Nova Scotia Examinations
Advanced Mathematics 12

Web Sample 3

NOVA SCOTIA
Education
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General Instructions to Students

This examination is composed of two sections with the following suggested time allotment:

Selected Response (Multiple Choice) Questions	Value 35 pts	(approx. 1 hour)
Constructed Response Questions	Value 67 pts	(approx. 2 hours)

Total time: 3 hours

Use these suggested times to guide you in the completion of the examination; however, you may not find it necessary to spend the suggested time on each section. Plan your time to enable you to complete the examination.

Your teacher must clear the memory of your own graphing calculator prior to this examination. The only graphing calculators permitted are: TI-82, TI-83, TI-83 Plus, TI-84, or TI-84 Plus.

When a question indicates that you may not use a graphing calculator, you are not permitted to use the graphing or regression functions, but you are still permitted to use the calculator to perform arithmetic operations.

Calculators are not to be shared.

Graph paper, scrap paper, and formula sheets are provided in a separate booklet. Please place this booklet along with your student response sheet inside the front cover of this examination booklet before handing it in.

Note: Diagrams are not necessarily drawn to scale.

Selected Response Questions
(Total Value: 35 points)

In this part of the examination, there are 35 selected response questions, each with a value of 1 point. Read each question carefully, and decide which one of the responses best answers the question being asked.

You are provided a separate student answer sheet. In the selected response section of the student answer sheet, fill in the bubble that corresponds to your choice, as shown in the example below. Use an HB pencil only.

Example:

1. What are the roots of $x^2 + 3x - 4 = 0$?

   A. 4 and 1 B. –4 and 3
   C. –4 and 1 D. 4 and 3

(On student answer sheet)

1. **C** (A) (B) (●) (D)

If you wish to change an answer, please ensure that you erase your first answer completely on the student answer sheet. Calculations or rough work on the selected response pages of the examination booklet will not be scored.

1. A quadratic function has a *y*-intercept at (0, –11) and a vertex at (4, –3), which one of the following is the transformational form of the function?

A. $-\frac{1}{2}(y+3)=(x-4)^2$

B. $-2(y+3)=(x-4)^2$

C. $-\frac{1}{2}(y-3)=(x+4)^2$

D. $-2(y-3)=(x+4)^2$

2. Which of the following best describes the discriminant if a quadratic equation has two equal roots?

A. The discriminant is an irrational number.

B. The discriminant is equal to zero.

C. The discriminant is a positive number.

D. The discriminant is a negative number.

3. If the x-intercepts of a quadratic function are (–3, 0) and (7, 0), then which of the following could be its vertex?

A. (10, 4)

B. (4, 10)

C. (2, 0)

D. (2, –25)

4. A golf ball is hit into the air. The height, h, of the path of the golf ball, in meters, as a function of time, t, in seconds, is described by the function $h=-5t^2+30t$. What is the total length of time the ball is in the air?

A. 3 s

B. 5 s

C. 6 s

D. 30 s

5. If the roots of $ax^2+bx+c=0$ are $x=\frac{-1\pm i\sqrt{19}}{10}$, then what is true of the graph of the function $y=ax^2+bx+c$?

A. it has no *x*-intercept

B. it has no *y*-intercept

C. it is not parabolic

D. it has 2 distinct *x*-intercepts

6. Celeste changed the function $\frac{1}{2}(y+3)=(x-2)^2$ from transformational form to general form. What should she have obtained?

A. $y=2x^2-8x+5$

B. $y=2x^2+5$

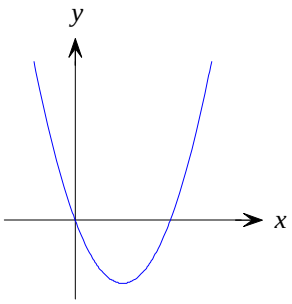
C. $y=2x^2+8x+11$

D. $y=4x^2-8x+11$

7. What is the minimum value of the function $-\frac{1}{3}(y+21)=(x-4)^2$?

A. 21
B. –21
C. 4
D. –4

8. A function $y=ax^2+bx+c$ is represented by the graph below.



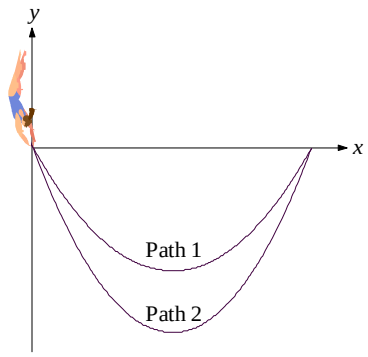
Which of the following statements is **true**?

A. $a<0$
B. $a=0$
C. $a>0$
D. a is an imaginary number

9. What function has $x=-\frac{q}{4p}$ as its axis of symmetry?

A. $y=4px^2-qx+k$
B. $y=2px^2+qx+k$
C. $y=2px^2-4qx+k$
D. $y=8px^2+qx+4k$

10. In the diagram below, Path 1 represents the path of a diver from the time she enters the water until she resurfaces. Path 2 represents the path of the diver on her second trial.



Her coach graphs the quadratic function $\frac{1}{a}(y-k)=(x-h)^2$ to model the parabolic Path 1 of the dive. The coach then changes certain values in the given equation to model the parabolic Path 2. Which value(s) did the coach **NOT** change?

