to help support assessment?

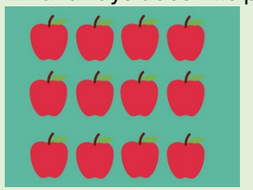
Cognitive Level	Grade 3	Grade 4	Grade 5	Grade 6
Knowledge	Rewrite the following number sentences using multiplication or division. 8 - 2 - 2 - 2 - 2 = 0 12 - 3 - 3 - 3 - 3 = 0 2 + 2 + 2 + 2 = 8 3 + 3 + 3 + 3 = 12 What number is four times larger than three? What number is three times smaller than fifteen?	Calculate: 8 x 365 = ___ 68 ÷ 3 = ___ 123 x 9 = ___ 96 ÷ 4 = ___ Can 3 people share 18 marbles fairly? How about 60? Describe the solution procedure for determining the product of a single digit and a 3-digit number. What numbers can be multiplied to give you a product of 24? What numbers are divisible by 4 and 5?	Calculate: 36 x 42 = ___ 123 ÷ 6 = ___ 18 x 9 = ___ 645 ÷ 5 = ___ Describe the solution procedure for determining the product of two 2-digt numbers. What number is double 8? What number is divisible by 18 and 3? Can 3 people share 183 marbles fairly?	Calculate: 5 x 0.27 = ___ 6.05 ÷ 5 = ___ 3 x 5.67 = ___ 14.50 ÷ 8 = ___ Estimate the following: 4 x 57.9 82.2 ÷ 9 1.62 x 5 1.16 ÷ 6 What happens when you divide by 0 or 1? Describe a solution procedure for determining the product of a whole number and a decimal. Describe a solution procedure for 18 x 5 using mental math.

Application

Write a multiplication and/or division sentence to describe this picture.

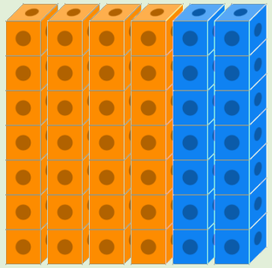
*Adapted from *Eyes on Math* M. Small g3-5

What arrays does this picture represent?

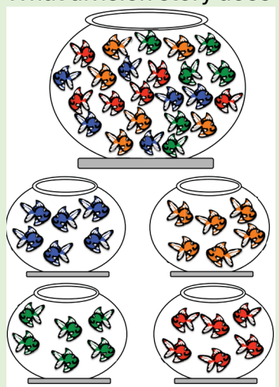


Three friends want to share 15 apples. How many apples will each friend get? Show how you solved it.

What multiplication or division (or both) does this picture represent?



What division story does this image show?



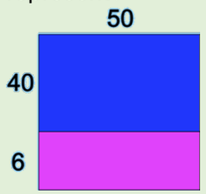
*Adapted from Eyes on Math M. Small g3-5

Dani has 145 cards.
Dani keeps 5 cards and shares the rest with three friends Arden, Tatum, and Onyx.
Arden receives 80 cards.
Tatum receives double the number of cards that are given to Onyx.
How many cards does Onyx get?

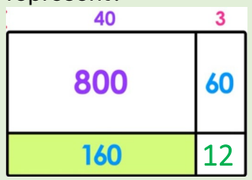
JJ has $400 to buy 8 games. Each game costs $37. Without calculating, does JJ have enough money? How do you know?

To multiply 12 X 5, Chris thinks “6 X 10”. Explain his thinking.

What multiplication sentence does this image represent?



What division sentence does this image represent?



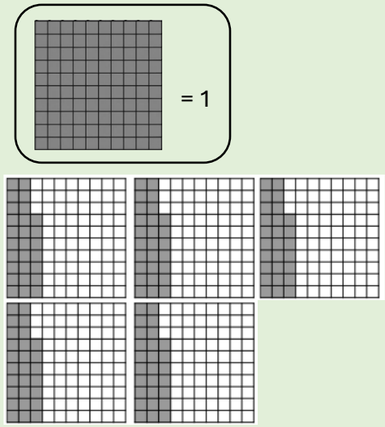
Pens come in packages of 3, 5, and 8. Mrs. O’Regan bought 26 pens for her class. How many packages of each type might she have bought?

The local arena is hosting an event. It can hold up to a maximum of 800 people. If 86 people can fit on a bus, and the organizers are expecting 9 full buses, will there be enough room in the arena for everyone? Why or why not?

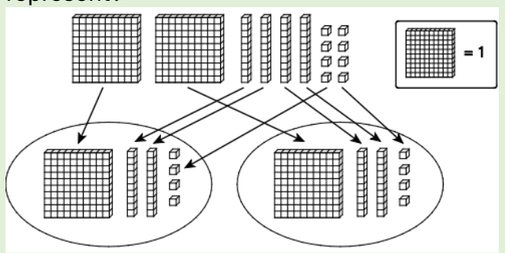
The soccer team collected 680 recyclable containers during the recycling drive. That was almost 3 times more than the hockey team collected. About how many recyclables did the hockey team collect?