A. h
B. h and a
C. a
D. k

11. Which one of the following represents the range of the function $-2(y-5)=(x-3)^2$?

A. $\{y \in \mathbb{R} \mid y \geq 5\}$
B. $\{y \in \mathbb{R} \mid y < 5\}$
C. $\{y \in \mathbb{R} \mid y \leq 5\}$
D. $\{y \in \mathbb{R} \mid y > 5\}$

12. What type of function would best model the data in the table below?

x	–15	–12	−9	−6	−3
y	64	32	16	8	4

A. linear
B. quadratic
C. exponential
D. logarithmic

13. Which expression is equivalent to $2\log\left(\frac{10}{x}\right)$ for all possible values of x?

A. $\log 10 - 2\log x$
B. $2 - \log x^2$
C. $\log 100 - \log 2x$
D. $(1-\log x)^2$

14. Which of the following is **<u>false</u>**?

A. $\dfrac{\log M}{\log N} = \log M - \log N$

B. $\dfrac{1}{2}\log M = \log\sqrt{M}$

C. $\log_5(5M) = 1 + \log_5 M$

D. $\log_M N = \dfrac{\log N}{\log M}$

15. $3^{-1} + 3^{-1} =$

A. $\dfrac{1}{9}$

B. -6

C. $\dfrac{2}{6}$

D. $\dfrac{2}{3}$

16. Which function is the inverse of the function $y = 3^x$?

A. $y = \log_3 x$

B. $y = \log_x 3$

C. $y = \left(\dfrac{1}{3}\right)^x$

D. $y = 3^{-x}$

17. Given $x = \log_5 25 - \log_5\left(\dfrac{1}{25}\right)$, what is the value of x?

A. 4

B. $\log_5 1$

C. 0

D. $\log_5\left(\dfrac{624}{25}\right)$

18. A particular bacteria population doubles 5 times a day. Which function determines N, the number of bacteria, after t days, given an initial amount of 500 bacteria?

A. $N = 500(2)^{\frac{t}{5}}$

B. $N = 500(5)^{\frac{t}{2}}$

C. $N = 500(5)^{2t}$

D. $N = 500(2)^{5t}$

19. What is the simplified form of $(\log_a b)(\log_b a)$?

A. $\log_{ab}(b + a)$

B. $\log_{(a+b)}(ab)$

C. -1

D. 1

20. What is the simplified form of $2\log_a 5 + \log_a 6 - \frac{1}{3}\log_a 8$?

A. $\log_a 29$

B. $\log_a 30$

C. $\log_a 75$

D. $\frac{8}{3}\log_a 3$

21. Given $\log_x y = 1.25$, what is the value of $\log_y x$?

A. –1.25

B. 0.80

C. –17.78

D. 17.78

22. Which of the following is **not** a geometric sequence?

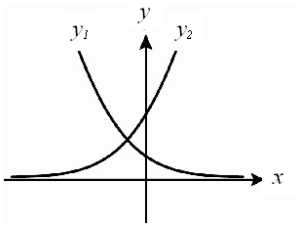
A. $\{3^1, 3^2, 3^3, 3^4\}$

B. $\{2, 4, 8, 14\}$

C. $\{\sqrt{2}, 2, 2\sqrt{2}, 4\}$

D. $\left\{\frac{1}{3}, \frac{1}{6}, \frac{1}{12}, \frac{1}{24}\right\}$

23. The functions $y_1 = ab^x$ and $y_2 = cd^x$ are graphed below.



Which of the following statements is **true**?

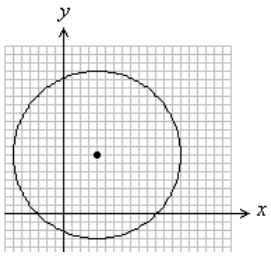
A. $0 < b < 1$ and $d > 1$

B. $b > 1$ and $0 < d < 1$

C. $b = d$

D. $b > d$

24. What is the equation of the circle drawn below?



A. $(x+4)^2+(y+7)^2=10$

B. $(x+4)^2+(y+7)^2=100$

C. $(x-4)^2+(y-7)^2=10$

D. $(x-4)^2+(y-7)^2=100$

25. Given the statement:

"If two chords of a circle are congruent, then they are equidistant from the centre of the circle."

Which of the following is the converse statement?

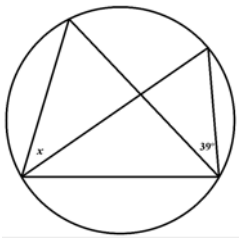
A. Two chords of a circle that are equidistant from the centre of the circle must also be equidistant from the circle.

B. If two chords of a circle are equidistant from the centre of the circle, then they are congruent.

C. Two chords of a circle are equidistant from the centre iff they are congruent.

D. If two chords of a circle are congruent, then they are not equidistant from the centre of the circle.

26. To prove that quadrilateral ABCD is a rectangle, Meghan showed that both pairs of opposite sides are congruent.



A. 39°

B. 19.5°

C. 78°

D. 141°

27. What is the range of the ellipse defined by $\left(\frac{x}{9}\right)^2+\left(\frac{y}{3}\right)^2=1$?

A. $\{y\in\mathbb{R}\,|\,-3\le y\le 3\}$

B. $\{y\in\mathbb{R}\,|\,-9\le y\le 3\}$

C. $\{y\in\mathbb{R}\,|\,-3\le y\le 9\}$

D. $\{y\in\mathbb{R}\,|\,-9\le y\le 9\}$

28. What is the length of the radius of the circle $x^2 + y^2 + 4x + 10y + 9 = 0$?

A. 29
B. $2\sqrt{5}$
C. 20
D. $\sqrt{38}$

29. When the circle represented by $x^2 + y^2 + 10x - 8y = 11$ is translated 7 units to the right and 4 units down, what are the coordinates of the new centre?

A. (12, –8)
B. (–12, 8)
C. (7, –4)
D. (2, 0)

30. ${}_nP_r - {}_nC_r =$

A. $r!$
B. ${}_nP_r\left(\frac{r!-1}{r!}\right)$
C. 0
D. $\frac{1}{r!}$

31. An electronic lock on a door has 6 buttons numbered 1–6. The door will open only when a certain 4-digit sequence is entered. The digits in the sequence must all be different. What is the probability that a random sequence of 4 digits will open the door?