To multiply 12 X 15, Chris thinks “6 X 30”. Explain this thinking.

What multiplication sentence does this image represent?



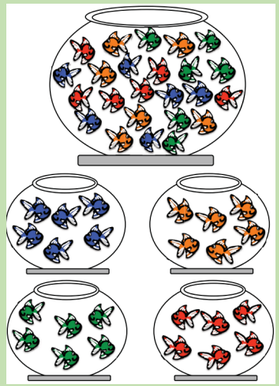
What division sentence does this image represent?



You can buy a 2 kg bag of apples for $8.46 or apples of your choice for $4.38 for every kilogram. Which option will you choose if you need 4 kg of apples?

The length of a killer whale on average is 9.3 m. The length of a cow is about 3.1 m. How many times bigger is the whale compared to the cow?

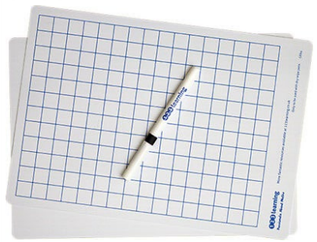
To multiply 12 X 1.5, Chris thinks “6 X 3”. Explain this thinking.

Analysis	You are planning a game day party. You decide to prepare mini sandwiches and fresh lemonade for you and your 4 friends. If you make 3 mini sandwiches each for everyone, how many sandwiches do you make? If you need 2 lemons for every cup of lemonade and you only have 8 lemons, how many cups of lemonade can you make? What numbers can be used to multiply to get the value of 12? 24? Justify your answer.	Suppose there are four more fish added to this picture. Would it still show a division story? How?  *Eyes on Math M. Small g3-5 Sophie builds a tower with green and blue blocks. There are 9 levels in the tower. If less than half of the blocks are green, how many could be green blocks and how many could be blue blocks? How do you know? Create a story problem and solve it using the values of 6 and 9. How many people could share 18 marbles fairly?	You decide to set up a stand over the summer on weekends to sell lemonade and giant cookies. You sell the lemonade for $1 a glass and the cookies for $2 each. After one day you make $62 and on the second day you make $56. If you made 4 dozen cookies and have none left, how many glasses of lemonade did you sell? Name a three-digit number where the last two digits is a number divisible by 4. Repeat for three more numbers. What do you notice? You divide a 3-digit number by a 1-digit number and get a whole number quotient. How many digits can the quotient have? You multiply two numbers, and the product is close to 2600. Both numbers are greater than 10. What could they be? How many people could share 60 marbles fairly?	You decide to set up a stand over the summer on weekends to sell lemonade and giant cookies. You sell the lemonade for $1.25 a glass and the cookies for $1.75 each. After one day you make $62.50 and on the second day you make $56.25. If you made 4 dozen cookies and have none left, how many glasses of lemonade did you sell? List lots of numbers greater than 50 that are multiples of 3. Add the digits of each number. What do you notice? Does the same thing happen for multiples of numbers other than 3? You multiply two numbers, and the product is close to 24.55. What could they be? You divide two numbers, and the quotient is smaller than 1. What could they be? Create a story problem and solve it using the values 3.46 and 7.

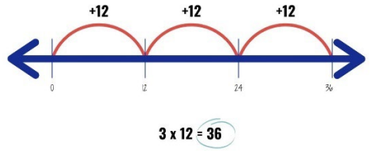
Supporting Resources

Manipulatives and Models to Support Learning

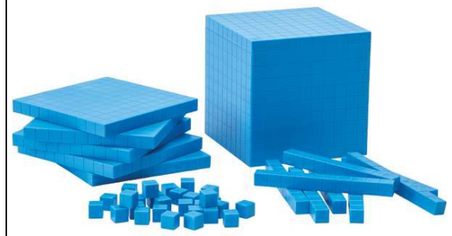
Grids



Open Number Line



Base-ten blocks

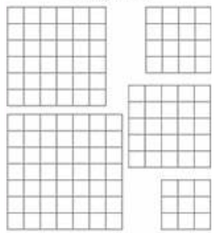


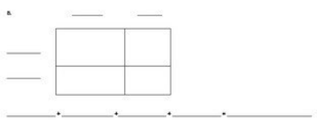
Place Value Chart

Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths

Play money

Arrays or Area Models





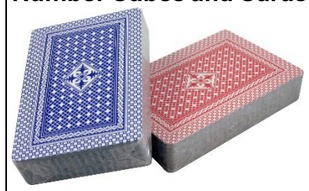
Hundreds Charts

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Multiplication Charts

X	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

Number Cubes and Cards



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