A. $6 \times 5 \times 4 \times 3$
B. $\frac{1}{{}_6C_4}$
C. $\frac{1}{{}_6P_4}$
D. $\frac{4}{6!}$

32. For all possible values of m, what is the simplified form of ${}_mP_{m-2}$?

A. $m(m-2)$
B. $\frac{m(m-1)}{2}$
C. $\frac{m!}{2}$
D. $m^2 - m$

33. Laura randomly grabs 2 marbles from a bag that contains 4 red marbles and 5 blue marbles. Which of the following represents the probability that she will grab two red marbles from the bag of marbles?

A. $\frac{1}{2}$

B. $\frac{2}{9}$

C. $\frac{{}_4C_2}{{}_5C_2}$

D. $\frac{{}_4C_2}{{}_9C_2}$

34. If an event can succeed in *s* ways and fail in *f* ways, then which expression is equal to *P*(*failure*)?

A. 1 – *P*(*success*)

B. $\frac{s}{s+f}$

C. *P*(*success*) – 1

D. $\frac{s+f}{f}$

35. A fair 8-sided die is rolled. The faces of the die are numbered 1 through 8. On any given roll, what is the probability of rolling a 1 or a number larger than 6?

A. $\frac{3}{8}$

B. $\frac{2}{64}$

C. $\frac{2}{8}$

D. $\frac{3}{16}$

Constructed Response Questions
(Total Value: 67 points)

Read each question carefully. Be sure to write your response in the box and space provided. When the answer box indicates that you are to show your work, then points will be awarded for your correct work and your correct final answer. The method used to solve a problem must clearly be shown even when using a graphing calculator. If the answer box requires that just a final answer be provided, then points will be awarded for the correct answer only.

You may round off decimal values to the hundredths place in your final answer only. If any decimal values are rounded prior to the final step of the solution, at least 4 decimal places must be kept.

All answers must be given in simplest form.

36. (a) Solve the following equation for x. Express your answer(s) in exact simplest form.

(3 points)

$x^2 = 2x + 7$

Show your work above and write your conclusion or final answer in the box below.

(b) Solve the following equation for x using a **<u>different</u>** method than that used in part (a). Express your answer(s) in exact simplest form.

(3 points)

$3x^2 - 5x - 2 = 0$

Show your work above and write your conclusion or final answer in the box below.

37. Express the function $y = -\frac{3}{4}x^2 + 12x - 15$ in transformational form? (3 points)

Show your work above and write your conclusion or final answer in the box below.

38. Algebraically solve for *x*. (3 points)

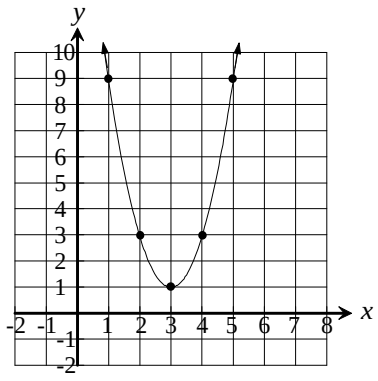
$$\frac{(x+3)!}{(x+1)!} = 20$$

Show your work above and write your conclusion or final answer in the box below.

39. For what value(s) of k does the equation $3x^2 = kx - 1$ have no real roots? (3.5 points)

Show your work above and write your conclusion or final answer in the box below.

40. (a) Without using regression, determine the function, in transformational form, represented by the following parabola. (3 points)



Show your work above and write your conclusion or final answer in the box below.

(b) Write the equation of the axis of symmetry. (1 point)

Final Answer	

(b) Write the equation of the *y*-intercept. (1 point)

Final Answer	

41. During a high school baseball tournament, Nellie hits a pitch and the baseball stays in the air for 4.423 seconds. The function $h = -5t^2 + 22t + 0.5$ describes the height over time, where h is its height, in meters, and t is the time, in seconds, from the instant the ball is hit.

(a) Algebraically determine the maximum height the ball reaches. (3 points)

Show your work above and write your conclusion or final answer in the box below.

(b) By the method of your choice, determine how long the ball will be at a height of **less than** 20 meters while it is in the air. (4 points)

Show your work above and write your conclusion or final answer in the box below.

42. Solve for x algebraically.

(a) $\log_3(x+4)+\log_3 6=2$ (2 points)

Show your work above and write your conclusion or final answer in the box below.

(b) $\log_5(7x-3)-\log_5(x+1)=1$ (2.5 points)

Show your work above and write your conclusion or final answer in the box below.

43. Suppose the cost of a parking permit increases by 4% annually. At this rate, how long will it take for the cost to triple? (4 points)

Show your work above and write your conclusion or final answer in the box below.

44. Solve for x in the following. (4 points)

$$\left(16^{x}\right)^{x+1} = \left(\frac{1}{64}\right)^{x-1}$$

Show your work above and write your conclusion or final answer in the box below.

45. Given the function: $y = 3(2)^{-x+1} - 4$

(a) Write the function in transformational form. (1 point)

Final Answer	

(b) State the mapping rule that maps $y = 2^x$ onto the given function. (2 points)

Final Answer	

(c) Write the equation of the horizontal asymptote. (1 point)

Final Answer	

(d) Write the coordinates of the *y*-intercept. (1 point)

Final Answer	

46. Graham rolls a fair 8-sided die.

(a) What is the probability that he rolls an 8 on the first roll and an odd number on the second roll? (2 points)

Show your work above and write your conclusion or final answer in the box below.

(b) What is the probability that he rolls an 8 on the first roll or the second roll? (3 points)

Show your work above and write your conclusion or final answer in the box below.

47. Dan chooses 5 marbles from a bag containing only red marbles and green marbles of identical size. If the bag contains 15 red marbles and 10 green marbles, what is the probability that he chooses 3 red marbles and 2 green marbles? (3 points)

Show your work above and write your conclusion or final answer in the box below.

48. Given: $\left[\frac{1}{3}(x-4)\right]^2+\left[\frac{1}{5}(y+1)\right]^2=1$

(a) State the domain and the range of the above ellipse. (2 points)

Domain:	***Final Answer***	
Range:	***Final Answer***	

(b) Give the coordinates of the endpoints of the major axis. (1 point)

Final Answer	

49. The following is the equation of an ellipse:

$4x^2 + 9y^2 - 16x + 18y - 11 = 0$

(a) Express the equation in transformational form. (3 points)

Show your work above and write your conclusion or final answer in the box below.

(b) State the coordinates of its centre. (1 point)

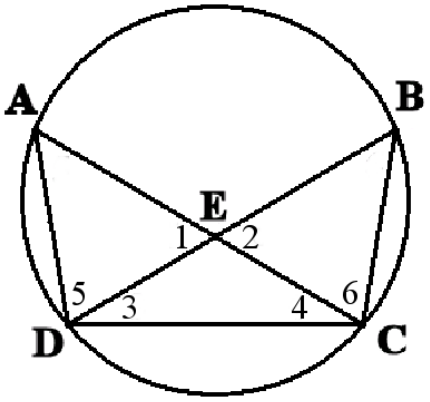
Final Answer	

(c) Determine the length of its minor axis. (1 point)

Final Answer	

50. Given: $\angle 3 \cong \angle 4$

Prove: $\overline{AE} \cong \overline{BE}$



Statement	Reason

Nova Scotia Examinations
Advanced Mathematics 12

Solutions - Web Sample 3

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Selected Response Answer Key

1. B
2. B
3. D
4. C
5. A
6. A
7. B
8. C
9. B
10. A
11. C
12. C
13. B
14. A
15. D
16. A
17. A
18. D
19. D
20. C
21. B
22. B
23. A
24. D
25. B
26. A
27. A
28. B
29. D
30. B
31. C
32. C
33. D
34. A
35. A

Question 36 (a) (3 points)

$$x^2 - x + 7 = 0$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(\ 7)}}{2(1)}$$ 1 pt

$$= \frac{1 \pm \sqrt{-27}}{2}$$ 0.5 pt

$$= \frac{1 \pm i\sqrt{27}}{2}$$ 0.5 pt

$$x = \frac{1 + 3i\sqrt{3}}{2}$$ 0.5 pt

$$x = \frac{1 - 3i\sqrt{3}}{2}$$ 0.5 pt

OR

$$x^2 - x = -7$$

$$x^2 - x + \frac{1}{4} = -7 + \frac{1}{4}$$ 1 pt

$$\left(x - \frac{1}{2}\right)^2 = -\frac{27}{4}$$ 0.5 pt

$$x - \frac{1}{2} = \pm\frac{\sqrt{-27}}{2}$$

$$x = \frac{1}{2} \pm \frac{i\sqrt{27}}{2}$$ 0.5 pt

$$x = \frac{1 + 3i\sqrt{3}}{2}$$ 0.5 pt

$$x = \frac{1 - 3i\sqrt{3}}{2}$$ 0.5 pt

Question 36(b) (3 points)

$3x^2-5x-2=0$

$(3x+1)(x-2)=0$ — 1 pt

$3x+1=0$ — 0.5 pt

$x=-\frac{1}{3}$ — 0.5 pt

$x-2=0$ — 0.5 pt

$x=2$ — 0.5 pt

$3x^2-5x-2=0$

$x=\frac{-(-5)\pm\sqrt{(-5)^2-4(3)(-2)}}{2(3)}$ — 1 pt

$x=\frac{5\pm\sqrt{49}}{6}$ — 1 pt

$x=\frac{5-7}{6}$

$x=-\frac{1}{3}$ — 0.5 pt

$x=\frac{5+7}{6}$

$x=2$ — 0.5 pt

OR

$3x^2-5x-2=0$

$x^2-\frac{5x}{3}=\frac{2}{3}$ — 0.5 pt

$x^2-\frac{5x}{3}+\frac{25}{36}=\frac{2}{3}+\frac{25}{36}$ — 0.5 pt

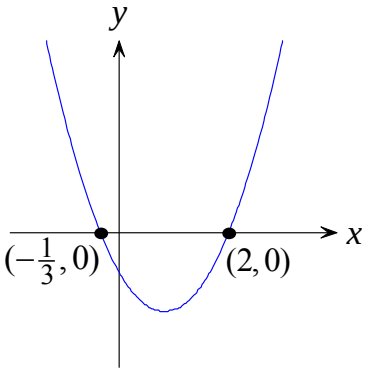
$\left(x-\frac{5}{6}\right)^2=\frac{49}{36}$ — 0.5 pt

$x-\frac{5}{6}=\pm\frac{7}{6}$ — 0.5 pt

$x=-\frac{1}{3}$ — 0.5 pt

$x=2$ — 0.5 pt

$y_1=3x^2-5x-2$



2 pts

$x=-\frac{1}{3}$ — 0.5 pt

$x=2$ — 0.5 pt

Question 37 (3 points)

$y = -\frac{3}{4}x^2 + 12x - 15$

$y + 15 = -\frac{3}{4}x^2 + 12x$ } 0.5 pt

$y + 15 = -\frac{3}{4}\left(x^2 - 16x\right)$ } 0.5 pt

$y + 15 - 48 = -\frac{3}{4}\left(x^2 - 16x + 64\right)$ } 1 pt

$y - 33 = -\frac{3}{4}\left(x - 8\right)^2$ } 0.5 pt

$-\frac{4}{3}\left(y - 33\right) = \left(x - 8\right)^2$

or

$-\frac{1}{\frac{3}{4}}\left(y - 33\right) = \left(x - 8\right)^2$ } 0.5 pt

OR

If $y = -\frac{3}{4}x^2 + 12x - 15$

Then the vertex is $\left(-\frac{b}{2a},\ f\left(-\frac{b}{2a}\right)\right)$

So $-\frac{b}{2a} = -\frac{12}{2\left(-\frac{3}{4}\right)} = 8$ } 1 pt

and

$f\left(-\frac{b}{2a}\right) = f\left(8\right) = \underbrace{-\frac{3}{4}\left(8\right)^2 + 12\left(8\right) - 15 = 33}$

1 pt

Therefore transformational form is

$-\frac{1}{\frac{3}{4}}\left(y - 33\right) = \left(x - 8\right)^2$

or

$-\frac{4}{3}\left(y - 33\right) = \left(x - 8\right)^2$ } 1 pt

$y = -\frac{3}{4}x^2 + 12x - 15$

$y + 15 = -\frac{3}{4}x^2 + 12x$ } 0.5 pt

$-\frac{4}{3}\left(y + 15\right) = x^2 - 16x$ } 0.5 pt

$-\frac{4}{3}\left(y + 15\right) + 64 = x^2 - 16x + 64$ } 1 pt

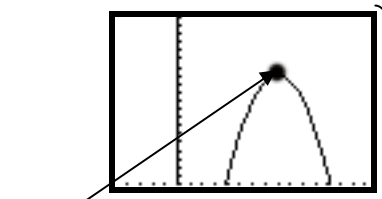
$-\frac{4}{3}y + 44 = x^2 - 16x + 64$ } 0.5 pt

$-\frac{4}{3}\left(y - 33\right) = \left(x - 8\right)^2$

or

$-\frac{1}{\frac{3}{4}}\left(y - 33\right) = \left(x - 8\right)^2$ } 0.5 pt

$y_1 = -\frac{3}{4}x^2 + 12x - 15$



(8, 33)

2 pts

Therefore transformational form is

$-\frac{1}{\frac{3}{4}}\left(y - 33\right) = \left(x - 8\right)^2$

or

$-\frac{4}{3}\left(y - 33\right) = \left(x - 8\right)^2$ } 1 pt

Question 38 (3 points)

$$\frac{(x+3)!}{(x+1)!} = 20$$

$$\frac{(x+3)(x+2)(x+1)!}{(x+1)!} = 20$$ } 0.5 pt

$$(x+3)(x+2) = 20$$ } 0.5 pt

$x^2 + 5x - 14 = 0$ } 0.5 pt

Solving quadratic(any method) } 1 pt

So ~~$x = -7$~~ and $x = 2$ } 0.5 pt
(Discard)

OR

Two consecutive whole numbers with a product of 20 must be 4 and 5. } 0.5 pt

So $x+3 = 5$ and $x+2 = 4$ 1 pt

$\therefore x = 2$ } 0.5 pt

Note: Deduct 0.5 pt if x = -7 is given as a solution and not discarded.

Question 39 (3.5 points)

$$3x^2 = kx - 1$$

$$3x^2 - kx + 1 = 0$$ } 0.5 pt

$$\therefore \underbrace{(-k)^2}_{1\text{ pt}} - \underbrace{4(3)(1) < 0}_{1\text{ pt}}$$

$$k^2 < 12$$

$k < 2\sqrt{3}$ or $k < 3.46$ } 0.5 pt

$k > -2\sqrt{3}$ or $k > -3.46$ } 0.5 pt

$$\therefore \left\{k \middle| -2\sqrt{3} < k < 2\sqrt{3},\ k \in \mathbb{R}\right\} \quad \text{or} \quad \left(-2\sqrt{3},\ 2\sqrt{3}\right)$$

Question 40(a) (3 points)

The vertex is at (3, 1).

$$\frac{1}{a}(y-1)=(x-3)^2$$

0.5 pt 0.5 pt

Since, from the vertex, it is over 1 and up 2×1 there is a vertical stretch of 2.

1 pt

$\therefore a = 2$ and transformational form is:

$$\frac{1}{2}(y-1)=(x-3)^2$$ 1 pt

OR

The vertex is at (3, 1).

$$\frac{1}{a}(y-1)=(x-3)^2$$

0.5 pt 0.5 pt

Since the parabola passes through the point (4, 3) we know that:

$$\frac{1}{a}(3-1)=(4-3)^2$$ 0.5 pt

$$\frac{1}{a}(2)=1$$

$$a=2$$ 0.5 pt

$\therefore$ the transformational form is:

$$\frac{1}{2}(y-1)=(x-3)^2$$ 1 pt

Question 40(b) (1 point)

$x=3$ 1 pt

Question 40(c) (1 point)

$(0, 19)$ 1 pt

Question 41(a) (3 points)

Note: Concluding that the maximum occurs at $\frac{4.423}{2} = 2.2115$ seconds is to be considered a conceptual error.

$h = -5t^2 + 22t + 0.5$

The vertex is $\left(-\frac{b}{2a},\ f\left(-\frac{b}{2a}\right)\right)$

$-\frac{b}{2a} = -\frac{22}{2(-5)} = 2.2$ — 1 pt

$f(2.2) = -5(2.2)^2 + 22(2.2) + 0.5 = 24.7$ — 1 pt

$\therefore$ The maximum height is 24.7 m. — 1 pt

OR

$h = -5t^2 + 22t + .5$

$h - .5 = -5t^2 + 22t$ — 0.5 pt

$h - .5 = -5(t^2 - 4.4t)$ — 0.5 pt

$h - .5 - 24.2 = -5(t^2 - 4.4t + 4.84)$ — 1 pt

$h - 24.7 = -5(t^2 - 4.4t + 4.84)$

$\therefore$ The maximum height is 24.7 m. — 1 pt

Note: There are several variations of this method. Use your professional judgment.

Question 41(b) (4 points)

$h = -5t^2 + 22t + 0.5$

Let $h = 20$ — 1 pt

$-5t^2 + 22t + 0.5 = 20$

$-5t^2 + 22t - 19.5 = 0$

$t = \frac{-(22) \pm \sqrt{(22)^2 - 4(-5)(-19.5)}}{2(-5)}$ — 0.5

$t = \frac{-22 \pm \sqrt{94}}{-10}$ — 0.5 pt

$t \doteq 1.2305$ and $t \doteq 3.1695$

The ball was at a height of more than 20 m during the interval between 1.2305 seconds and 3.1695 seconds.

$\therefore$ Duration above 20 m was: — 1 pt

$3.1695 - 1.2305 \doteq 1.939$ seconds.

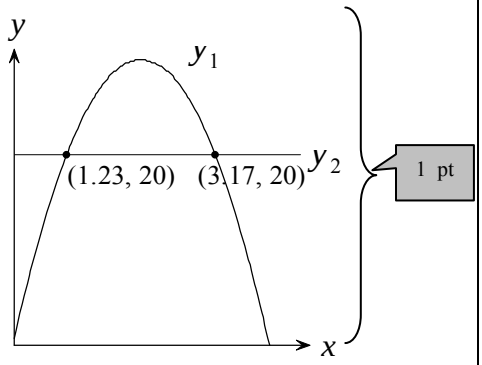
$\therefore$ the time at no more than 20 m was: — 1 pt

$4.423 - 1.939 \doteq 2.48$ seconds

OR

$y_1 = -5x^2 + 22x + 0.5$ — 1 pt

$y_2 = 20$



1 pt

The ball was at a height of more than 20 m during the interval between 1.2305 seconds and 3.1695 seconds.

$\therefore$ Duration above 20 m was: — 1 pt

$3.1695 - 1.2305 \doteq 1.939$ seconds.

$\therefore$ the time at no more than 20 m was: — 1 pt

$4.423 - 1.939 \doteq 2.48$ seconds

Question 42(a) (2 points)

Note: Deduct 0.5 pt if the answer is discarded or considered extraneous.

$\log_3(x+4)+\log_3 6=2$

$\log_3[6(x+4)]=2$ — 0.5 pt

$6(x+4)=3^2$ — 0.5 pt

$6x+24=9$ — 0.5 pt

$x=-\frac{15}{6}=-\frac{5}{2}$ — 0.5 pt

OR

$\log_3(x+4)+\log_3 6=2$

$\frac{\log(x+4)}{\log 3}+\frac{\log 6}{\log 3}=2$ — 0.5 pt

$\log(x+4)+\log 6=2\log 3$

$\log(x+4)=2\log 3-\log 6$

$\log(x+4)=0.1761$ — 0.5 pt

$x+4=10^{0.1761}$

$x=10^{0.1761}-4$ — 0.5 pt

$x=-2.5$ — 0.5 pt

OR

$\log_3(x+4)+\log_3 6=2$

$\log_3(x+4)=2-\log_3 6$ — 0.5 pt

$\log_3(x+4)=0.3691$ — 0.5 pt

$x+4=3^{0.3691}$ — 0.5 pt

$x=-2.5$ — 0.5 pt

OR

$\log_3(x+4)+\log_3 6=2$

$\log_3[6(x+4)]=2$ — 0.5 pt

$\log_3[6(x+4)]=\log_3 9$ — 0.5 pt

$6x+24=9$ — 0.5 pt

$x=-\frac{15}{6}=-\frac{5}{2}$ — 0.5 pt

Question 42(b) (2.5 points)

$\log_5(7x-3)-\log_5(x+1)=1$

$\log_5\left(\frac{7x-3}{x+1}\right)=1$ — 1 pt

$\frac{7x-3}{x+1}=5^1$ — 0.5 pt

$7x-3=5(x+1)$ — 0.5 pt

$7x-3=5x+5$

$2x=8$

$x=4$ — 0.5 pt

OR

$\log_5(7x-3)-\log_5(x+1)=1$

$\frac{\log(7x-3)}{\log 5}-\frac{\log(x+1)}{\log 5}=1$ — 0.5 pt

$\log(7x-3)-\log(x+1)=\log 5$ — 0.5 pt

$\log\left(\frac{7x-3}{x+1}\right)=\log 5$ — 0.5 pt

$\therefore \frac{7x-3}{x+1}=5$ — 0.5 pt

$7x-3=5x+5$

$x=4$ — 0.5 pt

Question 43 (4 points)

Let C represent the current cost of parking and n represent the number of years.

$$3C = C(1.04)^n$$

$$3 = (1.04)^n$$

1 pt

1 pt

$\log 3 = \log(1.04)^n$ 0.5 pt

$\log 3 = n\log(1.04)$ 0.5 pt

$n = \dfrac{\log 3}{\log(1.04)}$ 0.5 pt

$n = 28.01$ years 0.5 pt

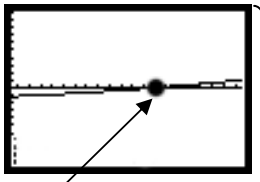
$\log_{1.04} 3 = n$ 0.5 pt

$n = \dfrac{\log 3}{\log(1.04)}$ 0.5 pt

$n = 28.01$ years 0.5 pt

OR

$y_1 = 1.04^x - 3$ 0.5 pt



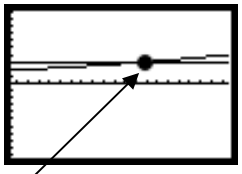
1 pt

(28.0110, 0)

Approximately 28.01 years. 0.5 pt

$y_1 = 1.04^x$ 0.5 pt

$y_2 = 3$



1 pt

(28.0110, 3)

Approximately 28.01 years. 0.5 pt

Question 44 (4 points)

$$\left(16^{x}\right)^{x+1} = \left(\frac{1}{64}\right)^{x-1}$$

$$\underbrace{\left[\left(4^{2}\right)^{x}\right]^{x+1}}_{\text{0.5 pt}} = \underbrace{\left(\frac{1}{4^{3}}\right)^{x-1}}_{\text{0.5 pt}}$$

$$\underbrace{4^{2x^{2}+2x}}_{\text{0.5 pt}} = \underbrace{4^{-3x+3}}_{\text{0.5 pt}}$$

$$\therefore 2x^{2}+2x = -3x+3 \quad \} \text{ 0.5 pt}$$

$$2x^{2}+5x-3 = 0 \quad \} \text{ 0.5 pt}$$

Using any method:

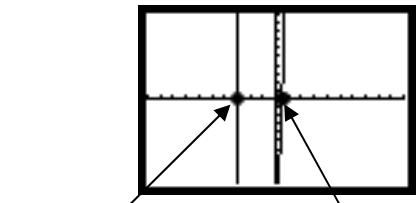
$$x = \frac{1}{2} \quad \} \text{ 0.5 pt}$$

Using any method:

$$x = -3 \quad \} \text{ 0.5 pt}$$

OR

$$y_{1} = \left(16^{x}\right)^{x+1} - \left(\frac{1}{64}\right)^{x-1} \quad \} \text{ 1 pt}$$



$\left(-3,\ 0\right)$ $\left(\frac{1}{2},\ 0\right)$ } 1 pt

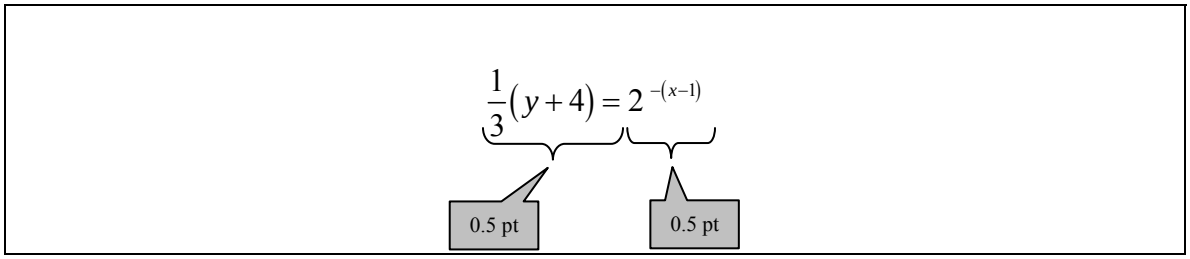
Using any method:

$$x = \frac{1}{2} \quad \} \text{ 1 pt}$$

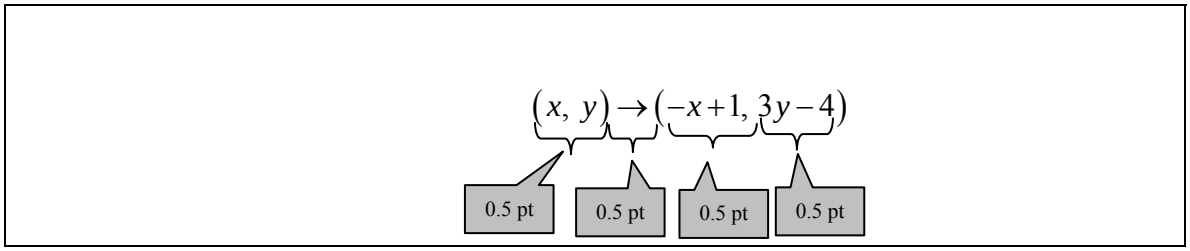
Using any method:

$$x = -3 \quad \} \text{ 1 pt}$$

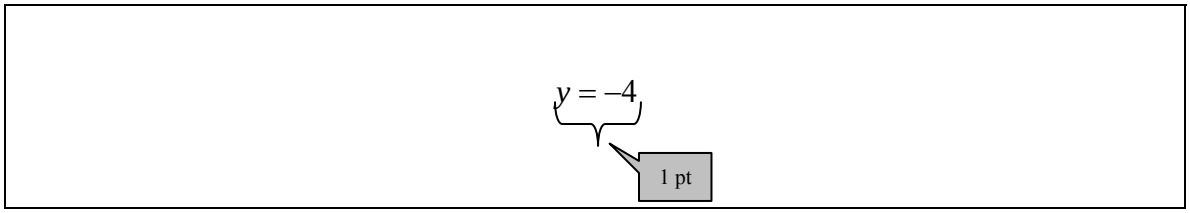
Question 45(a) (1 point)



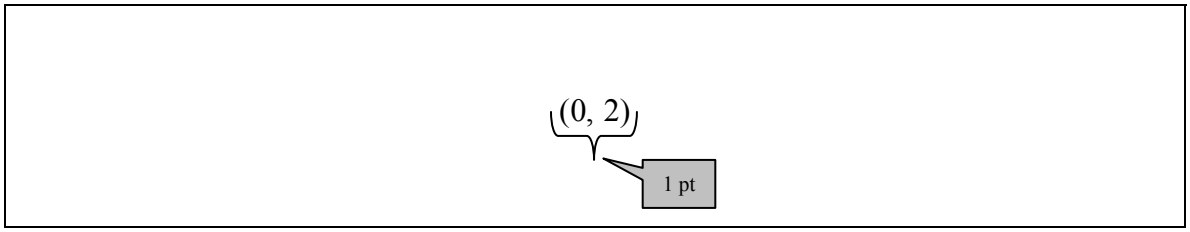
Question 45(b) (2 points)



Question 45(c) (1 point)



Question 45(d) (1 point)



Question 46(a) (2 points)

$\text{P(He rolls an 8 on first roll)} = \frac{1}{8}$

$\text{P(He rolls an odd number on second roll)} = \frac{4}{8}$

$\text{P(Both)} = \frac{1}{8} \times \frac{4}{8} = \frac{4}{64}$ or $\frac{1}{16}$

0.5 pt 0.5 pt 0.5 pt 0.5 pt

Question 46(b) (3 points)

$\text{P(He rolls an 8 on first roll)} = \frac{1}{8}$

$\text{P(He rolls an 8 on second roll)} = \frac{1}{8}$

$\text{P(Both)} = \frac{1}{8} \times \frac{1}{8} = \frac{1}{64}$

$\text{P(He rolls an 8 on first or second roll)} = \frac{1}{8} + \frac{1}{8} - \frac{1}{64} = \frac{15}{64}$

0.5 pt 0.5 pt 0.5 pt 0.5 pt 0.5 pt 0.5 pt

OR

1 1 2 3 4 5 6 7 8 | 2 1 2 3 4 5 6 7 8 | 3 1 2 3 4 5 6 7 8 | 4 1 2 3 4 5 6 7 8 | 5 1 2 3 4 5 6 7 8 | 6 1 2 3 4 5 6 7 8 | 7 1 2 3 4 5 6 7 8 | 8 1 2 3 4 5 6 7 8

2 pt

Any correct tree or counting scheme is worth 2 points.

Therefore the probability is $\frac{15}{64}$. 1 pt

Question 47 (3 points)

0.5 pt | 1 pt | 0.5 pt | 0.5 pt

$$\frac{{}_{15}C_3 \times {}_{10}C_2}{{}_{25}C_5} = \frac{20475}{53130} \text{ or } 0.39$$

0.5 pt

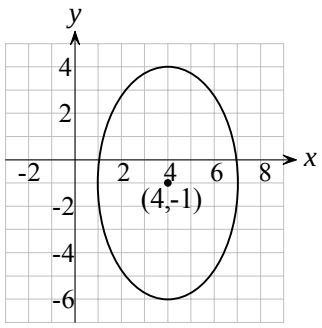
NOTE

$\frac{{}_{15}P_3 \times {}_{10}P_2}{{}_{25}P_5}$ is worth 1 point.

Question 48(a) (2 points)

Domain: $\{x \in \mathbb{R}, 1 \le x \le 7\}$ or $[1,7]$ — 1 pt

Range: $\{y \in \mathbb{R}, -6 \le y \le 4\}$ or $[-6,4]$ — 1 pt



Question 48(b) (1 point)

0.5 pt | 0.5 pt

$(4, 4)$ and $(4, -6)$.

Question 49(a) (3 points)

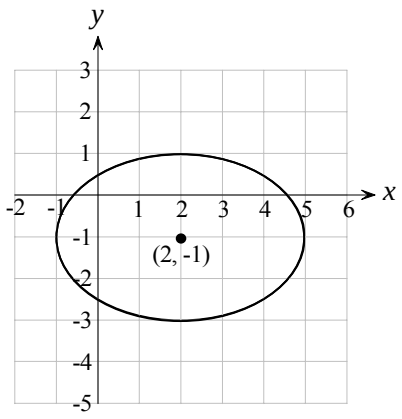
$4x^2+9y^2-16x+18y-11=0$

$4\left(x^2-4x\right)+9\left(y^2+2y\right)=11$ } 0.5 pt

$4\left(x^2-4x+4\right)+9\left(y^2+2y+1\right)=11+16+9$ } 1 pt

$\frac{(x-2)^2}{9}+\frac{(y+1)^2}{4}=1$ } 1 pt

$\left[\frac{x-2}{3}\right]^2+\left[\frac{y+1}{2}\right]^2=1$ } 0.5 pt



Question 49(b) (1 point)

$(2, -1)$ 1 pt

Question 49(c) (1 point)

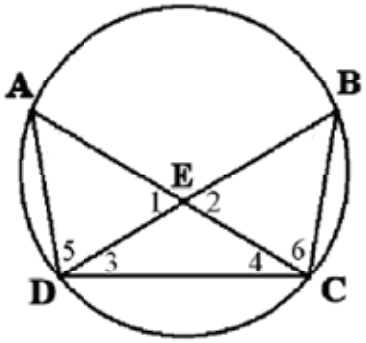
4 units } 1 pt

Question 50 (6 points)

(1 point for each correct relevant line – both statement and reason must be correct)

Given: $\angle 3 \cong \angle 4$

Prove: $\overline{AE} \cong \overline{BE}$



Statement	Reason
$\angle 3 \cong \angle 4$	Given
$\overline{DE} \cong \overline{CE}$	ITT converse
$\angle 1 \cong \angle 2$	Vertically opposite angles (X Theorem).
$\angle 5 \cong \angle 6$	Both subtended by $\overset{\frown}{AB}$
$\therefore \Delta ADE \cong \Delta BCE$	ASA
$\therefore \overline{AE} \cong \overline{BE}$	CPCTC

Or

Statement	Reason
$\angle 3 \cong \angle 4$	Given
$\overline{DE} \cong \overline{CE}$	ITT converse
$\angle 1 \cong \angle 2$	Vertically opposite angles (X Theorem).
$\angle DAE \cong \angle CBE$	Both subtended by $\overset{\frown}{DC}$
$\therefore \Delta ADE \cong \Delta BCE$	AAS
$\therefore \overline{AE} \cong \overline{BE}$	CPCTC

Or

Statement	Reason
$\angle 3 \cong \angle 4$	Given
$\overline{DE} \cong \overline{CE}$	ITT converse
$\angle 5 \cong \angle 6$	Both subtended by $\overset{\frown}{AB}$
$\angle DAE \cong \angle CBE$	Both subtended by $\overset{\frown}{DC}$
$\therefore \Delta ADE \cong \Delta BCE$	AAS
$\therefore \overline{AE} \cong \overline{BE}$	CPCTC

or

Construction: Connect A and B to form $\overline{AB}$ } 1 pt

Statement	Reason
$\angle 3 \cong \angle 4$	Given
$\angle 3 \cong \angle CAB$	Both subtended by $\overset{\frown}{BC}$
$\angle 4 \cong \angle DBA$	Both subtended by $\overset{\frown}{AD}$
$\angle CAB \cong \angle DBA$	Axiom of Equality
$\therefore \overline{AE} \cong \overline{BE}$	ITT converse

(Other valid, some longer, proofs are possible. Use your professional judgment.)